Optimal Power Allocation Techniques using SVD for MIMO-OFDM Multimedia Mobile Networks

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Abstract - Energy efficiency in MIMO-OFDM technology is one of the hot topics in mobile multimedia mobile communication. An energy efficient power allocation for MIMO-OFDM systems using Singular Value Decomposition (SVD) of channel matrix is taken as reference for the development of this article. SVD of the channel matrix is computed for each MIMO sub-channel for all sub-carriers. In the reference article a marginal probability density function (pdf) based technique is used to calculate the optimal power allocation threshold. This method involves complex integration requiring a lot of computing recourses. A closed form solution for threshold calculation equation is non-existing in many scenarios. Another limitation of the existing method is that this does not guarantee Shannon Capacity.

A simplified power allocation using well known Water Filling technique is proposed as an alternative. In water filling method, more power is allocated to channels with higher gain, maximizing total capacity with a peak power constraint. A closed form solution for power allocation threshold using water filling at each sub-channel in all OFDM sub-carriers is derived. A "2D" water filling algorithm considering all the sub-channels in all OFDM sub-carriers is derived to further enhance the capacity and thereby the energy efficiency. The advantage of water filling methods is that these can guarantee Shannon capacity. The effective capacity and energy efficiency using average power allocation, water-filling power allocation and "2D" water filling algorithms has been simulated. The comparison shows significant improvement in effective capacity and energy efficiency using the proposed methods compared to the traditional average power allocation.

Keywords — *MIMO, OFDM, Singular Value decomposition, Water Filling, QoS Constraints.*

I. INTRODUCTION

Wireless communication, in particular, mobile communications has become increasingly popular with consumers in the last decades. The extensive growth in the number of users, new products and services, and rising service usage times are resulting in increased demand for energy consumption in information and communication technology (ICT) industry. This causes about 2% of world-wide CO_2 emission yearly [1]. Motivated by the demand for improving the energy efficiency in mobile multimedia communication systems, various resource allocation optimization schemes aiming at enhancing energy efficiency have become one of the mainstreams in mobile multimedia communication systems. Some of such optimization schemes are transmission power allocation, bandwidth allocation, sub-channel allocation and Multiple-input–multiple-output (MIMO) systems.

MIMO technologies can create independent parallel channels to transmit data streams, which improves spectral efficiency and system capacity without increasing the bandwidth requirement. One of the attractive features of MIMO system is the spatial multiplexing gain and consequently a higher capacity performance over single-input single-output system. In spatial multiplexing, data transmission is carried out by multiple parallel channels between transmitter and receiver. Total capacity of MIMO system is given by sum of individual capacity of all parallel channels.

Orthogonal frequency-division multiplexing (OFDM) is a technology widely used in communication systems to eliminate the multipath effect by transforming frequency-selective channels into flat channels. The combination of MIMO and OFDM technologies are widely used in multimedia mobile networks.

Quality of Service (QoS) constraints is another parameter which can affect the energy efficiency of MIMO-OFDM multimedia communication channel.

Different models of energy efficient power allocation schemes for MIMO-OFDM communication systems have been proposed in the last decade. An energy efficient power allocation technique with QoS constraint using marginal probability density function (pdf) of channel gain [1] is studied in this work and an alternate simplified power allocation using well known water filling power allocation is proposed. A modified version of water filling power allocation with improved effective channel capacity is also presented in this work.

The remaining sections of this article are organized as follows. The system model is explained in Section II and a brief explanation of SVD for channel matrix is provided in section III. The studies on the reference article are presented in section IV. The proposed alternate power allocation using water filling method is explained in section V. In section VI the modified "2D" water filling power allocation method is explained. Simulation algorithm is explained in section VII and the simulation results are provided in section VIII. Conclusions and future work is provided in section IX.

