MHD Free Convective Heat and Mass transfer of fluid flow past a moving variable surface in porous media

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Abstract: This paper is focused to developed a mathematical model and comparative study of combined effects of free convective heat and mass transfer on the steady two-dimensional, laminar fluid flow past a moving permeable vertical surface in a porous media subject to a transverse uniform magnetic field. The surface is maintained at linear temperature and concentration variations. Using a similarity transformation, the nonlinear coupled differential equations governing the boundary layer flow, heat and mass transfer are first reduced to a system of coupled ordinary differential equations and then solved numerically using an implicit finite difference technique. The effects of different parameters such as magnetic field strength parameter (M), Schmidt number (Sc), buoyancy ratio (N), suction /blowing parameter (fw), darcy number (Da) are examined in detail to show the interesting aspects of the solution. Numerical result for the dimensionless velocity profiles, the temperature profiles, the concentration profiles, the local Nusselt number, sherwood number, skin friction coefficient are presented. The interesting numerical results indicate that the buoyancy parameter, magnetic parameter and suction/blowing parameter have a retarding effect on the velocity, temperature and concentration profiles.

I. Introduction

The Fluid flow and heat transfer through a porous medium has been extensively studied in the past because of its relevance to nuclear waste disposal, solid matrix heat exchanger, thermal insulation and other practical application. Natural convective flows are frequently encountered in physical and engineering problems such as chemical catalytic reactors, nuclear waste materials, geothermal system etc. Simultaneous heat and mass transfer from different geometries embedded in porous Medium has many Engineering and Geophysical applications such as geothermal reservoirs, drying of porous solids, thermal insulation, enhanced oil recovery, packedbed catalytic reactors, cooling of nuclear reactors, and underground energy transport. The science of magneto hydrodynamics (MHD) was concerned with geophysical and astrophysical problem for a number of years. In recent years, the possible use of MHD is to affect a flow stream of an electrically conducting fluid for the purpose of thermal protection, braking, propulsion and control. From the point of applications, model studies on the effect of magnetic field on the convection flow have been made by several investigators. From technological point of view, MHD convection flow problems are also very significant in the field of stellar and planetary magnetospheres, aeronautics, chemical engineering and electronics. Model studies of the above phenomena of MHD convection have been made by many.

Convection in porous media has applications in geothermal energy recovery, oil extraction, thermal energy storage and flow through filtering devices, Nield and Bejan. From technological point of view, MHD free convection flows have significant applications in the field of stellar and planetary magnetosphere, aeronautics, chemical engineering and electronics on account of their varied importance. Bejan and Khair (1985) investigated the free convection boundary layer flow in a porous medium owing to combined heat and mass transfer. Lai and Kulacki (1990) used the series expansion method to investigate coupled heat and mass transfer in natural convection from a sphere in a porous medium. The suction and blowing effects on free convection coupled heat and mass transfer over a vertical plate in a saturated porous medium were studied by Raptis et al.(1981) and Lai and Kulacki (1991).

Recently K. Jayarami Reddy et al(2012) analyzed a steady two-dimensional MHD free convection flow viscous dissipating fluid past a semi-infinite moving vertical plate in a porous medium with Soret and Dufour effects. A study on MHD heat and mass transfer free convection flow along a vertical stretching sheet in the presence of magnetic...
field with heat generation was carried out by Samad and Mohebujjaman [2009]. Saravana et al. [2011] studied the mass transfer effects on MHD viscous flow past an impulsively started infinite vertical plate with constant mass flux.

In further studies by Satya Sagar Saxena et al. [2011] investigates the unsteady two-dimensional magnetohydrodynamic heat and mass transfer free convection flow of an incompressible, viscous, electrically conducting polar fluid via a porous medium past a semi-infinite vertical porous moving plate in the presence of a transverse magnetic field with thermal diffusion and heat generation is considered. The plate moves with a constant velocity in the longitudinal direction, and the free stream velocity follows an exponentially increasing or decreasing small perturbation law. A uniform magnetic field acts perpendicularly to the porous surface which absorbs the polar fluid with a suction velocity varying with time.

