Soft Pre Generalized - Closed Sets in a Soft Topological Space

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Abstract

This paper introduces soft pre-generalized closed set in soft topological spaces. The notations of soft pre interior and soft pre closure are generalized using these sets. In a soft topological space, a soft set F_A is said to soft pre generalised –closed if $p(\overline{F_A}) \tilde{\subseteq} F_O$ whenever $F_A \tilde{\subseteq} F_O$ where F_O is a soft P-open set . A detail study is carried out on properties of soft Pre generalized closed sets and soft $PT_{1/2}$ Space.

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Introduction. Soft set theory was first introduced by Molodtsov [5] in 1999 as a general mathematical tool for dealing uncertain fuzzy, not clearly defined objects. He has shown several applications of this theory in solving many practical problems in economics, engineering, social science, medical science, and so on. Modern topology depends strongly on the ideas of set theory. In 2010 Muhammad shabir, Munazza Naz [6] used soft sets to define a topology namely Soft topology. J.Subhashini and C.Sekar defined soft pre-open sets [11] in a soft topological space. In General topology the concept of generalized closed set was introduced by Levine [3] plays a significant. This notation has been studied extensively in recent years by many topologies. The investigation of generalized closed sets has led to several new and interesting concepts, e.g. new covering properties and new separation axioms. Some of these separation axioms have been found to be useful in computer science and digital topology. Soft generalised closed set was introduced by K.Kannan[2] in 2012. In this paper we introduce soft pre-generalized closed set and soft $PT_{1/2}$ – Space in a soft topological space and a detailed study of some of its properties.

2. Preliminaries

For basic notations and definitions not given here, the reader can refer [1-12].

2.1. Definition. [11] A soft set F_A on the universe U is defined by the set of ordered pairs $F_A = \{(e, f_A(e)): e \in E, f_A(e) \in P(U)\}$, where E is a set of parameters, $A \subseteq E$, P (U) is the power set of U, and $f_A: A \to P(U)$ such that $f_A(e) = \emptyset$ if $e \notin A$. Here, f_A is called an approximate function of the soft set F_A. The value of $f_A(e)$ may be arbitrary, some of them may be empty, some may have non-empty intersection. Note that the set of all soft set over U is denoted by S (U).

2.2. Example. Suppose that there are five cars in the universe. Let U={c_1, c_2, c_3, c_4, c_5} under consideration,and that $E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\}$ is a set of decision parameters. The e_i $(i = 1,2,3,4,5,6,7,8)$ stand for the parameters "expensive", "beautiful", "manual gear", "cheap", "automatic gear", "in good repair", "in bad repair" and "costly" respectively. In this case, to define a soft set means to point out expensive cars, beautiful cars and so on. It means that, Consider the mapping f_E given by "cars (.)", where (.) is to be filled in by one of the parameters $e_i \in E$. For instance, $f_E(e_1)$ means "car (expensive)", and its functional value is the set {c $\in U$: c is an expensive car} and so,

Let $A \subseteq E$, the soft set F_A that describes the "attractiveness of the cars" in the opinion of a buyer say Ram, may be defined like A={e_2, e_3, e_4, e_5, e_7}, $f_A(e_2)$= {c_2, c_3, c_5}, $f_A(e_3)$= {c_2, c_4 }, $f_A(e_4)$= {c_1}, $f_A(e_5)$= {U} and $f_A(e_7)$= {c_3,c_5}. We can view this soft set F_A as consisting of the following collection of approximations:

$F_A = \{(e_2, \{c_2, c_3, c_5\}), (e_3, \{c_2, c_4\}), (e_4, \{c_1\}), (e_5, \{U\}), (e_7, \{c_3, c_5\})\}$.

2.3. Definition. [11] The soft set $F_A \tilde{\in} S(U)$ is called a soft point in F_E, denoted by (e_{f_A}), if for the element $e \in A$ and $f_A(e) \neq \emptyset$ and $f_A(e') = \emptyset$ for all $e' \in A - \{e\}$.The soft point (e_{f_A}) is said to be in the soft set F_B, denoted by $(e_{f_A}) \tilde{\in} F_B$ if for the element $e \in A$ and $f_A(e) \subseteq f_B(e)$.

2.4.Definition. [10] Let $F_A \tilde{\in} S(\mathrm{U})$. The soft power set of F_A is defined by $\tilde{P}(F_A) = \{F_{A_i}: F_{A_i} \tilde{\subseteq} F_A, i \epsilon I \subseteq N\}$ and its cardinality is defined by $|\tilde{P}(F_A)| = 2^{\sum_{e \epsilon E}|f_A(e)|}$, where $|f_A(e)|$ is the cardinality of $f_A(e)$.