II. SYSTEM MODEL

A simplified model of MIMO-OFDM system is shown in Fig. 1. The list of major parameters considered in the development of this work is listed in Table 1.**Error! Reference source not found.**

Table 1 : Major System Parameters

Si. No.	Symbol	Description
1	M_T	No. of Transmit Antennas
2	M_R	No. of Receive Antennas
3	M	Min (M_T, M_R)
4	N	No. of OFDM Subcarriers
5	S	No. of OFDM symbols
6	B	System Bandwidth
7	T_f	Frame Duration
8	θ	QoS Statistical Exponent

The OFDM symbols are assumed to be transmitted within the frame duration. The received signal can be expressed as

$$y_k[i] = H_k x_k[i] + n$$

Where $y_k[i]$ and $x_k[i]$ are the received signal vector and transmitted signal vector at the k^{th} sub-carrier of the i^{th} OFDM symbol respectively, H_k is the frequency domain channel matrix at k^{th} sub-carrier and n is the additive noise vector.

The channel in consideration is block fading channel with channel coherence time greater than the frame duration. With this assumption the channel gain is considered constant within the frame duration, but varies independently from frame to frame [2]. Transmitters are assumed to obtain perfect Channel State Information (CSI) from receivers via feedback channels without delay. The gain of each subchannel for power allocation technique is calculated using Singular Value Decomposition (SVD) of the channel matrix. Power constraint is another important parameter used in power allocation techniques, some models use peak power constraint while some use average power constraint. Peak power constraint is considered in the development of this work.

III.SINGULAR VALUE DECOMPOSITION

Singular Value Decomposition (SVD) is matrix decomposition technique in which a rectangular matrix is broken down to the product of three matrices – two orthogonal matrices and a diagonal matrix [3],[11].

$$A_{mn} = U_{mm} S_{mn} V_{nn}^H$$

where A_{mn} is a rectangular matrix, U_{mm} and V_{nn} are orthogonal square matrices and S_{mn} is a diagonal matrix with the Eigen values of A_{mn} as the diagonal elements.

From the expression it can be clearly seen that SVD transforms correlated variables into a set of uncorrelated variables [4]. Eigen values of channel matrix directly give the gain of the subchannels in the MIMO system. The orthogonal matrices can be easily nullified in transmitter and receiver by multiplying with the Hermitian transpose of the corresponding matrix. Thus SVD can be used to calculate uncorrelated subchannel gains of the MIMO system from the correlated channel matrix. SVD of a MIMO-OFDM channel gives M x N parallel Single Input Single Output (SISO) channels [5] converting the complex multi channel joint optimization problem for power allocation of MIMO system to simple multi target single channel optimization problem.

IV.RELATED WORK

Energy Efficient Optimal Power Allocation (EEOPA) technique proposed by Xiaohu Ge, et. al in [1] is taken as the reference in the development of this work. A marginal pdf based power allocation technique with average power constraint is considered in this system. The SVD of the channel matrix is computed at each sub-carrier giving MxN space frequency sub-channels. These subchannels are sorted in the decreasing order of gains. The sorted subchannels with the same order position in all the subcarriers are assigned to the same group. Total M groups are obtained after sub-channel grouping.

The subchannel groups follow a marginal pdf. Optimal power allocation threshold for each subchannel group is calculated considering the marginal pdf of the subchannel in consideration and the QoS exponent as given in the equation below.

$$\int_{\Delta_n}^{\infty}\left(\frac{1}{\Delta_n^{\frac{1}{\beta+1}}\lambda^{\frac{\beta}{\beta+1}}}-\frac{1}{\lambda}\right)p\Gamma_n(\lambda)d\lambda \le \overline{P}$$

