DFT-based channel estimation for OFDM system and comparison with LS and MMSE over Rayleigh and Rician fading channel

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Abstract- Orthogonal Frequency Division multiplexing is a high data rate transmission scheme for wireless communication systems. The received signal is not only coming directly from the transmitter, but the combination of multipath components of transmitted signal. One of the most intriguing aspects in wireless communication is fading which is present when there are multipath components. To cancel the effect of fading, channel estimation is required at the receiving end. In this paper a DFT-based channel estimation approach is simulated and comparing with conventional channel estimation techniques, Least Square (LS) and Minimum Mean Square Error (MMSE) over Rayleigh and Rician fading Channel. The simulation results show that DFT-based channel estimation technique reduces the Mean Square Error (MSE) and the performance of DFT-based is better than LS and MMSE.

Keywords- OFDM; DFT; MSE; Channel Estimation; LS; MMSE.

1. INTRODUCTION-

Wireless communication system is mainly affected by the wireless channel environment by which the performance of the system will degrade. The wireless channel is rather influential and fickle, by which the exact analysis of any wireless communication system is difficult. The received signal is generally distorted by the channel. To recover the transmitted data, the channel characteristics must be estimated and compensated in the receiver [1–3].

There are so many channel estimation techniques. Three channel estimation algorithms have been presented and compared in [4] for OFDM system. Least Square (LS) technique introduced in [5] is the simplest algorithm it has low complexity, but larger Mean Square Error (MSE) and easily affected by noise. A low complexity Minimum Mean Square Error(MMSE) algorithm is proposed in [6] which little attenuate the Mean Square Error [MSE] but it need the channel statistics which are usually unknown in real system. In this paper DFT-based channel estimation algorithm is simulated and comparing with LS and MMSE algorithms over Rayleigh and Rician Fading Channel for a OFDM system. This algorithm improve the performance of OFDM system and reduce the MSE as compared to LS or MMSE.

The outline of the paper is as follow. In Section 2 OFDM System is described. In Section 3 Rayleigh and Rician Fading channel described. In Section 4 Introduced LS and MMSE algorithms. In Section 5 The DFT-based channel estimation technique is Explained. in Section 6 Simulation Results are presented and finally Section 7 provides the conclusion.

2. OFDM SYSTEM

The OFDM message is generated in the complex baseband. Each symbol is modulated onto the corresponding subcarrier using Quadrature Amplitude Modulation (QAM). The data symbols are converted from serial to parallel before data transmission. The subcarriers spacing is achieved using the inverse discrete Fourier transform (IDFT), and it can easily implemented by the inverse fast fourier transform (IFFT) operation . As a result, the OFDM symbol generated for an N-subcarrier system translates into N samples, with the i^{th} sample being. The key components of an OFDM system are the Inverse DFT in the transmitter and the DFT in the receiver [7].

$$x_l[n] = \frac{1}{N}\sum_{k=0}^{N-1} X_l[k]e^{\frac{j2\pi kn}{N}} \qquad (1)$$

$$for\ n = 0, 1, \ldots N-1$$

Eq. (1) represent the N-point IDFT of QAM data symbols $\{X_l[k]\}_{k=0}^{N-1}$ and can be computed efficiently by using Inverse Fast Fourier Transform (IFFT) algorithm.

At the receiver, the OFDM symbol goes through the exact reverse operation in the discrete Fourier transform (DFT) to recover the corrupted symbols from a time domain symbol into the frequency domain. The baseband OFDM receiver performs the fast Fourier transform (FFT) of the receive message to recover the information that was originally sent.

$$Y_l[k] = \sum_{n=0}^{N-1} y_l[n] e^{-j2\pi kn/N} \quad (2)$$

Eq. (2) represent the N-point DFT of received OFDM symbol $y_l[n]$.

A general N-to-N point linear transformation requires multiplications and additions. This would be true of the DFT and IDFT if each output symbol were calculated separately. However, by calculating the outputs simultaneously and taking advantage of the cyclic properties of the multipliers $e^{\pm j2\pi nk/N}$,. Fast Fourier Transform (FFT) techniques reduce the number of computations to the order of $N \log N$. The FFT is most efficient when N is a power of two. Several variations of the FFT exist, with different ordering of the inputs and outputs, and different use of temporary memory.

