A Comparative Study of Filter Cake Yield Using Lmt Dimensional Analysis and Other Models

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Abstract:
A comparative study of filter cake yields has been made by comparing a model developed from the Buckingham-π method of LMT Dimensional Analysis, Cake yield values from experimental work and previous researches in cake yield analyses. The model equation developed shows that the cake yield from a pressure filter (filter press) is directly proportional to the filter area of the pressure vessel, applied pressure and initial solids content of the sludge while being inversely proportional to specific resistance of the filtrate, viscosity of filtrate, compressibility coefficient of the sludge and pressing time. The new equation enables performance of a pressure filter (Filter Press) to be predicted from a simple laboratory determination of cake yields. Curves derived from the models compared show how filter cake yield depends on investigated parameters as highlighted above. A comparative study of the effects of specific resistance on the cake formed at the operating pressure of 2039.41g/cm² shows that both the new equation and Carman’s modified equation exhibiting similar trends with specific resistance. For instance, reducing specific resistance from 1.485 Cm/g to 1.341 Cm/g increased cake yields by 1.146 times, 1.258 times and 1.082 times respectively for the new model Y_{LMT}, Carman Modified equation, Y_{CAR} and the experimental values, Y_{exp}. Considering the differences in the parameters tested, it is significant that they all responded similarly when compared with previous research works. Particularly, results showed that the newly derived equation gave the best result. Values obtained from Carman’s model were higher than the experimental yield values. This implied that Carman’s equation cannot be validly used to predict compressible sludge filtration process. Overall, the predicted values using the newly derived model demonstrated conformity with the experimental data with a correlation coefficient of 0.993 while correlation between values from Carman’s modified equation and experimental values was found to be 0.92.

Keywords: Cake yield, Sludge, Prediction, Pressure filter, Dimensional Analysis, Filtration

I. INTRODUCTION

Sludge is often generated from the treatment of wastewater and in order to reduce the cost of disposal, it is essential to reduce its volume before discharge into the environment through dewatering processes, [1]. Cake filtration is a process widely used in numerous industrial applications such as chemical, medical, agriculture, food manufacture, process, minerals treatment and water treatment industries. Matters pertaining to key environmental issues have led to more stringent demands on industries and as result more emphasis has been placed towards better understanding of filtration processes, [2]. The management of sludge from both domestic and industrial process operations occasioned by modern advancement in technology is highly complex and cost effective, hence, if poorly accomplished, may jeopardize the environmental and sanitary advantages expected from the treatment. Moreover, due to the low indices of wastewater and other industrial processes resulting from high population increase and industrialization especially in developing countries, there is need to explore and adopt more scientific options to meet the demand. As a consequence, the amount of sludge produced is expected to increase posing serious environmental and health concerns to the developing nations, [3], [4]. A recent study conducted by [5] revealed that despite the fact that the volume of sludge tends to be less than 1% of the total plant influent, sludge handling costs ranges between 21-50% of total plant operating and maintenance costs., Dewatering of sewage sludge is not only found in removal of the excess moisture but to render the sludge odourless and nonputrescible, [6]. Similarly, [7] and [8] expressed that dewatering of sewage sludge prior to drying or disposal is an important step because the lower the water content of the sludge, the lesser the transport cost. Proper waste management system should be established and enhanced in view of menace imposed in our community due to improper handling and disposal of wastes to our environment.

Dimensional Analyses is a conceptual tool often applied in physics, chemistry and engineering to
understand physical solution involving a mixture of different kinds of physical quantities. Dimensional formulae provide a useful catalogue system for physical quantities according to [9]. The principle of dimensional homogeneity states that in a physical equation consisting of an algebraic sum of two or more terms, the exponent of the dimension of Length, Mass and Time in any term of the equation must be the same as that in any other term. The system of fundamental units commonly used in Newtonian mechanics is the LMT System. In this study, Buckingham π-method was used in developing the new model.

II. A REVIEW OF PREVIOUS MODELS ON CAKE FILTRATION

[10] while modifying the previous work of [11] for sludge undergoing rotary filtration process assumed that for a yield equation to be fully described as parabolic, the initial specific resistance must be assumed as zero. Sludge compressibility coefficient was not accounted for in his derivation.