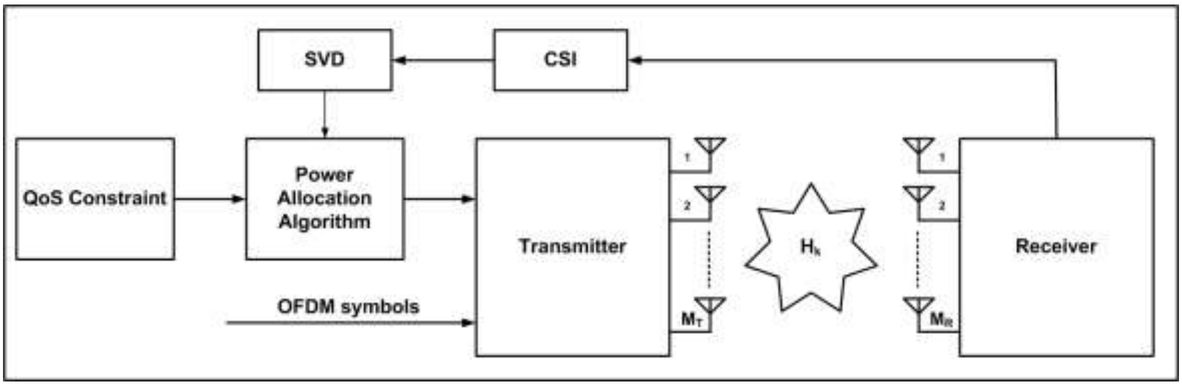

Fig. 1 : System Model

where Δ_n is the power allocation threshold for group n, $\beta = \frac{\theta T_f B}{\log 2}$ is the normalized QoS constraint, λ is the subchannel gain, $p\Gamma_n(\lambda)$ is the marginal pdf of group n and $\overline{P}$.

Power is allocated to sub-channels in each group with gain higher than the calculated threshold for the corresponding subchannel group using the equation below.

$$\mu_{\mathrm{opt\_n}}(\theta,\lambda) = \begin{cases} \dfrac{1}{\Delta_n^{\frac{1}{\beta+1}}\lambda^{\frac{\beta}{\beta+1}}} - \dfrac{1}{\lambda}, & \lambda \geq \Delta_n \\ 0, & \lambda < \Delta_n \end{cases}$$

The optimized effective capacity for each group can be calculated as given in the equation below.

$$C_e(\theta)_{\mathrm{opt\_n}} = \frac{N}{\theta}\log\left(\int_0^{\infty} e^{-\theta T_f B \log_2\left(1+\mu_{\mathrm{opt\_n}}(\theta,\lambda)\lambda\right)}\, p\Gamma_n(\lambda)\,d\lambda\right)$$

The energy efficiency of the MIMO-OFDM system can be calculated as follows

$$\eta_{\mathrm{opt}} = -\frac{1}{\theta \times \overline{P} \times M} \times \sum_{n=1}^{M}\log\left(\int_0^{\infty} e^{-\theta T_f B \log_2\left(1+\mu_{\mathrm{opt\_n}}(\theta,\lambda)\lambda\right)}\, p\Gamma_n(\lambda)\,d\lambda\right)$$

In this method, power allocation threshold is calculated using marginal pdfs of the grouped subchannels. This method involves complex integration of lengthy equations which require a lot of computing resources. In many instances a closed form solution for threshold calculation equation does not exist. Another drawback of this system is that this does not always guarantee Shannon Capacity. In low QoS regime, more power is allocated to subchannels with lower gains in this method to maintain the average power constraint, the reason for not achieving Shannon capacity.

V. WATER FILLING POWER ALLOCATION

As an alternate to the Energy Efficient Optimal Power Allocation (EEOPA) technique proposed by Xiaohu Ge, et. al in [1], a simplified power allocation using well known water filling algorithm is explained in this section.

The advantage of water filling technique is that optimal power is allocated to each transmission stream to maximize capacity thereby achieving Shannon capacity. In water filling method the optimal power allocation threshold is calculated a constraint maximization problem given in equation below

$$max_{p_i}\left(\sum_{i=1}^{t}\log_2\left(1+\frac{\sigma_i^2 P_i}{\sigma_N^2}\right)\right)$$

subject to the constraint

$$\sum_{i=1}^{t} P_i = P_{max}$$

where σ_i^2 is the gain, P_i is the power allocated, σ_N^2 is the noise power of channel i and P_{max} is the maximum power constraint per subcarrier.

