Fuzzy Clustering and ILB Methods Using Level Set Method for Image Segmentation

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Abstract—*A stochastic parallel program is developed, the lattice Boltzmann method (LBM) has involved much concentration as a fast alternative method for solving PDE. In this article, first we compute energy function which is based on the Improved Kernel Fuzzy C-Means (IKFCM) object function which involves the bias field that the intensity inhomogeneity of the digital image. The level set equation from which we realize a fuzzy external force for the ILBM by using the gradient descent method. The approach is quick fast, strong against noise, self-governing to the position of the initial snake, efficient in the non-absence of intensity inhomogeneity, extremely parallel and can spot the objects with or without edges. Demonstration of the Experiments on medical images is performing of the estimated method in terms of speed and efficiency.*

Keywords— Improved Kernel Fuzzy *C*- Means (IKFCM), segmentation, intensity inhomogeneity, Improved lattice Boltzmann method (ILBM), level set Method, Partial Differential Equation (PDE).

I. INTRODUCTION

Segmentation is plays a vital role in the medical imaging whose object is to partition a given digital image into numerous regions or to spot an object of importance from the milieu. This job is more difficult, actually imaging devices produce images degraded by intensity inhomogeneity. The modified level set method is a part of the whole family of snake (Active Contour) methods. The Idea behind the level set started by Hamilton –Jacobi approach, the equation affecting surface on the timely needs. This work was carried and proposed by Osher and Sethian [1]. In 2D space, the Level set method represents a curl (curve)which is closed in the plane surface as the null level set of a 3-D function φ. Consider an example, first using a curvature around the object to be identify, the curvature moves towards the interior normal and has to end on the boundary of the object. There are two approaches are usually consider to stop the embryonic curl on the edge of the desired object; the primarily first one uses an boundary (edge) indicator depends on the gradient of the digital image like in classical snakes or Active Contour and ACMs, and the second approach is used some area attributes to end the evolving curl (curve) on the definite boundary. The robust approach against the noise and can detect the objects without boundary or edges. The objection of the model of Chan–Vese (CV) method is not appropriate for parallel training program because, at each and every iteration, the average intensities values inside and outside the snake or contour should be computed, which improves the performance of the CPU time by rising communications between processors. For this idea, we propose new innovative methods which overcome the above mentioned drawbacks. Proposed method is based on a novel initiative which objective to stop the evolving curl according to the degree of membership of the existing pixel to be inside or outside of the snake or active contour, which is done with the help of the improved fuzzy C-means (IFCM) objective function obtained in [2] which also takes into reflection the shading image due to the intensity inhomogeneity.

In this article, the Lattice Boltzmann Method is used to solve the Level set Equation. The idea behind this proposed method is on the approach of the Lattice Boltzmann Method Partial Difference Equation solver defined in [6]. In our proposed method, using a modified IKFCM function, we design a new innovative Fuzzy External Force.

II. INNOVATIVE METHOD

This part details firstly the concept of the Fuzzy C Means based on Energy function we can deduce the corresponding Level Set Equation from the set of Fuzzy External Force.

2.1. Design of Energy Function

In the segmentation context of a Digital Image, the standard Fuzzy C Means algorithm is an optimization difficulty for partitioning an digital image of N no of pixels, $X = \{x_i\}\ N_i=1$, into c classes. It highly aim to minimize a clustering criterion.

$$J(U,V,X)=\sum_{k=1}^{c}\sum_{i=1}^{N}u_{ki}^{p}\|x_i-v_k\|^2$$
$$\sum_{k=1}^{c}u_{ki}=1,\forall i \qquad (1)$$
$$0\le u_{ki}\le 1,\ \forall\ k,i$$

Where U is the *partition matrix* whose element u_{ik} is the membership of the i^{th} voxel for k^{th} class. V is the vector centroid whose element v_k is the centroid (or prototype) of k^{th} class. The parameter p, called fuzzy index, is a weighting exponent on each membership of fuzzy and determine the sum of "fuzziness" of the resulting partition. The norm operator $|.|$ represents the standard Euclidean distance. The bias field is integrated into the Fuzzy C Means framework by modeling the image as follows:

$$Y_i=X_iG_i,\ \forall\ i\in\{1,2,3,4,5,..........N\} \qquad (2)$$

Where Y_i, X_i, and G_i are observed intensity, or true intensity, and gain field at i^{th} pel, respectively .N is the total number of pels in the magnetic resonance image. The artifact can be modeled as an additive bias field by applying a logarithmic transformation to both sides of (2)