A research was carried out by [12] on the effects of skeleton builders on oily sludge using filter press apparatus. They found out that filter press raw sludge yield increased continuously with added conditioner dose. They also maintained that even though the yield curves represent the rate of raw sludge solids capture during the filtration process, they cannot be used directly to determine the practical optimum conditioner dose. They thereafter developed a filter press yield model at 90% completion of filtration.

In deriving their model [13] maintained that Carman’s modified cake yield equation can only be applied to determine a cake filtration process if sludge conditioning does not appreciably alter the solids content of the original slurry, as is usually with chemical coagulants. He stated that if the main objective of sludge conditioning is to improve filter yield, it is better to express filterability as yield. He thereafter modified [14] equation to account for both the original sludge solids and conditioner solids.

However, it is important to note that the traditional Carman’s equations, Rebhun, Jones, Gale and co-workers did not account for the compressibility coefficient in the formulation of their equations as highlighted earlier. The mere absence of the compressibility attribute of the sludge in the above models necessitated [15], comment on the need to define minimum and maximum specific resistance as a means of justifying the filterability of sludge cakes even though he did not add compressibility coefficient S as part of his modified equation. The comparative analyses was made between the LMT Model (Equation 1) and Carman’s modified model (equation 3) since the latter has been always been relied upon by previous researchers in developing their cake filtration models.

A. The Basis for the Comparative Analyses Between the LMT Model and other Models

In this study, a cake yield model derived using the Buckingham-π method of LMT Dimensional Analysis by [16] was compared with other previous research works and in a detailed form with the model developed by [17]. The compressibility attribute as a measure of filterability was incorporated in the newly derived equation. Its validity was further proved through comparative analyses of the parameters affecting cake dewater ability. The incorporation of the compressibility coefficient ‘S’ is against the traditional cake yield filtration equations already suggested by [10], [12], [13] and [17] where the sludge compressibility effects on filterability were obviously unaccounted for. It has been discovered in literature that the traditional equations were embedded with uncertainties in the areas of formulating them. [18] stressed that since the literature is replete of dewatering operations which have unsatisfactory performance predictions and formulations and also in view of the controversies among prominent researchers to the present knowledge of filtration equations, it is justified that an acceptable equation which characterize the filtration process has to be derived. Such model must contain the compressibility coefficient ‘S’ as an attribute of the sludge. The incorporation of ‘S’ will make such equation acceptable to the previous researchers. Based on the above, [16] developed a cake yield model given as;

\[ Y_{LMT} = 0.007082 \frac{C^{0.770}}{P^{0.827}} \frac{A^{0.149}}{G^{0.431}} \]  \hspace{1cm} (1)

Carman derived his equation based on non-compressible sludge cakes. In his equation, which was a modification of Darcy’s equation, he stressed that the specific resistance is constant throughout the filtration process. Hence, [17] proposed the equation;

\[ \frac{dV}{d\theta} = \frac{P.A^2}{\mu(RCV + RmA)} \]  \hspace{1cm} (2)

Where:
\[ A = \text{filter area, m}^2 \]
\[ R_m = \text{septum resistance, kg/m} \]
\[ V = \text{volume of filtrate, m}^3 \]
\[ P = \text{pressure drop, Kg/m}^2 \]
\[ C = \text{concentration of solids in the feed, kg/m}^3 \]
\[ R = \text{specific cake resistance, Kg/m} \]
\[ \mu = \text{liquid viscosity, poise} \]
\[ \theta = \text{filtration time, S} \]

Integrating the above and neglecting septum resistance, a modified form of the equation is obtained by relating specific resistance, \( R \) with cake yield as:

\[ Y = \left( \frac{2PC}{\mu R\theta} \right)^{\frac{1}{2}} \]  

(3)