The solution is given by the calculation of power allocation threshold $\frac{1}{\mu}$ according to the equation below [5], [6].

$$\sum_{i=1}^{t}\left(\frac{1}{\mu}-\frac{\sigma_N^2}{\sigma_i^2}\right)^{+} = P_{max}$$

where

$$X^{+} = \begin{cases} x, & x > 0 \\ 0, & x \leq 0 \end{cases}$$

In MIMO-OFDM systems with N subcarriers and M subchannels, power allocation threshold for each subcarrier is calculated as follows

$$\sum_{i=1}^{M}\left(\frac{1}{\mu_k}-\frac{\sigma_N^2}{\lambda_{k,i}}\right)^{+} = P_{max}$$

where $\lambda_{k,i}$ is the gain of i^th^ subchannel in k^th^ subcarrier, $\frac{1}{\mu_k}$ is the threshold for k^th^ subcarrier, $(i = 1,2,\ldots M, k = 1,2,\ldots N)$.

Once threshold is calculated, next step is to allocate power to subchannels in each subcarrier as per the equation below.

$$P_{k,i} = \begin{cases} \left(\dfrac{1}{\mu_k}-\dfrac{\sigma_N^2}{\lambda_{k,i}}\right), & \dfrac{1}{\mu_k} > \dfrac{\sigma_N^2}{\lambda_{k,i}} \\ 0, & \dfrac{1}{\mu_k} \leq \dfrac{\sigma_N^2}{\lambda_{k,i}} \end{cases}$$

where $P_{k,i}$ is the power allocated to i^th^ subchannel in k^th^ subcarrier.

Effective capacity after water filling power allocation can be calculate as per the equation below.

$$C_{eff} = \sum_{k=1}^{N}\sum_{i=1}^{M}\log_2\left(1+\frac{P_{k,i}\lambda_{k,i}}{\sigma_N^2}\right)$$

Energy efficiency can be calculated as given below.

$$\eta = \frac{C_{eff}}{N \times P_{max}}$$

In the proposed model, SVD of subchannel matrix is computed for each subcarrier in MIMO-OFDM system and water filling is performed at each subcarrier. Thus Shannon capacity is achieved at subcarrier level.

Water filling power allocation algorithm is given in the **Error! Reference source not found.**.

Table 2: Water Filling Power Allocation Algorithm

```
Water Filling Power Allocation Algorithm
Inputs : $H_k$, $\sigma_N^2$, $P_{max}$
Assuming noise power $\sigma_N^2$ same in all subchannels
1) For k = 1:N
2) Sort M subchannels in each subcarrier in the increasing order of $\frac{\sigma_N^2}{\lambda_{k,i}}$, where $\lambda_{k,i}$ is the gain of i^th subchannel in k^th subcarrier, i = 1:M
3) For m = M:1
4) Calculate power allocation threshold as $\frac{1}{\mu_k} = \frac{P_{max} + \sum_{n=1}^{m} \frac{\sigma_N^2}{\lambda_{k,n}}}{m}$
5) If $\frac{1}{\mu_k} > \frac{\sigma_N^2}{\lambda_{k,m}}$, allocate power to n sorted subchannels as $P_{k,n} = \frac{1}{\mu_k} - \frac{\sigma_N^2}{\lambda_{k,n}}$ where n = 1: m
6) Else, m = m-1 and go to step 3.
7) Calculate effective capacity
   $C_{eff} = \sum_{k=1}^{N}\sum_{i=1}^{M} \log_2\left(1 + \frac{P_{k,i}\lambda_{k,i}}{\sigma_N^2}\right)$
8) Calculate energy efficiency $\eta = \frac{C_{eff}}{N \times P_{max}}$
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VI. “2D” Water Filling Power Allocation

For the water filling power allocation discussed in the section above, power allocation is done for the subchannels in each subcarrier. To further enhance the effective capacity, a modified “2D” water filling algorithm is proposed. In this method water filling power allocation is performed considering all the subchannels in all subcarriers together, thereby performing water filling in two dimensions – along the subchannels and along the subcarriers. This method gives the additional advantage of selecting subchannels with higher gain in all subcarriers thereby enhancing the effective capacity.

In this method, power allocation threshold for all subcarriers is calculated as follows

$$\sum_{\substack{i=1:M\\k=1:N}} \left(\frac{1}{\mu} - \frac{\sigma_N^2}{\lambda_{k,i}}\right)^+ = N \times P_{max}$$

where $\lambda_{k,i}$ is the gain of i^th sub channel in k^th subcarrier, $\frac{1}{\mu}$ is the threshold for all subcarriers, $(i = 1,2,\ldots M, k = 1,2,\ldots N)$.

Once threshold is calculated, next step is to allocate power to subchannels in all subcarriers as per the equation below.

$$P_{k,i} = \begin{cases} \left(\frac{1}{\mu} - \frac{\sigma_N^2}{\lambda_{k,i}}\right), & \frac{1}{\mu} > \frac{\sigma_N^2}{\lambda_{k,i}} \\ 0, & \frac{1}{\mu} \le \frac{\sigma_N^2}{\lambda_{k,i}} \end{cases}$$

where $P_{k,i}$ is the power allocated to i^th subchannel in k^th subcarrier.

Effective capacity after water filling power allocation can be calculate as per the equation below.

$$C_{eff} = \sum_{k=1}^{N}\sum_{i=1}^{M} \log_2\left(1 + \frac{P_{k,i}\lambda_{k,i}}{\sigma_N^2}\right)$$

Energy efficiency can be calculated as given below

$$\eta = \frac{C_{eff}}{N \times P_{max}}$$

In this method, SVD of subchannel matrix is computed for each subcarrier in MIMO-OFDM system and water filling is performed for all subcarriers together. Thus Shannon capacity is achieved at system level.

“2D” water filling power allocation algorithm is given in the **Error! Reference source not found.Error! Reference source not found.**.

Table 3: “2D” Water Filling Power Allocation Algorithm