$$y_i=x_i+\beta_i,\ \forall\ \ i\in\{1,2,.....,N\} \qquad (3)$$

Where x_i and y_j are the observed and true log transformed intensities at the i^{th} voxel, respectively, and β_i is the bias field at the i^{th} voxel. By integrating the bias field model into an FCM framework, we will be capable of iteratively approximating both the true intensity and the bias field from the observed intensity. By substituting (3) into (1), the clustering criterion to minimize in the presence of bias field becomes a constrained optimization problem

$$J(U,V,B,Y)=\sum_{k=1}^{c}\sum_{i=1}^{N}u_{ki}^{p}\|y_i-\beta_i-v_k\|^2$$
$$s.t.\sum_{k=1}^{c}u_{ki}=1\forall i \qquad (4)$$
$$0\le u_{ki}\le 1\forall k,i$$

Where$Y=\{y_i\},N_i=1$ is the observed image and $B=\{\beta_i\}N_i=1$ is the bias field image.

In a continuous form, the aforementioned criterion can be written as

$$J(U,V,B,Y)=\sum_{k=1}^{c}\int_{\Omega_k}U_k^p(x,y)\|Y(x,y)-B(x,y)-v_k\|^2 d_xd_y$$
$$s.t.\sum_{k=1}^{c}U_k(x,y)=1\forall x,y \qquad (5)$$
$$0\le U_k(x,y)\le 1\forall k,x,y.$$

Consider the two-phase level set although the method can be easily extended to more than two phases. The image domain Ω is segmented into two disjoint regions Ω1 and Ω2, i.e., c = 2. In this case, we can introduce a level set function as follows:

$$J(U,V,B,Y,\phi)=\int_{\Omega}U_1^p(x,y)\|Y(x,y)-B(x,y)-v_1\|^2H(\phi)d_xd_y$$
$$+\int_{\Omega}U_2^p(x,y)\|Y(x,y)-B(x,y)-v_2\|^2(1-H(\phi))d_xd_y$$
$$s.t.U_1(x,y)+U_2(x,y)=1\forall x,y \qquad (6)$$
$$0\le U_k(x,y)\le 1\forall k,x,y$$

Whereφ is a signed distant function. The aforementioned term $J(U, V,B, Y, \varphi)$ is used as the data link in our energy functional which is defined as follows:

$$E(U,V,B,Y,\phi)=J(U,V,B,Y,\phi)+v|C| \qquad (7)$$

Where$v|C|$ is a regularization term with $v>0$ being a fixed parameter and C being a given curve which is represented implicitly as the zero level of φ and $|C|$ is the length of C and can be expressed by the following equation.

$$|C|=\int_{\Omega}|\nabla H(\phi)|d_xd_y \qquad (8)$$

2.2. LSE

To obtain the LSE, we minimize $E(U, V,B, Y, \varphi)$ with respect to φ. For fixed U, V , and B, we use the gradient descent method

$$\frac{\partial\phi}{\partial t}=\frac{\partial E}{\partial\phi} \qquad (9)$$

Where$\partial E/\partial\varphi$ is the Gateauxderivative of$E$. We obtain the following LSE:

$$\frac{\partial\phi}{\partial t}=\delta(\phi)\begin{pmatrix}U_1^p(x,y)\|Y(x,y)-B(x,y)-v_1\|^2-\\U_2^p(x,y)\|Y(x,y)-B(x,y)-v_2\|^2\end{pmatrix}$$
$$+v\delta(\phi)div\left(\frac{\nabla\phi}{|\nabla\phi|}\right) \qquad (10)$$
$$s.t.U_1(x,y)+U_2(x,y)=1\forall x,y$$
$$0\le U_k(x,y)\le 1\forall k,x,y$$

However, for solving the minimization problem of *E* (*U*, *V*, *B*, *Y*, *φ*), we should also compute the first derivatives of *E* (*U*, *V*, *B*, *Y*, *φ*) with respect to u_{ki}, v_k, and $\beta_i$and set them equal to zero. We thus obtain three necessary conditions

$$U_k^*(x,y)=\frac{1}{\sum_{l=1}^{c}\left(\frac{\|Y(x,y)-B(x,y)-v_k\|}{\|Y(x,y)-B(x,y)-v_l\|}\right)^{\frac{2}{(p-1)}}} \quad (11)$$

$$v_k^*=\frac{\int_\Omega U_k^p(x,y)\left(Y(x,y)-B(x,y)\right)d_xd_y}{\int_\Omega U_k^p(x,y)d_xd_y} \quad (12)$$

$$B^*(x,y)=\left[Y(x,y)-\frac{\sum_{k=1}^{c}U_k^p(x,y)v_k}{\sum_{k=1}^{c}U_k^p(x,y)}\right]^{1/2} \quad (13)$$