2.5. Example. [10] Let U={u_1,u_2}, E={e_1,e_2,e_3}, $A \subseteq E$ A={e_1,e_2} and $F_A = \{ (e_1, \{h_1, h_2\}), (e_2, \{h_1, h_2\}) \}$. Then
F_{A_1}= F_A, $F_{A_2} = F_\emptyset$,
F_{A_3} ={(e_1,{h_1,h_2})}, F_{A_4}={(e_1,{h_1})},F_{A_5}={(e_1,{h_2})},
F_{A_6}={(e_1,{h_1}),(e_2,{h_1,h_2})},F_{A_7}={(e_1,{h_2}),(e_2,{h_1,h_2}},
F_{A_8}={(e_1,{h_2}),(e_2,{h_2})}, F_{A_9}={(e_1,{h_1}),(e_2,{h_1})},$F_{A_{10}}$={(e_2,{h_1,h_2})},
$F_{A_{11}}$={(e_2,{h_1})},$F_{A_{12}}$={(e_2,{h_2})},$F_{A_{13}}$={(e_1,{h_2,h_3}),(e_2,{h_1})},
$F_{A_{14}}$={(e_1,{h_1,h_2}),(e_2,{h_2})}, $F_{A_{15}}$={(e_1,{h_2}),(e_2,{h_1})},
$F_{A_{16}}$ = {(e_1,{h_2}), (e_2,{h_2})} are all soft subset of F_A. So $|\tilde{P}(F_A)|$ = 2^4 = 16.

2.6. Definition. [11] Let $F_E \tilde{\in} S(U)$. If $f_E(e) = U$ for all $e \in E$, then F_E is called a soft universal set (soft absolute set) denoted by $F_{\tilde{E}}$ or $\tilde{U}$.

2.7. Definition. [6] Let $F_E \tilde{\in} S(U)$. A soft topology on F_E denoted by $\tilde{\tau}$ is a collection of soft subsets of F_E having the following properties:

(i). $F_\emptyset$, $F_E \in \tilde{\tau}$

(ii). $\{F_{E_i} \tilde{\subseteq} F_E: i \in I \subseteq N\} \subseteq \tilde{\tau} \Longrightarrow \bigcup_{i \epsilon I} F_{E_i} \epsilon \tilde{\tau}$

(iii).$\{F_{E_i} \tilde{\subseteq} F_E: 1 \leq i \leq n, n \in N\} \subseteq \tilde{\tau} \Longrightarrow \bigcap_{i=1}^{n} F_{E_i} \epsilon \tilde{\tau}$.

The pair $(F_E, \tilde{\tau})$ is called a soft topological space.

2.8. Example. Let us consider the soft subsets of F_A that are given in Example 2.5. Then $\tilde{\tau}_1 = \{F_\emptyset, F_A, F_{A_3}, F_{A_4}\}$, $\tilde{\tau}_2 = \{F_\emptyset, F_A, F_{A_5}\}$, $\tilde{\tau}_3 = \{\tilde{P}\ (F_A)\}$ are soft topologies on F_A.

2.9. Definition. [5] Let $(F_E, \tilde{\tau})$ be a soft topological space .Then, every element of $\tilde{\tau}$ is called a soft open set. Clearly $F_\emptyset$ and F_E are soft open sets. The collection of all soft open set is denoted by $G_s(F_E)$. Let $F_C \tilde{\subseteq} F_E$. Then F_C is said to be soft closed if the soft set $F_C^{\tilde{c}}$ is soft open in F_E. The collection of all soft closed set is denoted by $F_s(F_E)$.

2.10. Definition. A soft topology whose soft open sets are all soft subsets of F_E is called soft discrete topology.

2.11. Definition. [1] Let F_E be a soft set over U and $x \in U$, we say that $x \tilde{\in} F_E$ read as x belongs to the soft set F_E whenever $x \in f_E(e)$ for all $e \in E$. Not that for any $x \in U$, $x \tilde{\notin} F_E$ if $x \notin f_E(e)$ for some $e \in E$.

2.12. Definition. [1] The soft set F_E over U such that $f_E(e) = \{x\}$ for all $e \in E$ is called soft singleton and is denoted by x_E.

2.13. Definition. [5] Let $(F_E, \tilde{\tau})$ be a soft topological space over U, $F_E \tilde{\in} S(U)$ and Let V be a non null subset of U. Then the soft subset of F_E over V denoted by vF_E, is defined as follows: $f_E(e) = V \cap f_E(e)$, for all $e \in E$. In other words $vF_E = \tilde{V} \tilde{\cap} F_E$.

2.14. Definition. [9] Let $(F_E, \tilde{\tau})$ be a soft topological space over U. A soft set F_E is called a soft generalized closed in U if $\overline{F_E} \tilde{\subseteq} F_O$ whenever $F_E \tilde{\subseteq} F_O$ and F_O is soft open in U.

2.15. Definition. [12] Let $(F_E, \tilde{\tau})$ be a soft topological space, a soft set F_A is said to be soft pre-open set (soft P-open) if there exists a soft open set F_O such that $F_A \tilde{\subseteq} F_O \tilde{\subseteq} \overline{F_A}$. The set of all soft P-open set of F_E is denoted by $G_{sp}(F_E, \tilde{\tau})$ or $G_{sp}(F_E)$ **.** Then $F_A^{\tilde{c}}$ is said to be soft pre-closed. The set of all soft P-closed set of F_E is denoted by $F_{sp}(F_E, \tilde{\tau})$ or $F_{sp}(F_E)$.

2.16. Remark. [11] A soft set F_A which is both soft P-open and soft P-closed is known as soft P-clopen set. Clearly $F_\emptyset$ and F_E are soft P-clopen sets.

2.17. Proposition. [11]

(i) Every soft open set is a soft pre-open set.