```
“2D” Water Filling Power Allocation Algorithm
Inputs : $H_k$, $\sigma_N^2$, $P_{max}$
Assuming noise power $\sigma_N^2$ same in all subchannels
1) Sort NxM subchannels in all subcarrier in the increasing order of $\frac{\sigma_N^2}{\lambda_{k,i}}$, where $\lambda_{k,i}$ is the gain of i^th subchannel in k^th subcarrier, i = 1:M
2) For m = NxM : 1
3) Calculate power allocation threshold as $\frac{1}{\mu} = \frac{N \times P_{max} + \sum_{n=1}^{m} \frac{\sigma_N^2}{\lambda_n}}{m}$, where $\lambda_n$ is the gain of n^th subchannel in the sorted list.
4) If $\frac{1}{\mu} > \frac{\sigma_N^2}{\lambda_n}$, allocate power to m sorted subchannels as $P_n = \frac{1}{\mu} - \frac{\sigma_N^2}{\lambda_n}$ where n = 1: m
5) Else, m = m-1 and go to step 2.
6) Calculate effective capacity
   $C_{eff} = \sum_{k=1}^{N}\sum_{i=1}^{M} \log_2\left(1 + \frac{P_{k,i}\lambda_{k,i}}{\sigma_N^2}\right)$
7) Calculate energy efficiency $\eta = \frac{C_{eff}}{N \times P_{max}}$
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VII. SIMULATIONS

The different simulations performed in the study and development of this work is explained in this section. All simulations are done in MATLAB.

The first step in the simulation is to generate a random channel matrix. Rician flat fading channel is considered in the development. In MIMO-OFDM systems, there are a total of N subchannel matrices each with dimension M_T x M_R. To generate a Rician channel matrix, each element of the channel matrix is generated as complex numbers with real and imaginary parts each following an independent Guassian distribution [8],[9],[10]. Rician channels are often characterized using Rice factor 'K' from which the mean (μ) and variance (σ^2) of the Guassian variables can be calculated as

$$\mu = \sqrt{\frac{K}{1+K}},\ \sigma^2 = \frac{K+1}{2}$$

The next step is to compute the SVD of channel matrix at each subcarrier which is done using the built-in SVD function of MATLAB. The diagonal matrix after SVD contains the Eigen values of the subchannel matrices which give the subchannel gains.

Once the gains of subchannels at each subcarrier are available, optimal power threshold for water filling is calculated and power allocation is done as per water filling algorithm explained in Table 2 and effective capacity and energy efficiency are calculated.