Recently, V. Ravikumar et al. [2012] investigated the heat and mass transfer effects on MHD flow of viscous incompressible and electrically conducting fluid through a non-homogeneous porous medium in the presence of heat source, oscillatory suction velocity. A uniform transverse magnetic field is applied in the direction of the flow perpendicular to the plates. S. K. Ghosh [2012] study the boundary layer flow of a steady incompressible and visco-elastic fluid with short memory (obeying Walters’ B fluid model) passing over a hot vertical porous plate in the presence of transverse magnetic field. In addition Tiwari and Kapoor [2011] has done some work in this direction but this work is basically the extension of Rawat and Kapoor [2012] in which they considered the viscous model of the above problem they also shown the effect of radiation in there study. Here we have taken the porous model without the taken the external effect.

MATHEMATICAL MODEL
Let us consider two-dimensional free convective effect on steady incompressible laminar MHD heat and mass characteristics of a linearly moving permeable vertical surface in porous media when the velocity of the fluid far away from the plate is equal to zero. The variation of surface temperature and concentration are linear. All the fluid properties are assumed to be constant except for the density variations in the buoyancy force term of linear momentum. The magnetic Reynolds number is assumed to be small, so that the induced magnetic field is neglected. No electric field is assumed to exists and both viscous and magnetic dissipation are neglected. The Hall Effect, viscous dissipation and the joule heating term are neglected. Under these assumption along with the Boussinesq approximation, the boundary layer equation for the problem.

II. GOVERNING EQUATION

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]

\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{2}{\sqrt{\xi}} \left( 2 \frac{\partial u}{\partial x} + g \beta T \frac{\partial T}{\partial x} + g \beta_s \frac{\partial S}{\partial x} - S_{x} \right) - \frac{1}{\kappa} \frac{1}{\kappa} u
\]

\[
u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\partial^2 T}{\partial y^2}
\]

\[
u \frac{\partial S}{\partial x} + v \frac{\partial S}{\partial y} = \frac{\partial^2 S}{\partial y^2}
\]

Where \( u \) and \( v \) are the velocity components in X-and Y-directions, respectively, \( T \) is the temperature, \( \beta_x \) is the volumetric coefficient of thermal expansion, \( \alpha_x \) is the thermal diffusivity, \( g \) is the acceleration due to gravity under the influence of \( \xi \) is the kinematic viscosity, \( D \) is the coefficient of diffusion in the mixture, \( c \) is the species, \( \sigma \) is the electrical conductivity, \( B_0 \) is the externally imposed magnetic field in the Y-direction. The relevant boundary conditions can be written as

\[
Y = 0, v = -\nu_{\infty}, u = B X, T = T_{x} + a X, S = S_{x} + b X
\]

\[
Y \rightarrow \infty, u = 0, T = T_{\infty}, S = S_{\infty}
\]

Where \( \nu_{\infty} \) is the uniform surface mass flux, \( B, a, b \) are prescribed constant.

we introduce now the following non-dimensional variables

\[
u = \frac{\xi}{\beta x}, \eta = \frac{\sqrt{\xi}}{X}, \text{F(}\eta\text{)} = \frac{1}{\sqrt{\xi}} \frac{\eta}{X}
\]

\[
\Theta(\eta) = \frac{T - T_{\infty}}{T_{\infty} - T_{x}},c(\eta) = \frac{S - S_{\infty}}{S_{\infty} - S_{x}}
\]

With a new set of independent variable defined by Eq(7), Eq(1), is identically satisfied, and the partial differential equations (2)-(4) transform in to ordinary differential equation (8)-(10).
\[ F'''' + F F' - \left( M + F' + \frac{1}{D_{a}} \right) F' = \frac{G_{r\tau}}{R^2(\Theta + N c)} \]  

(8)

\[ \Theta' + P_{r}(F\Theta' - F'\Theta) = 0 \]  

(9)

\[ C' + S_{c}(F C' - F' C) = 0 \]  

(10)

Where primes denotes differentiation with respect to \( \eta \). The appropriate flat plate, free convection boundary conditions are also transformed into the form,

\[ \eta = 0 : F = F_{w}, F' = 1, \Theta = 1, C = 1 \]  

(11)

\[ \eta \rightarrow \infty : F' = 0, \quad \Theta = 0, C = 0 \]  

(12)

The velocity components are

\[ u = B X F', \quad v = -\sqrt{B_{x}F} \]

Where \( M = \sigma B_{x}B^{-1} \rho^{-1} \) is the magnetic law. In addition, In order to verify the accuracy of our presented method, a comparison of wall velocity gradient \( F(0) \) for various value of Hartmenn number(M) with those reported Takhar and yih The result of comparison is given in Table-1.