3. CHANNEL

3.1. Rayleigh Fading

The received signal can be considered as the sum of received signals from an infinite number of scatters in the propagation environment for a wireless channel According to the central limit theorem, the received message can be represented by a Gaussian random variable. In mean, a wireless channel subject to the fading environments can be represented by a complex Gaussian random variable, $W_1 + jW_2$ where W_1 and W_2 are the independent and identically-distributed Gaussian random variables with a zero mean and variance of σ^2. Let X denote the amplitude of the complex Gaussian random variable $W_1 + jW_2$ such that $X = \sqrt{W_1^2 + W_2^2}$. Then, note that X is a Rayleigh random variable. Constructive and destructive nature of multipath components in flat fading channels can be represented by Rayleigh distribution if there is no any direct path between transmitter and receiver which means it is no line of sight path communication.

The sum of two equal independent orthogonal Gaussian random variables is represent a Rayleigh distribution and the probability density function is given by

$$p(r) = \begin{cases} \frac{r}{\sigma^2} e^{\left(-\frac{r^2}{2\sigma^2}\right)} & (0 \le r \le \infty) \\ 0 & (r < 0) \end{cases} \quad (3)$$

Where σ^2 is the time-average power of the received signal[8].

3.2. Ricean Fading

In the direct path communication where there exists a line-of-sight (LOS) environment which is not subject to any loss due to scattering, diffraction and reflection, the amplitude of the received signal can be expressed as $X = c + W_1 + jW_2$ where c represents the LOS component while W_1 and W_2 are the Gaussian random variables with a zero mean and variance of σ^2 as in the non-Line Of Sight(NLOS) environment and X is the Rician random variable. This type of signal is approximated by Ricean distribution. As the main component run into more fade the signal characteristic and deviates from Ricean to Rayleigh distribution.

The Ricean distribution is given by

$$p(r) = \begin{cases} \frac{r}{\sigma^2} e^{\left(-\frac{r^2+A^2}{2\sigma^2}\right)} I_0\left(\frac{Ar}{\sigma^2}\right) & \text{for } (A \ge 0, r \ge 0) \\ 0 & (r < 0) \end{cases} \quad (4)$$

where A denotes the peak amplitude of the dominant signal and $I_0(.)$is the modified Bessel function of the first kind and zero-order. If K is Rician factor then defined as

$$K = \frac{A^2}{2\sigma^2}$$

4. CHANNEL ESTIMATION TECHNIQUES

4.1 LS Channel Estimation

The least-square (LS) channel estimation method finds the channel estimate $\hat{H}$ in such a way that the following cost function is minimized[9].

$$J(\hat{H}) = \left\|Y - X\hat{H}\right\|^2$$

$$= \left(Y - X\hat{H}\right)^H \left(Y - X\hat{H}\right)$$

$$= Y^H Y - Y^H X\hat{H} - \hat{H}^H X^H Y + \hat{H}^H X^H X\hat{H}$$

Derivative of the function with respect to $\hat{H}$ to zero,

$$\frac{\partial J(\hat{H})}{\partial \hat{H}} = -2(X^H Y)^* + 2(X^H X\hat{H})^* = 0$$

We have $X^H X\hat{H} = X^H Y$, which gives the solution to the LS channel estimation as

$$\hat{H}_{LS} = (X^H X)^{-1} X^H Y = X^{-1} Y \qquad (5)$$

LS Channel Estimation can be represent as

$$\hat{H}_{LS}[k] = \frac{Y[k]}{X[k]}, k = 0,1,2,\ldots N-1 \qquad (6)$$

The mean square error (MSE) of this LS channel estimate is given as

$$MSE_{LS} = E\left\{\left(H - \hat{H}_{LS}\right)^H (H - H_{LS})\right\} \qquad (7)$$

4.2 MMSE Channel Estimation

Consider the LS solution in Equation(5), $\hat{H}_{LS} = X^{-1}Y \triangleq \tilde{H}$. Using the weight matrix W,