(ii) Every soft closed set is a soft pre-closed set.

2.18. Theorem. [10]

(i) Arbitrary soft union of soft P-open sets is a soft P-open set.

(ii) The soft intersection of any two soft P-open set need not be a soft P-open set.

(iii) Arbitrary soft intersection of soft P-closed sets is soft P-closed set.

(iv) The soft union of any two soft P-closed set need not be a soft P-closed set.

2.19. Definition. [11] Let $(F_E, \tilde{\tau})$ be a soft topological space and $F_A \tilde{\subseteq} F_E$. Then the soft pre- interior (soft P-interior) of F_A denoted by $p(F_A)^o$ is defined as the soft union of all soft P-open subsets of F_A. Note that $p(F_A)^o$ is the biggest soft P- open set that contained in F_A.

2.20. Definition. [11] Let $(F_E, \tilde{\tau})$ be a soft topological space and $F_A \tilde{\subseteq} F_E$.Then the soft pre closure (soft P-closure) of F_A denoted by $p(\overline{F_A})$ is defined as the soft intersection of all soft P-closed supersets of F_A . Note that, $p(\overline{F_A})$ is the smallest soft P- closed set that containing F_A.

2.21. Proposition. [11] A soft set is soft P-open iff $p(F_A)^o = F_A$.

A soft set is soft P-closed iff $p(\overline{F_A}) = F_A$.

2.22. Definition. [3] A subset B of a topological space X is said to be:

(i) a generalized closed set if $\bar{B} \subseteq U$ whenever $B \subseteq U$ and U is open in X.

(ii) a pre generalized closed set if $p(\bar{B}) \subseteq U$ whenever $B \subseteq U$ and U is pre-open in X.

3. Soft P-generalized closed set

3.1 Definition. Let $F_A \tilde{\in} S(U)$ is said to be soft pre generalized closed set (soft Pg-closed set) if $p(\overline{F_A}) \tilde{\subseteq} F_O$ whenever $F_A \tilde{\subseteq} F_O$ and $F_O \tilde{\in} G_{sp}(F_E)$. The collection of all soft Pg- closed sets is denoted by $F_{spg}\ (F_E)$.

3.2. Example. Let as consider the soft subsets in Example 2.5. Let $(F_A, \tilde{\tau})$ be a soft topological space where $U = \{h_1, h_2\}$, $A = \{e_1, e_2\}$, $F_A = \{(e_1, \{h_1, h_2\}), (e_2, \{h_1, h_2\})\}$ and $\tilde{\tau} = \{F_\emptyset, F_A, F_{A_3}, F_{A_4}\}$. $\tilde{\tau}^{\tilde{c}} = \{F_\emptyset, F_A, F_{A_{10}}, F_{A_7}\}$, $G_{sp}(F_A) = \{F_\emptyset, F_A, F_{A_3}, F_{A_4}, F_{A_6}, F_{A_9}, F_{A_{13}}, F_{A_{14}}, F_{A_{16}}\}$.
$F_{sp}(F_A) = \{F_\emptyset, F_A, F_{A_{10}}, F_{A_7}, F_{A_{12}}, F_{A_8}, F_{A_{12}}, F_{A_{11}}, F_{A_{15}}\}$. Clearly for the soft sub sets F_{A_5}, $p(\overline{F_{A_5}}) = F_{A_{15}} \tilde{\subseteq} F_{A_{13}}$ whenever $F_{A_5} \tilde{\subseteq} F_{A_{13}}$ and $F_{A_{13}} \tilde{\in} G_{sp}(F_A)$ and for the soft subset F_{A_7}, $p(\overline{F_{A_7}}) = F_{A_7} \tilde{\subseteq} F_A$ whenever $F_{A_7} \tilde{\subseteq} F_A$ and $F_A \tilde{\in} G_{sp}(F_A)$. Therefore the soft subsets F_{A_5} and F_{A_7} are soft Pg-closed sets but the soft set F_{A_4}, $p(\overline{F_{A_4}}) = F_A$ is not contained in any soft P-open set. Therefore F_{A_4} is not a soft Pg- closed set.

3.3. Proposition. Every soft P-closed set is a soft Pg-closed set.

Proof: Let F_C be a soft P-closed set, then for every soft P-closed set F_C, $F_C = p(\overline{F_C})$, we have every soft p closed set is soft Pg-closed set. But the converse is not true in general.

3.4. Remark. A soft Pg-closed set need not be a soft P-closed set. The following example supports our claim.

3.5. Example. Let as consider the soft subsets in Example 2.5. Let $(F_A, \tilde{\tau})$ be a soft topological space where $U = \{h_1, h_2\}$, $A = \{e_1, e_2\}$, $F_A = \{(e_1, \{h_1, h_2\}), (e_2, \{h_1, h_2\})\}$ and $\tilde{\tau} = \{F_\emptyset, F_A, F_{A_3}, F_{A_4}\}$. $\tilde{\tau}^{\tilde{c}} = \{F_\emptyset, F_A, F_{A_{10}}, F_{A_7}\}$,$F_{sp}(F_A) = \{F_\emptyset, F_A, F_{A_{10}}, F_{A_7}, F_{A_{12}}, F_{A_8}, F_{A_{12}}, F_{A_{11}}, F_{A_{15}}\}$.Then $F_{spg}(F_A) = \{F_\emptyset, F_A, F_{A_3}, F_{A_5}, F_{A_7}, F_{A_8}, F_{A_{10}}, F_{A_{11}}, F_{A_{12}}, F_{A_{15}}\}$. Then by Example 3.2 soft subset F_{A_5} is a soft Pg-closed set but it is not a soft P- closed set.