In Fig-2/3, we shown the effect of suction and blowing parameter on velocity and temperature parameter while fixing some of the parameter at \( Re=100; \ N=1; \ Da=10^{-2}; \ Sc=70; \ Pr=7; \ M=1; \ G_{r\tau}=5 \). Here we have seen that the suction and blowing parameter is decelerate the fluid particle which is expected from this parameter. Here the graphical representation shows that velocity is maximum in the domain of characteristics length \( \eta \in [0,1] \) which is maximum up to 1. And as the length is increases the velocity is seems to converges asymptotically somewhere around 8. Here it is also be point to be noted that as we increases the parameter value of \( F_{w} \) the magnitude of the velocity is decreases smoothly. Here the point of separating the fluid velocity is also being negligible .Fig 3 shows as the fluid particle decelerate then the temperature of the fluid particle is laid down which is expected from this parameter. Here the graphical representation shows that temperature is maximum in the domain of characteristics length \( \eta \in [0,1,5] \) which is maximum up to 1. And as the length is increases the temperature is seems to converge but not asymptotically like the velocity somewhere around 8. Here it is also be point to be noted that as we increases the parameter value of \( F_{w} \) the magnitude of the temperature is decreases smoothly. This flatness characteristics is also be found due the hardness of the porous media , due the porous medium the drag force is also decelerate the fluid motion it makes the flow as a creeping flow.

In Fig4/5, we shown the effect of Buoyancy parameter (N) on the non dimensional velocity temperature and concentration parameter while fixing some of the parameter at \( Re=100; \ F_{w}=1; \ Da=10^{-2}; \ Sc=70; \ Pr=7; \ M=1; \ G_{r\tau}=5 \). Buoyancy parameter (N) is basically the ratio of solute source intensity to heat source intensity the definition is basically shows the domination of the actual physical situation like heat or mass. The low buoyancy means the high heat source intensity is dominated and the high buoyancy means the high solute source intensity is dominated. Here we have taken the range of buoyancy from 0.5 to 2. The parametric value \( N=0.5 \) means that heat source intensity is twice the solute source intensity as we found that increases the N means increases the velocity. Here we have taken \( Pr=7 \) means that the fluid is water but \( Sc=70 \) means the water is not the saline water i.e the double diffusive components are not
binary mixture. It means that the heat source intensity is more effective than the solute which also be depreciated in the Fig. in Fig-5. The low buoyancy means the high heat source intensity is dominated and the high buoyancy means the high solute source intensity is dominated. Here we have taken the range of buoyancy from 0.5 to 2. The parametric value N=0.5 means that heat source intensity is twice the solute source intensity as we found that increases the N means decreases the temperature. In this case the physical condition shows how heat transfer is affected on increasing the buoyancy. But in case of Fig-6, the behavior of the concentration is just opposite. Here we found that as we increase the value of N the concentration is decreases but not rapidly as found in the case of temperature.

In Fig-7, we shown the effect of Schimdt number (Sc) on the non dimensional parameter concentration while fixing some of the parameter at Re=100; Fr=1; Da=10^-2; N=1; Pr=7; M=1; Gr=5.. Schimdt number is basically viscous diffusion rate divided by molecular mass diffusion rate. It means the high schimdt number shows that the low mass diffusion rate is in action if Pr is fixed. This parameter shows that as we increases the value Sc it decelerate the

For the onset convection in hydrodynamics condition, some parameter plays an important role. Sizable influence on the flow and thermal fields can be produce with moderate magnetic field strengths only for the liquid metal flow while the effect of induced magnetic fields and Joule heating are very small. The magnetic field strength is to reduce the value of wall shear stress. Magnetic force is called retardation effect, similarly Blowing effect is effected here. On the other hand suction has the opposite effect and its incersed the wall shear stress. For the physical point of view different value of Darcy number, Hartmen number, Schimdt number, Nusselet and Sherwood number is presented.

Fig8 depicted the rate of heat transfer, for the different value of N at small range of darcy number. Here the range of darcy number is taken between 0.001 to 1. Here as we know from the definition of darcy number the rate of heat transfer is expected. It is seems to be constant in the entire domain. Since in the large amount of drag force the rate change of heat transfer is seems to constant. In this case the Reynolds number is also taken small somewhere around 500, in which we also found that the buoyancy is also increases the rate of heat transfer.