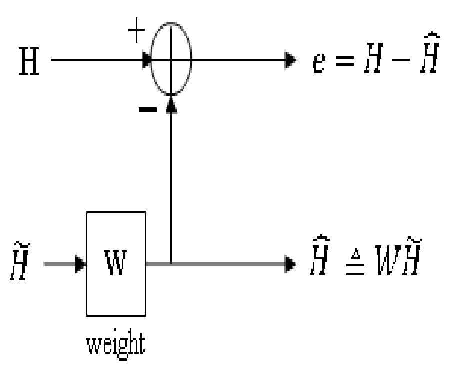

Figure 1 MMSE channel Estimation

$\hat{H} \triangleq W\tilde{H}$ which corresponds to the MMSE estimate.

$$J(\hat{H}) = E\{\|e\|^2\} = E\{\|H - \hat{H}\|^2\} \qquad (8)$$

The orthogonality principle states that the estimation vector $e = H - \hat{H}$ is orthogonal to $\tilde{H}$, such that

$$E\{e\tilde{H}^H\} = E\{(H - \hat{H})\,\tilde{H}^H\}$$

$$= E\{(H - W\tilde{H})\,\tilde{H}^H\}$$

$$= E\{H\tilde{H}^H\} - WE\{\tilde{H}\tilde{H}^H\}$$

$$= R_{H\tilde{H}} - WR_{\tilde{H}\tilde{H}} = 0$$

Where $W = R_{H\tilde{H}} R_{\tilde{H}\tilde{H}}^{-1}$

Where $R_{\tilde{H}\tilde{H}}$ is the autocorrelation matrix of $\tilde{H}$ and $R_{H\tilde{H}}$ is the cross-correlation matrix between the true channel vector and temporary channel estimate vector in the frequency domain. $\tilde{H}$ is the LS channel estimate given as

$$\tilde{H} = X^{-1}Y = H + X^{-1}Z \qquad (9)$$

So the MMSE, channel estimate follows as

$$\hat{H} = W\tilde{H} = R_{H\tilde{H}} R_{\tilde{H}\tilde{H}}^{-1}\,\tilde{H} \qquad (10)$$

5. DFT-BASED ALGORITHM

The DFT-based channel estimation technique has been derived to improve the performance of LS or MMSE channel estimation by eliminating the effect of noise outside the maximum channel delay. Let $\hat{H}[k]$ denote the estimate of channel gain at the k^th subcarrier, obtained by either LS or MMSE channel estimation method. Taking the IDFT of the channel estimate $\{\hat{H}[k]\}_{K=0}^{N-1}$,

$$IDFT\{\hat{H}[k]\} = h[n] + z[n] \triangleq \hat{h}[n] \qquad (11)$$

$$n = 0\,1\,2\ldots.N-1$$

Ignoring the coefficients $\{\hat{h}[n]\}$ that contain the noise only. Define the coefficients for maximum Channel delay L

$$\hat{h}_{DFT}[n] = \begin{cases} h[n] + z[n], & n = 0\,1\,2\,..\,L-1 \\ 0, & otherwise \end{cases}$$

Frequency Domain

$$\hat{H}_{DFT}[k] = DFT\{\hat{h}_{DFT}(n)\} \qquad (12)$$

6. SMULATION RESULTS

6.1 Performance of DFT-based channel estimation over Rayleigh fading channel in terms of MSE.

Figure 2 Shows that the DFT-based channel estimator have less Mean square error (MSE) than LS and MMSE Channel estimation in a Rayleigh fading environment. So from the figure it can say that the DFT-based channel estimation method improves the performance of channel estimation.