3.6. Proposition.

(i) If F_A and F_B are soft pre generalized closed sets then so is $F_A \tilde{\cap} F_B$.

(ii) Let F_A be a soft pre generalized closed set and suppose that F_B is a soft P-closed set. Then $F_A \tilde{\cap} F_B$ is a soft pre generalized closed set.

3.7. Definition. Let $(F_E, \tilde{\tau})$ be a soft topological space and $F_A \tilde{\subseteq} F_E$.Then the soft pre generalized closure (soft Pg-closure) of F_A denoted by $pg(\overline{F_A})$ is defined as the soft intersection of all soft pre generalized closed supersets of F_A .

3.8. Remark. Since the arbitrary soft intersection of soft pre generalized closed sets is a soft pre generalized closed set, $pg(\overline{F_A})$ is soft pre generalized closed set. Note that, $pg(\overline{F_A})$ is the smallest soft pre generalized closed set that containing F_A.

3.9. Theorem. Let $(F_E, \tilde{\tau})$ be a soft topological space and let F_A and F_B be a soft sets over U. Then

(a) $F_A \tilde{\subseteq} pg(\overline{F_A})$

(b) F_A is soft pre generalized closed iff $F_A = pg(\overline{F_A})$

(c) $F_A \tilde{\subseteq} F_B$, then $pg(\overline{F_A}) \tilde{\subseteq} pg(\overline{F_B})$

(d) $pg(\overline{F_\emptyset}) = F_\emptyset$ and $pg(\overline{F_E}) = F_E$.

(e) $pg(\overline{F_A \tilde{\cap} F_B}) \tilde{\subseteq} pg(\overline{F_A}) \tilde{\cap} pg(\overline{F_B})$

(f) $pg(\overline{F_A \tilde{\cup} F_B}) = pg(\overline{F_A}) \tilde{\cup} pg(\overline{F_B})$

(g) $pg\overline{(pg(\overline{F_A}))} = pg(\overline{F_A})$

Proof: Refer theorem 4.16 [11]

3.10. Theorem. A soft subset F_A of a soft topological space $(F_E, \tilde{\tau})$ is soft Pg-closed set iff $p(\overline{F_A})\backslash F_A$ does not contain any non-empty soft pre-closed set. (That is $p(\overline{F_A})\backslash F_A$ contains only null soft pre-closed set.)

Proof: Necessity, Suppose that F_A is soft Pg-closed set. Let F_C be a soft P-closed and $F_C \tilde{\subseteq} p(\overline{F_A})\backslash F_A$. Since F_C is soft P-closed, we have its relative complement $F_C^{\tilde{c}}$ is soft P-open. Since $F_C \tilde{\subseteq} p(\overline{F_A})\backslash F_A$, we have $F_C \tilde{\subseteq} p(\overline{F_A})$ and $F_C \tilde{\subseteq} F_A^{\tilde{c}}$. Hence $F_A \subseteq F_C^{\tilde{c}}$. Consequently $p(\overline{F_A}) \subseteq F_C^{\tilde{c}}$. [Since F_A is soft Pg-closed set]. Therefore, $F_C \tilde{\subseteq} p(\overline{F_A^{\tilde{c}}})$. Hence $F_C = F_\emptyset$. Hence $p(\overline{F_A})\backslash F_A$ contains only null soft P-closed set.

Sufficiency, Let $F_A \tilde{\subseteq} F_O$, where F_O is soft pre open in $(F_E, \tilde{\tau})$. If $p(\overline{F_A})$ is not contained in F_O, then $p(\overline{F_A}) \tilde{\cap} (F_E\backslash F_O) \neq F_\emptyset$. Now, since

$p(\overline{F_A}) \tilde{\cap} (F_E\backslash F_O) \tilde{\subseteq} p(\overline{F_A})\backslash F_A$ and $p(\overline{F_A}) \tilde{\cap} (F_E\backslash F_O)$ is a non-empty soft pre closed set, then we obtain a contradiction and therefore F_A is soft Pg-closed set.

3.11. Theorem (Sufficient condition for soft Pg-closed set is soft P-closed set.)

If F_A is a soft Pg-closed set of a soft topological space $(F_E, \tilde{\tau})$, then the following are equivalent

(i) F_A is soft P-closed set.

(ii) $p(\overline{F_A})\backslash F_A$ is soft P-closed set.

Proof:

Let $(F_E, \tilde{\tau})$ be a soft topological space. Let F_A be soft Pg-closed set.

(i)→(ii)

Let F_A be soft P-closed set also it is a soft Pg-closed set, and then by Theorem 3.10, $p(\overline{F_A})\backslash F_A = F_\emptyset$ which is soft P-closed set.