2.3. Lattice Boltzmann Solver for LSE

By using the gradient projection method of Rosen , we can replace $\delta(\varphi)$ by $|\nabla\varphi|$ in the proposed LSE, and as *φ* is a distance function, we have $|\nabla\varphi|=1$ and will stay at each step since an adaptive approach is not used and the distant field is valid in the whole domain. Thus, the proposed LSE becomes

$$\begin{aligned}\frac{\partial\phi}{\partial t}&=U_1^p(x,y)\|Y(x,y)-B(x,y)-v_1\|^2-\\ &U_2^p(x,y)\|Y(x,y)-B(x,y)-v_2\|^2+v\,div(\nabla\phi)\\ &s.t.\,U_1(x,y)+U_2(x,y)=1\forall x,y\\ &0\le U_k(x,y)\le 1\forall k,x,y.\end{aligned} \quad (14)$$

Replacing *ρ* by the signed distance function *φ*, (11) becomes

$$\frac{\partial\phi}{\partial t}=\gamma\,div(\nabla\phi)+F \quad (15)$$

By setting the external force

$$F=\lambda\begin{pmatrix}U_1^p(x,y)\|Y(x,y)-B(x,y)-v_1\|^2-\\ U_2^p(x,y)\|Y(x,y)-B(x,y)-v_2\|^2\end{pmatrix} \quad (16)$$

Where*λ*is a positive parameter; we can see that (14) is only a variation formula of (15) and, thus, can be solved by the LBM with the above-defined FEF. The choice of parameter *p* is at great importance for the segmentation result. Different values for *p* will result in the different results, as stated below.
1) If $p>2$, then the exponent $2/(p-1)$ in (11) decreases the membership value of the pixels that are closed to the centroid. The segmentation result will therefore be wrong since it is intuitively better that the membership value be high for those pixels who are closed to the centroid.
2) If $p\to\infty$, all the membership values tend to $1/c$. This implies that the

$$FEF\to\lambda\begin{pmatrix}\left(\frac{1}{c}\right)^p\|Y(x,y)-B(x,y)-v_1\|^2-\\ \left(\frac{1}{c}\right)^p\|Y(x,y)-B(x,y)-v_2\|^2\end{pmatrix}\to 0 \quad (17)$$

There is, therefore, no link with the image data in the LSM process. Therefore, segmentation is impossible.
3) If $p\to 1$, the exponent $2/(p-1)$ increases the membership values of the pixels who are closed to the centroid. As $p\to 1$, the membership tends to one for the closest pixels and tends to zero for all the other pixels. This case is equivalent to the use of the *k*-means objective function instead of the FCM one. The segmentation is therefore rigid, and we lose the advantage of FCM over *k*-means.

For all these reasons, a suitable choice of the parameter *p* can be the value of two, which is therefore used in all our experiments.

2.4. Implementation

When using LBM to resolve the convection–diffusion equation,the particle density is set as *φ* which is a signed distance function. Since the particle number of the cell cannot be negative, we modify the distance function as $\varphi'=\varphi-\min(\varphi)$. The contour is those pixels which satisfy $\varphi'=-\min(\varphi)$. The steps for the computation are outlined as follows.
1) Initialize the distance function *φ* and class centroid values *v*1 and *v*2. Initialize *B* with zeros.
2) Compute $U_1^p(x,y)$ and $U_2^p(x,y)$ with (11).
3) Compute *v*1 and *v*2 with (12).
4) Compute *B* with (13).
5) Compute the external force with (16).
6) Include the external force.
7) Resolve the convection–diffusion equation with LBM.
8) Accumulate the $f_i(r, t)$ values at each grid point by (6),which generates an updated distance value at each point.
9) Find the contour.
10) If the segmentation is not done, increase the value of *λ* and go back to step (5).