Where \( Y \) = solids yield (dry cake production), Kg/m\(^2\)/s

III. MATERIALS AND METHOD

The filtration process was started by connecting the compressed air line to the top of the sample holder using an easy push through arrangement. Thus, the suspension was forced to flow through the filter cell producing a filter cake at the surface of the filter paper. As the filter cake deposited, the flow rate kept declining. The filtrate was collected in a graduated cylinder placed in a tilted position so that the filtrate traveled along the walls of the cylinder without causing a splash, enabling accurate determinations of equal increments of filtrate volume. Constant pressure filtration experiments were carried out respectively on 0.0128g/Cm\(^3\), 0.0194g/Cm\(^3\), 0.0220g/Cm\(^3\), 0.02465g/Cm\(^3\) and 0.0393g/Cm\(^3\) samples using different doses of Ferric Chloride suspensions. The filtration pressure ranges investigated was between 2039.43g/Cm\(^2\)-6628.155g/Cm\(^2\). Filtration was allowed to proceed and stopped once deliquoring, which was determined when the \( t/V \) against \( V \) plot experienced a sudden change in accordance to traditional filtration behavior was deemed to be beginning. On the conclusion, the filter cake in the filter cell was removed and put in a pre-weighed beaker. It was then dried over a period of twenty four hour sand reweighed. The raw data from the laboratory, pilot filter runs made to evaluate the effect of pressure drop, initial solids contents, conditioner dosages, specific resistance and compressibility on cake yields were accordingly tabulated.

After about 60 ml of the suspension had filtered through and had formed a cake on the surface of the filter paper, time increments required to produce successive equal increments of filtrate volume were observed. The apparatus was uncoupled. The filter paper and the deposited solid mass was carefully transferred into a pre-weighed beaker and oven dried at 105°C. The beaker with the dry mass on filter paper was cooled and the new weight recorded as \( W_2 \). The weight of the deposited solid mass per filtration area per each filtration cycle was thereafter calculated as actual cake yield, \( Y_A \):

\[ Y_A = \frac{W_3-W_1}{\text{Filter area x Cycle time}} (\text{Kg/M}^2\text{.S}) \]  

(4)

where \( W_1 \) is the weight of empty beaker and filter paper.

The data were plotted in the appropriate manner and the specific resistance to filter pressing were accordingly evaluated using appropriate equation.

IV. RESULTS AND DISCUSSION

A. Comparative Analyses of the Effects of Pressing Time on Cake Yield Models

The comparative effects of pressing time with actual, filter cake yield model and Carman’s modified yield are presented on figure 2. Both yields increased with decreasing pressing time at various conditioner doses.

From the new model, cake yield is inversely proportional with the pressing time. Hence, from equation (1), \( Y_{LMT} \propto \frac{1}{\theta^{0.14995}} \).

In other words, \( Y_{LMT} = \frac{K}{\theta^{0.14995}} \)

Where K is proportionality constant given as

\[ 0.007082 \frac{0.770}{R^{0.795}} \frac{C^{0.1494}}{S^{0.69}} \]
The variation of cake yields with time is however affected by chemical dosages which tend to decrease the amount of period needed for the filtration process to complete.

For instance, if the pressing time is increased from 70 seconds to 100 seconds, cake yield would decrease from 3.0441 to 1.6015 g/cm², S, 3.3450 to 2.8059 g/cm², S and 4.2974 to 3.096 g/cm², S respectively for the experimental yield, the new model and Carman’s modified model. It is important to note that in pressure filters, pressing time varies as cake thickness increases,[19].

B. Comparative Analyses of the Effects of Pressures on Cake Yield Models

According to the relationship derived from Darcy’s law which relates pressure drop to dry solids yield (equation 3), an increase in pressure drop should result in an increase in dry cake production. This is the case if the filter cake is not highly compressible such that the specific cake resistance increases with pressure drop, [19]. It is also beneficial to gradually increase the pressure until a constant pressure is reached. This is because the solids are non-homogeneous and a high initial pressure drop can result in particles plugging the interstices of the cloth. With the assumption that the cake was not highly compressible, the applied pressure was thus set to a maximum of 6.5bars for all runs in this study. It was also necessary to have the applied pressure as high as possible to enable maximum pressure drop over the filter. This is to ensure that the cake was held firmly onto the filter medium during filtration.

Comparatively, both model cake yield, experimental and yield values from Carman’s modified equation increased at higher pressures at various doses of ferric chloride conditioner in line with the predictions of [20] and other Research works on the effects of operating parameters on sludge filterability. A closer look at the curves revealed that the developed model and Carman modified model values exhibit similar responses to the pressure effects. Similarly, increasing the operating pressure from 2039.43 g/cm² to 4078.87 g/cm² increased cake yield production by 1.38 times, 1.25 times and 1.32 times at pilot testing, using the developed model and using Carman modified equation respectively.