(ii)→(i)

Let $p(\overline{F_A})\backslash F_A$ be a soft P-closed set and F_A be soft Pg-closed. Then by Theorem 3.10, $p(\overline{F_A})\backslash F_A$ contains only null soft P-closed subset. Since, $p(\overline{F_A})\backslash F_A$ is soft P-closed and $p(\overline{F_A})\backslash F_A = F_\emptyset$. This shows that F_A is soft P-closed set.

3.12. Theorem. For a soft topological space $(F_E, \tilde{\tau})$, the following are equivalent

(i)Every soft subset of F_E is soft Pg-closed set.

(ii)$G_{sp}(F_E) = F_{sp}(F_E)$.

Proof: (i)→(ii)

Let $F_O \tilde{\in} G_{sp}(F_E)$. Then by hypothesis, F_O is soft Pg-closed which implies that $p(\overline{F_O}) \tilde{\subset} F_O$. So $p(\overline{F_O}) = F_O$, Therefore $F_O \tilde{\in} F_{sp}(F_E)$. Also let $F_C \tilde{\in} F_{sp}(F_E)$. Then $F_E \setminus F_C \tilde{\in} G_{sp}(F_E)$, hence by hypothesis $F_E \setminus F_C$ is soft Pg-closed and then $F_E \setminus F_C \tilde{\in} F_{sp}(F_E)$, thus $F_C \tilde{\in} G_{sp}(F_E)$ according above we have $G_{sp}(F_E) = F_{sp}(F_E)$.

(ii)→(i)

If F_A is a soft subset of a soft topological space $(F_E, \tilde{\tau})$ such that $F_A \tilde{\subseteq} F_O$ where $F_O \tilde{\in} G_{sp}(F_E)$, then $F_O \tilde{\in} F_{sp}(F_E)$ and therefore $p(\overline{F_A}) \tilde{\subseteq} F_O$ which shows that F_A is soft Pg-closed set.

3.13. Theorem . If F_A is soft Pg-closed in $(F_E, \tilde{\tau})$ and $F_A \tilde{\subseteq} F_B \tilde{\subseteq} p(\overline{F_A})$, then F_B is soft Pg- closed set.

Proof: Suppose that F_A is soft Pg- closed set in $(F_E, \tilde{\tau})$ and $F_A \tilde{\subseteq} F_B \tilde{\subseteq} p(\overline{F_A})$, Let $F_B \tilde{\subseteq} F_O$ and F_O is soft P-open in $(F_E, \tilde{\tau})$. Since $F_A \tilde{\subseteq} F_B$ and $F_B \tilde{\subseteq} F_O$, we have $F_A \tilde{\subseteq} F_O$. Hence $p(\overline{F_A}) \tilde{\subseteq} F_O$ (since F_A is soft Pg-closed set). Since $F_B \tilde{\subseteq} p(\overline{F_A})$, we have $p(\overline{F_B}) \tilde{\subseteq} p(\overline{F_A}) \tilde{\subseteq} F_O$. Therefore, F_B is soft Pg- closed set.

4. **Soft pre generalized open set**

4.1. Definition. Let $(F_E, \tilde{\tau})$ be a soft topological space. A soft set F_A is called a soft pre generalized open set (soft Pg-open set) if the complement $F_A^{\tilde{c}}$ is soft Pg-closed set in $(F_E, \tilde{\tau})$. The collection of all soft Pg-open sets is denoted by $G_{spg}(F_E)$. Equivalently, a soft set F_A is called a soft Pg-open in a soft topological space $(F_E, \tilde{\tau})$ if and only if $F_C \tilde{\subseteq} p(F_A)^o$ whenever $F_C \tilde{\subseteq} F_A$ and F_C is soft P-closed set in $(F_E, \tilde{\tau})$.

4.2. Example. Let as consider the soft subsets in Example 2.5. Let $(F_A, \tilde{\tau})$ be a soft topological space where $U = \{h_1, h_2\}$, $A = \{e_1, e_2\}$, $F_A = \{(e_1, \{h_1, h_2\}), (e_2, \{h_1, h_2\})\}$ and $\tilde{\tau} = \{F_\emptyset, F_A, F_{A_3}, F_{A_4}\}$. $\tilde{\tau}^{\tilde{c}} = \{F_\emptyset, F_A, F_{A_{10}}, F_{A_7}\}$,$G_{sp}(F_E) = \{F_\emptyset, F_A, F_{A_3}, F_{A_4}, F_{A_6}, F_{A_9}, F_{A_{13}}, F_{A_{14}}, F_{A_{16}}\}$. Clearly for the soft subset $F_{A_{10}}$, $F_\emptyset \tilde{\subseteq} p(F_{A_{10}})^o$ whenever $F_\emptyset \tilde{\subseteq} F_{A_{10}}$ and $F_\emptyset \tilde{\in} F_{sp}(F_A)$ and for the soft subset F_{A_6}, $F_{A_{12}} \tilde{\subseteq} p(F_{A_6})^o = F_{A_6}$ whenever $F_{A_{12}} \tilde{\subseteq} F_{A_6}$ and $F_{A_{12}} \tilde{\in} F_{sp}(F_A)$. Therefore $F_{A_{10}}$, and F_{A_6} are soft Pg-open sets but the soft subset F_{A_5} is not a soft Pg- open set.