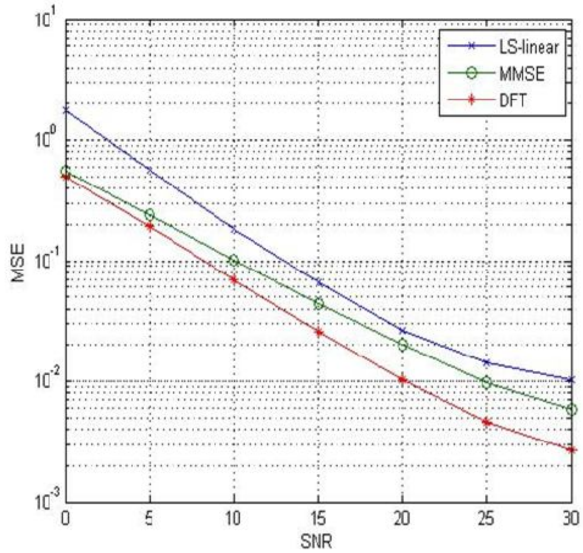

Figure 2 MSE in Rayleigh fading channel

Table .1 show the MSE/SNR Results comparison of three different channel estimations in a Rayleigh fading channel.

S. No.	Signal to Noise Ratio	MSE in LS	MSE in MMSE	MSE in DFT
1.	0 [dB]	1.7668	0.5569	0.4968
2.	5 [dB]	0.5662	0.2428	0.1930
3.	10 [dB]	0.1858	0.1026	0.0698
4.	15 [dB]	0.0650	0.0445	0.0259
5.	20 [dB]	0.0266	0.0199	0.0103
6.	25 [dB]	0.0143	0.0097	0.0046
7.	30 [dB]	0.0103	0.0058	0.0027

6.2 Performance of DFT-based channel estimation over Rician fading channel in terms of MSE.

Figure 3 show the mean-square-errors (MSEs) of DFT-based is less than LS and MMSE channel estimators. So it is also better in Rician fading environment.

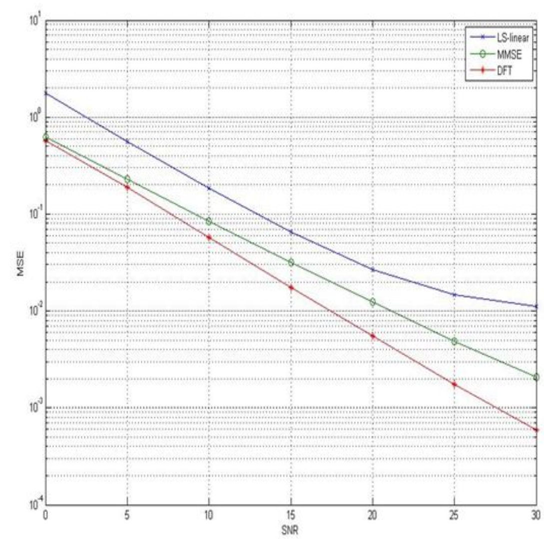

Figure 3 MSE in Rician fading channel

Table 2 show the MSE/SNR Results comparison of three different channel estimations in a Rician fading channel.

S. No.	Signal to Noise Ratio	MSE in LS	MSE in MMSE	MSE in DFT
1.	0 [dB]	1.7580	0.6245	0.5772
2.	5 [dB]	0.5623	0.2296	0.1896
3.	10 [dB]	0.1842	0.0836	0.0572
4.	15 [dB]	0.0646	0.0318	0.0175
5.	20 [dB]	0.0268	0.0123	0.0055
6.	25 [dB]	0.0148	0.0049	0.0018
7.	30 [dB]	0.0111	0.0021	0.0006

7. CONCLUSION

In this paper DFT-based Channel Estimation approach for OFDM system has been simulated using MATLAB. The performance of DFT-based Channel Estimation approach for OFDM system in Rayleigh and Rician fading channel has been studied. It has been found that the Mean Square Error is going to reduce of OFDM system in DFT-based Channel Estimation process in both the channels Rayleigh and Rician.

This work also compares the DFT-based channel Estimation approach with two conventional channel estimation technique Least Square (LS) and Minimum Mean Square Error (MMSE). Simulation results show that the MMSE estimation shows better performance than the LS estimation does at the cost of requiring the additional computation and information on the channel characteristics. But the DFT-based channel estimation method can reduce the leakage energy efficiently. And the MSE performance of the DFT-based method is better than LS estimation and MMSE channel estimation methods.

Hence it could be concluded that DFT-based channel estimation is a suitable technique for channel estimation in terms of reduction in Mean Square Error (MSE) as compared to LS and MMSE techniques.

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