Fig9 Indicate the rate of mass transfer, for the different value of N at small range of darcy number. Here the range of darcy number is taken between 0.001 to 1. Here as we know from the definition of darcy number the rate of heat transfer is expected. It is seems to be constant in the entire domain. Since in the large amount of drag force the rate change of mass transfer is seems to constant. It is also be important to mention that the rate of mass transfer is greater then the rate of heat transfer. From the fig we have found that the rate of mass transfer is almost 3 times of the rate of heat transfer.

Fig10 shows the rate of heat transfer. It is clearly seen that the influence of M on the local Nusslet number is considerable. The results of figure indicate that, a larger M is experienced for a system with a larger M. That is, the heat transfer rate increases with the increase in magnetic number. This is due to the fact that a larger Magnetic number corresponds to a thinner heating on boundary point relative to the flow boundary layer thickness. This results in a larger heat transfer rate at the duct wall or a larger Nusslet number. There is large amount of fluctuation is noted down in the domain of M when \( 0 < M < 45 \). In which heat transfer is rapidly grow up and its fall down and remain constant for \( 45 < M \). This phenomena is repeat for all different cases of buoyancy N=0 to 10.

Fig11 shows the rate of mass transfer. It is clearly seen that the influence of Sc on the local Sherwood number is considerable. The results of figure indicate that, a larger Sh is experienced for a system with a larger Sc. That is, the mass transfer rate increases with the increase in Schmidt number. This is due to the fact that a larger Schmidt number corresponds to a small binary diffusion coefficient for a given mixture and to a thinner concentration boundary layer thickness relative to the flow boundary layer thickness. This results in a larger mass transfer rate at the duct wall or a larger Sherwood number. As Buoyancy number increases in it. There is some fluctuation is also noted down in the domain of M when \( 0 < M < 15 \). In which mass transfer is rapidly down a then grow up this is happen in the case of when buoyancy N=0 it means that the heat – solutal ratios is neglected same is happen but its lesser then at the case N=1 it shows that as the buoyancy increased that fluctuation is die out and the rate of mass transfer is seems to be constant for the large value of M.

<table>
<thead>
<tr>
<th>Magnetic</th>
<th>M=0</th>
<th>M=0.5</th>
<th>M=1.0</th>
<th>M=1.5</th>
</tr>
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Table-1: Comparison of non dimensionall wall velocity gradient for the value of Hartmen number (M)
number(M)  
Takhar[14]  -1.0  -1.22  -1.41  -1.57  
Yih[15]  -1.0  -1.2247  -1.412  -1.501  
Adalak[16]  -1.0  -1.2356  -1.4142  -1.497  
Present work  -1.0  -1.23573  -1.4136  -1.5011

Fig 2: Effect of Suction parameter on velocity at Re=100; N=1; Da=10^{-2}; Sc=70; Pr=7; M=1; Grt=5

Fig 3: Effect of Suction parameter on Temperature at Re=100; N=1; Da=10^{-2}; Sc=70; Pr=7; M=1; Grt=5

Fig 4: Effect of Buoyancy parameter on velocity at Re=100; F_w=1; Da=10^{-2}; Sc=70; Pr=7; M=1; Grt=5

Fig 5: Effect of Buoyancy parameter on Temperature at Re=100; F_w=1; Da=10^{-2}; Sc=70; Pr=7; M=1; Grt=5

Fig 6: Effect of Buoyancy parameter on Concentration at Re=100; F_w=1; Da=10^{-2}; Sc=70; Pr=7; M=1; Grt=5

Fig 7: Effect of Schimdt parameter on Concentration at Re=100; F_w=1; Da=10^{-2}; N=1; Pr=7; M=1; Grt=5
V. CONCLUSIONS

In this work our objective is to develop the mathematical model of, the heat and mass transfer in MHD flow of moving permeable vertical surface in porous media and solve it numerically using FDM. Here for large value of darcy number, in presence of magnetic field, the velocity is found to be decreases, associated with reduction in the velocity gradient at the wall. Also, the applied magnetic filed tends to decreases the wall temperature gradient and concentration.

The results are presented: for the magnetic parameter, the Prandtl number, the dimensionless suction/blowing coefficient, Schmidt number and buoyancy ratio a systematic study on the effects of the various parameters on flow, heat and mass transfer characteristics is carried out.

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VI. REFERENCES