4.3. Proposition. Every soft P-open set is a soft Pg-open set

Proof: Let F_O soft P-open set**,** then for every soft P-open set F_O, $F_O = p(F_O)^o$, we have every soft P-open set is soft Pg- open set. But the converse is not true in general.

4.4. Remark. A soft Pg-open set need not be soft P-open set. The following example supports our claim.

4.5. Example. Let as consider the soft subsets in Example 2.5. Let $(F_A, \tilde{\tau})$ be a soft topological space in Example 4.2 where $U = \{h_1, h_2\}$, $A = \{e_1, e_2\}$, $F_A = \{(e_1, \{h_1, h_2\}), (e_2, \{h_1, h_2\})\}$ and $\tilde{\tau} = \{F_\emptyset, F_A, F_{A_3}, F_{A_4}\}$. $\tilde{\tau}^{\tilde{c}} = \{F_\emptyset, F_A, F_{A_{10}}, F_{A_7}\}$,$G_{sp}(F_E) = \{F_\emptyset, F_A, F_{A_3}, F_{A_4}, F_{A_6}, F_{A_9}, F_{A_{13}}, F_{A_{14}}, F_{A_{16}}\}$. Then $G_{spg}(F_E) = \{F_A, F_\emptyset, F_{A_{10}}, F_{A_6}, F_{A_4}, F_{A_9}, F_{A_3}, F_{A_{14}}, F_{A_{13}}, F_{A_{16}}\}$. The soft set $F_{A_{10}}$is soft Pg-open set but it is not a soft P-open set.

4.6. Proposition

(i)If F_A and F_B are soft Pg-open sets then so is $F_A \tilde{\cup} F_B$.

(ii)The soft union of two soft Pg-open set is generally not a soft Pg- open set.

4.7. Proposition

$G_{sp}(F_E, \tilde{\tau}) \tilde{\subseteq} G_{spg}(F_E, \tilde{\tau})$.

Proof: Let F_A be any soft P-open set. Then, $F_A^{\tilde{c}}$ is soft P-closed and hence soft Pg-closed. This implies that F_A is soft Pg-open. Hence $G_{sp}(F_E, \tilde{\tau}) \tilde{\subseteq} G_{spg}(F_E, \tilde{\tau})$.

4.8. Definition. Let $(F_E, \tilde{\tau})$ be a soft topological space and $F_A \tilde{\subseteq} F_E$. Then the soft pre generalized interior (soft Pg-interior) of F_A denoted by $pg(F_A)^o$ is defined as the soft union of all soft pre generalized open subsets of F_A.

4.9. Remark. Since arbitrary soft union of soft pre generalized-open sets is soft pre generalized open set, $pg(F_A)^o$ is a soft pre generalized open set. Note that $pg(F_A)^o$ is the biggest soft pre generalized open set that contained in F_A.

4.10. Theorem. Let $(F_E, \tilde{\tau})$ be a soft topological space and F_A, F_B be a soft sets over U. Then

(a) $pg(F_A)^o \tilde{\subseteq} F_A$.

(b) F_A is soft pre generalized open iff $F_A = pg(F_A)^o$.

(c) $F_A \tilde{\subseteq} F_B$, then $pg(F_A)^o \tilde{\subseteq} pg(F_B)^o$.

(d) $pg(F_\emptyset)^o = F_\emptyset$ and $pg(F_E)^o = F_E$.

(e) $pg(F_A \tilde{\cap} F_B)^o = pg(F_A)^o \tilde{\cap} pg(F_B)^o$.

(f) $pg(F_A \tilde{\cup} F_B)^o \tilde{\supseteq} pg(F_A)^o \tilde{\cup} pg(F_B)^o$.

(g) $pg\,(pg(F_A)^o)^o = pg(F_A)^o$.

(h) (i) $(pg(\overline{F_A}))^{\tilde{c}} = pg(F_A^{\tilde{c}})^o$

(ii) $(pg(F_A)^o)^{\tilde{c}} = pg(\overline{F_A^{\tilde{c}}})$

Proof: Refer Theorem 3.23 [11].

4.11. Theorem . A soft set F_A is soft Pg-open in $(F_E, \tilde{\tau})$ and $p(F_A)^o \tilde{\subseteq} F_O \tilde{\subseteq} F_A$, then F_O is soft Pg- open set.

Proof: Suppose that F_A is soft pre generalized open in $(F_E, \tilde{\tau})$ and $p(F_A)^o \tilde{\subseteq} F_O \tilde{\subseteq} F_A$. Let $F_B \tilde{\subseteq} F_O$ and F_B is soft p-closed set. Since $F_O \tilde{\subseteq} F_A$ and $F_B \tilde{\subseteq} F_O$, we have $F_B \tilde{\subseteq} F_A$. Hence $F_B \tilde{\subseteq} p(F_A)^o$. (Since F_A is soft pre generalized open). Since $p(F_A)^o \tilde{\subseteq} F_O$, we have $F_B \tilde{\subseteq} p(F_A)^o \tilde{\subseteq} p(F_O)^o$. Therefore, F_O is soft pre generalized open.

4.12. Note. Soft Pg-open set implies soft P-open set if it is a soft $PT_{1\backslash 2}$ space.

5. Soft $PT_{1/2}$ – Spaces

In this section, we introduce the new soft lower separation axioms, namely soft pre $T_{1/2}$-space in soft topological space with the help of soft Pg- generalized closed sets.