Mathematically, the new model and Carman’s modified model show that yield increases proportionally with pressure as respectively illustrated.

\[ Y_{LMT} \propto P^{0.77}, \quad Y_{CARM} \propto P^{0.50} \]

Furthermore, the deterioration of filtrate quality as the pressures were increased cannot be ignored as was the case with previous researchers. However, physically, it is quite easy to explain. As the operating pressures were increased, sludge flocs were ruptured accounting for the poor filtrate quality.
C. Comparative Analyses of the Effects of Conditioner Dosages on Cake Yield between the New Model, Actual Yield and Yield from Carman’s Model.

Ferric chloride was used to test the effect of conditioning on yield. A comparative assessment of the effects of chemical dosages on cake yields on figure 4 showed cake yield values from the new model and Carman’s equation increasing with the dosages in consonance with the experimental values. The graph also shows that a 5% increment in ferric chloride dosage resulted in additional increase in cake yields as follows: 1.03g/cm².s using the new LMT model prediction, 1.04g/cm².s using Carman modified equation and 1.17 g/cm².s at practical laboratory experiment. There is an overlap between the values from the experimental values and the new model showing that predicted values from the derived equation are more accurate when compared with Carman’s model values.

In summary, the new model conforms favourably with previous works cited above while the specific resistance of the sludge decreases.

D. Variation of Cake Yield with Initial Solids Content at Various Pressures

Many Researchers including [13], [21]-[23] have indicated this filterability dependence on initial solids content especially when considering the effects on specific resistance on filtration, but the effect was never reported to be as great as observed in this study. Cake yield increases with initial solids content at higher pressures. From the developed model, cake yield is mathematically related to operating initial solids content, C as shown below:

\[ Y_{LMT} = K C^{0.149} \]

while in Carman’s modified equation, yield is related to initial solids content as:

\[ Y_{Carm} = K C^{0.5} \]

Where \( K \) is proportionality constant.

The result of the experiment was displayed graphically in Figure 5. These graph shows that cake yields increases with increasing solid content at any height of the sludge cake.

By observation, when comparing Carman modified equation and the derived LMT equation, all the slopes have increasing cake yields at increasing solid contents. For instance, increasing solid contents from 0.0128g/cm³ to 0.0247g/cm³ would increase cake yields by 1.64times for practical plant operation, 1.32 times for the new model and 1.41times using Carman modified model respectively. The curves showed that the predicted values using the LMT model were closer to the experiment yield values as against values from Carman’s model.

![Fig. 4: Comparison of the Variation of Conditioner Dosages on Yields Between The Developed Cake Yield Model \( Y_{LMT} \), Actual Cake Yield \( Y_A \) and Carman’s Modified Equation \( Y_{CARM} \) For 0.02465g/Cm³ Sludge Sample.](image-url)
E. Overall Correlation

In most of the experiments, the measured values (actual cake yield) were within the theoretically predicted yield especially operating between 2039.43g/cm² to 4078.87g/Cm². The correlation coefficients between the predicted yield values and experimental values was 0.993 while between the predicted values and yield from Carman’s modified equation was 0.92.

V. CONCLUSION

The LMT Model equation developed (Equation 1) shows that the cake yield from a pressure filter (filter press) is directly proportional to the filter area of the pressure vessel, applied pressure and initial solids content of the sludge while being inversely proportional to specific resistance of the filtrate, viscosity of filtrate, compressibility coefficient of the sludge and pressing time. The cake yield values from Carman’s modified model were far above the actual cake yield, a confirmation that the model cannot validly predict the filterability of compressible sludges under constant pressure condition. This is in agreement with scientific reasoning and experimental observation. Equation 1 enable performance of a pressure filter (Filter Press) to be predicted from a simple laboratory determination of cake yields.

Moreover, experimental verification of the equation and the derived curves has been described and it may be concluded that for practical purposes, the predicted performance agrees with measured values. The negligible variations between the actual and the predicted values may be due to the conditioner dosages used.

REFERENCES