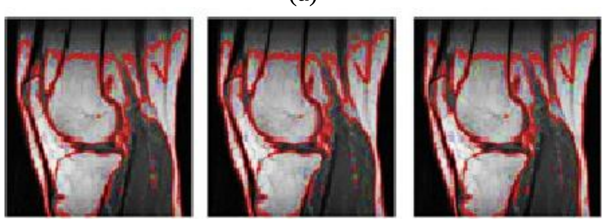
We should notice that the *B* obtained from (13) is a “residual” image but not necessarily the bias field image.The adaptive fuzzy *c*-means (AFCM) algorithm by Pham and Prince solved the problem by introducing regularization terms into the objective function that ensure the resulted bias field image to be smooth. The regularization terms, however, make the estimation of the bias field a computationally intensive process. As done in, another solution is to estimate the bias field by filtering the residual image *B* in (13) using an iterative low-pass spatial filter. This filtering strategy is based on the fact that the bias field is of low spatial frequency and the assumption that other components in the residual image are of higher frequency.

In the implementation of our method, the value of the *fuzzy index p* is set to two, the class centroid values $v1$ and $v2$ are randomly initialized at zero and one, respectively, and the diffusion coefficient γ is set to 15. All the methods have been implemented using Matlab R2010b installed on a PC Advanced Micro Devices (AMD) Athlon [trademark (tm)] 5200 processor with a clock speed of 2.31 GHz and 2 GB of RAM.

Fig.1.shows the proposed method on Magnetic Resonance Imaging (MRI) image of the knee. Intensity inhomogeneities can be clearly seen in the image. Fig.1.(a) shows the initial contour, Fig.1.(b) shows the segmentation result using the proposed method.

Fig.1.shows the segmentation result obtained on a real-world image with different initial contours. In terms of quality and accuracy, it can be seen that the proposed method gives better results whatever the shape and the position of the initial contour.

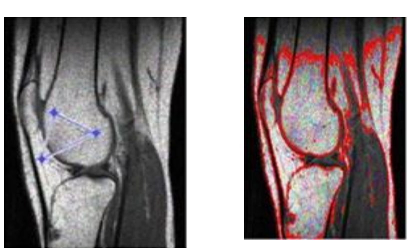
(a)



(b)

Fig. 1. Segmentation of an MRI image of the knee. (a) Shows the initial contours, and (b) shows the segmentation result of the proposed method.

Fig.2.shows the segmentation result of an MRI image of the knee corrupted by a multiplicative noise. We used the Matlab speckle function with $v = 0.04$ to insert the multiplicative noise. It can be seen that the proposed method is more robust to noise and gives the best results.



(a) (b)

Fig.2. Segmentation of an MRI image of the knee corrupted by a multiplicative noise. (a) Initial contour. (b) Segmentation result using the proposed method.

III.DISCUSSIONS

The proposed method is takes less time for the segmentation with efficient results.In the graph below the Elapsed time 1 indicates the time taken by the LBM method to evaluate the segmentation result, the Elapsed time 2 indicates the time taken by the Improved LBM method to evaluate the segmentation result.It can be observed that the proposed method Improved LBM is faster and gives accurate results compared to the previous LBM method.

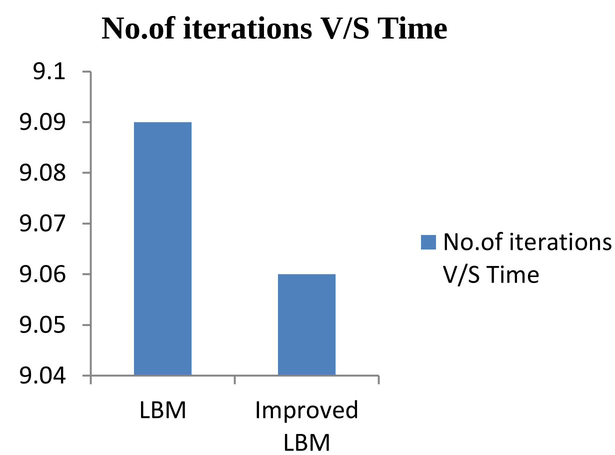


Fig.3. LBM and Improved LBM in terms of elapsed time

IV.CONCLUSION

In conclusion, the results of this study confirmed that the combination of Improved LBM with the level set methods could be used for the fast and efficient segmentation of images. The method has the advantages of more-initialization, automation, and reducing the number of iterations. The LSE is solved by using the powerful, simple and highly parallelizable Improved LBM which gives promising results.

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