5.1. Definition. A soft topological space $(F_E, \tilde{\tau})$ is said to be a soft pre $T_{1/2}$ Space ($PT_{1/2}$ –space) if every soft Pg-closed set is soft P-closed set. The spaces where the class of soft P-closed sets and the soft Pg-closed sets coincide.

5.2. Example. Any soft discrete topological space$(F_E, \tilde{\tau})$ is soft $PT_{1/2}$ -space. Since every soft subset of F_E is a soft P-closed sets in a soft discrete topology. Hence every soft subset of F_E is a soft Pg-closed set in a soft discrete topological space.

5.3. Example. Consider a soft topological space $(F_A, \tilde{\tau})$ in Example 2.5. Where $U = \{h_1, h_2\}$, $A = \{e_1, e_2\}$, $F_A = \{(e_1, \{h_1, h_2\}), (e_2, \{h_1, h_2\})\}$ and $\tilde{\tau} = \{F_\emptyset, F_A, F_{A_3}, F_{A_{10}}\}$. The soft sets are defined as follows $F_{A_3} = \{(e_1, \{U\})\}$, $F_{A_{10}}\{(e_2, \{U\})\}$. $G_{sp} = \{F_\emptyset, F_A, F_{A_2}, F_{A_3}, F_{A_4}, F_{A_5}, F_{A_6}, F_{A_7}, F_{A_8}, F_{A_9}, F_{A_{10}}, F_{A_{11}}, F_{A_{12}}, F_{A_{13}}, F_{A_{14}}, F_{A_{15}}, F_{A_{16}}\}$.$F_{sp} = F_{spg} = \{F_\emptyset, F_A, F_{A_2}, F_{A_3}. F_{A_4}, F_{A_5}, F_{A_6}, F_{A_7}, F_{A_8}, F_{A_9}, F_{A_{10}}, F_{A_{11}}, F_{A_{12}}, F_{A_{13}}, F_{A_{14}}, F_{A_{15}}, F_{A_{16}}\}$

Therefore $(F_A, \tilde{\tau})$ is a soft $PT_{1/2}$ -space over U.

5.4. Remark

Every soft $PT_{1/2}$ -space is a soft topological space.

5.5. Remark.

A soft topological space need not be a soft $PT_{1/2}$ -space.

5.6. Example

Let as consider the soft subsets in Example 2.5. Let $(F_A, \tilde{\tau})$ be a soft topological space where $U = \{h_1, h_2\}$, $A = \{e_1, e_2\}$, $F_A = \{(e_1, \{h_1, h_2\}), (e_2, \{h_1, h_2\})\}$ and $\tilde{\tau} = \{F_\emptyset, F_A, F_{A_3}, F_{A_4}\}$. $\tilde{\tau}^{\tilde{c}} = \{F_\emptyset, F_A, F_{A_{10}}, F_{A_7}\}$,$G_{sp}(F_E)\{F_\emptyset, F_A, F_{A_3}, F_{A_4}, F_{A_6}, F_{A_9}, F_{A_{13}}, F_{A_{14}}, F_{A_{16}}\}$. $F_{sp}(F_E) = \{F_\emptyset, F_A, F_{A_{10}}, F_{A_7}, F_{A_{12}}, F_{A_8}, F_{A_{12}}, F_{A_{11}}, F_{A_{15}}\}$. Then $F_{spg}(F_E) = \{F_\emptyset, F_A, F_{A_3}, F_{A_5}, F_{A_7}, F_{A_8}, F_{A_{10}}, F_{A_{11}}, F_{A_{12}}, F_{A_{15}}\}$. $G_{spg}(F_E) = \{F_A, F_\emptyset, F_{A_{10}}, F_{A_6}, F_{A_4}, F_{A_9}, F_{A_3}, F_{A_{14}}, F_{A_{13}}, F_{A_{16}}\}$. The soft topological space $(F_A, \tilde{\tau})$ is not a soft PT$_{1/2}$ space, because the soft Pg- closed F_{A_5} is not a soft P-closed set.

5.7. Theorem. A soft topological space $(F_E, \tilde{\tau})$ is a soft PT$_{1/2}$-space if and only if $G_{sp}(F_E) = G_{spg}(F_E)$ holds.

Proof: Necessity, Since the soft P-closed sets and the soft Pg-closed sets coincide by the assumption, $p\overline{(F_C)} = pg\overline{(F_C)}$ holds for every soft subsets F_C of $(F_E, \tilde{\tau})$. Therefore, we have that $G_{sp}(F_E) = G_{spg}(F_E)$.

Sufficiency, we have that $G_{sp}(F_E) = G_{spg}(F_E)$. Let F_C be a soft Pg closed set. Then, we have $F_C = pg\overline{(F_C)}$ and hence $F_C^{\tilde{c}} \tilde{\in} G_{sp}(F_E)$. Thus F_C is soft P-closed set. Therefore $(F_E, \tilde{\tau})$ is soft PT$_{1/2}$ –space.

5.8. Theorem. A soft topological space $(F_E, \tilde{\tau})$ over U is a soft PT$_{1/2}$ -space if and only if, for each $x \in U$, the soft singleton x_E is either soft P-open or soft P-closed.

Poof: Necessity, Suppose that for some $x \in U$, x_E is not soft P-closed. Since F_E is the only soft P-open set containing$x_E^{\tilde{c}}$, the soft set $x_E^{\tilde{c}}$ is soft Pg-closed and so it is soft P-closed in the soft PT$_{1/2}$ –space $(F_E, \tilde{\tau})$. Therefore x_E is soft P-open set.

Sufficiency, Since $G_{sp}(F_E) \tilde{\subseteq} G_{spg}(F_E)$ holds, by Theorem 5.7, it is enough to prove that $G_{spg}(F_E) \tilde{\subseteq} G_{sp}(F_E)$. Let $F_O \tilde{\in} G_{spg}(F_E)$. Suppose that $F_O \tilde{\notin} G_{sp}(F_E)$. Then, $pg\overline{(F_O^{\tilde{c}})} = F_O^{\tilde{c}}$ and $p\overline{(F_O^{\tilde{c}})} \neq F_O^{\tilde{c}}$ hold. There exists $x \in U$ such that $x \tilde{\in} p\overline{(F_O^{\tilde{c}})}$ and $x \tilde{\notin} F_O^{\tilde{c}} \left(= pg\overline{(F_O^{\tilde{c}})}\right)$. Since $x \tilde{\notin} pg\overline{(F_O^{\tilde{c}})}$ there exist a soft Pg-closed set F_C such that $x \notin F_C$ and $F_C \tilde{\supseteq} F_O^{\tilde{c}}$. By the hypothesis, the soft singleton x_E is soft P-open or soft P-closed set.

Case (i)

The soft singleton x_E is soft P-closed set: Since $x_E^{\tilde{c}}$ is a soft P-closed set with $F_O^{\tilde{c}} \tilde{\subset} x_E^{\tilde{c}}$, we have $p\overline{(F_O^{\tilde{c}})} \tilde{\subset} x_E^{\tilde{c}}$. That is $x \tilde{\notin} p\overline{(F_O^{\tilde{c}})}$. This contradicts the fact that $x \tilde{\in} p\overline{(F_O^{\tilde{c}})}$. Therefore $F_O \tilde{\in} G_{sp}(F_E)$.

Case (ii)

If the soft singleton x_E is soft P-closed: Since $x_E^{\tilde{c}}$ is a soft P-open set containing the soft Pg-closed set F_C $\left(\tilde{\supseteq} F_O^{\tilde{c}}\right)$, we have $x_E^{\tilde{c}} \tilde{\supseteq} p\overline{(F_C)} \tilde{\supseteq} p\overline{(F_O^{\tilde{c}})}$. Therefore $x \tilde{\notin} p\overline{(F_O^{\tilde{c}})}$. This is a contradiction. Therefore $F_O \tilde{\in} G_{sp}(F_E)$. Hence in both cases, we have $F_O \tilde{\in} G_{sp}(F_E)$, that is $G_{spg}(F_E) \tilde{\subseteq} G_{sp}(F_E)$.

5.9. Theorem. If $(F_E, \tilde{\tau})$is a soft topological space, then the following statements are equivalent

(i)$(F_E, \tilde{\tau})$ is a soft PT$_{1/2}$ –space.

(ii)Every soft subset of S (U) is the soft intersection of soft P- open and soft P-closed set containing it.

Proof: (i)→(ii)

If F_E is a soft $PT_{1/2}$ -space with $F_A \tilde{\subseteq} F_E$, then $F_A = \tilde{\cap} \{F_E \setminus x_E : x \notin F_A\}$ is the intersection of soft P-open and soft P-closed set containing it.

(ii)→(i)

For each $x \in U, F_E \setminus x_E$ is the soft intersection of all soft P-open and soft P-closed sets containing it. Hence $F_E \setminus x_E$ is either soft P-open or soft P-closed. Therefore by Theorem 5.8, $(F_E, \tilde{\tau})$ is a soft $PT_{1/2}$ -space.

5.10. Proposition. The soft subspace of a soft $PT_{1/2}$ –space is soft $PT_{1/2}$-space.

Proof: Let V is a soft subspace of a soft pre-$T_{1/2}$ space over U and $x \in V \subseteq U$. Then x_E is soft p-open or soft pre closed in$(F_E, \tilde{\tau})$. (by Theorem 5.8) Therefore x_E is either soft P-open or soft P-closed set in V. Hence V is a soft $PT_{1/2}$ -space.

Conclusion: The initiation of notation of soft topological space was introduced by D.Molodtsov [5] in 1999. Many Mathematicians turned their attention to the various concepts of soft topological space. By this way, [6] Muhammad Shabir and Munazza Naz introduced the concept soft topological spaces. In 2013 J.Subhashini and C.Sekar [11] introduced the concept of soft pre-open sets. In this paper, we continue this work and introduce soft pre generalised -closed sets taking help of the soft pre open sets. Also we introduce soft $PT_{1/2}$ -Space. We discuss the relation between soft pre generalised closed sets and soft $PT_{1/2}$ spaces. Finally we prove every soft topological space need not be a soft $PT_{1/2}$ – Spaces.

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