Modified "2D" water filling is performed as the next step. The subchannel gains calculated using SVD for all subcarriers are taken as input and the optimal power allocation threshold is calculated as per the "2D" water filling algorithm explained in Table 3.

Simulation results for EEOPA were not obtained since a closed form solution for threshold calculation equation could not be found. Average power allocation method is taken as a benchmark to compare the results obtained with water filling and "2D" water filling. In average power allocation method, equal power is allocated to all subcarriers.

VIII. SIMULATION RESULTS

The different simulation results obtained are explained in this section. A 4x4 MIMO system is considered in the simulations. The values of different parameters used for simulations are $M_T = 4$, $M_R = 4$, $M = 4$, $N = 8$, $S = 2$, $B = 1MHz$, $T_f = 1ms$, $K = 1$, $P_{max} = 0.2W$.

The subchannel gains obtained after SVD is shown in Fig. 2.

Sorted subchannels in the decreasing order of gains is shown in Fig. 3.

A pictorial illustration of average power allocation is shown in Fig. 4.

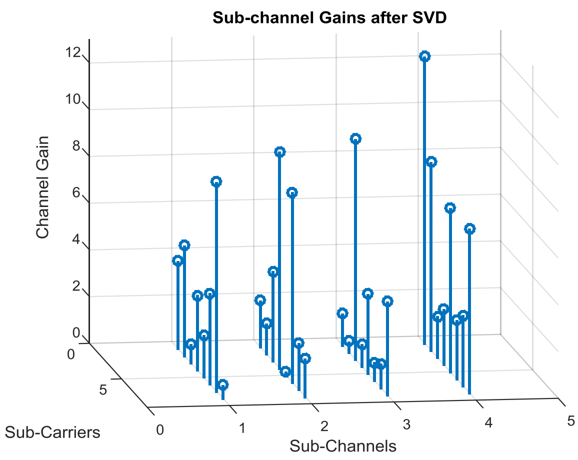

Fig. 2: Subchannel Gains after SVD

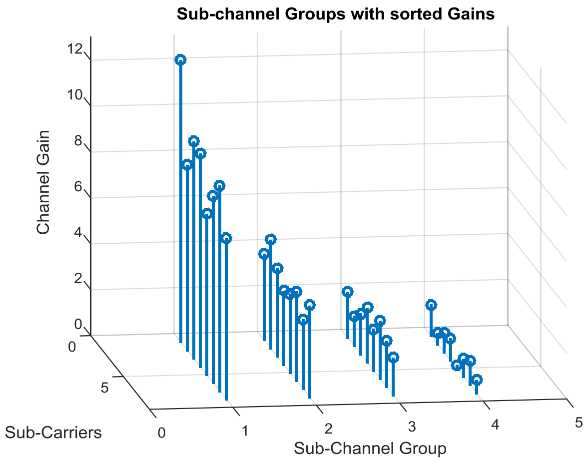

Fig. 3 : Sorted Subchannel gains

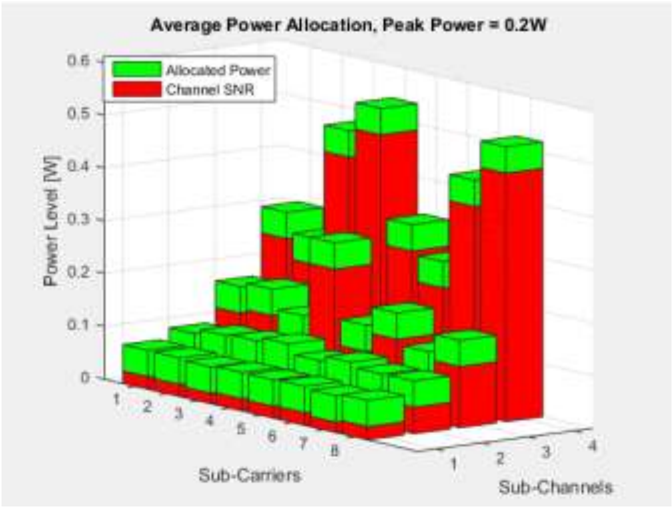

Fig. 4 : Average Power Allocation

Power allocation with water filling algorithm is shown in Fig. 5.

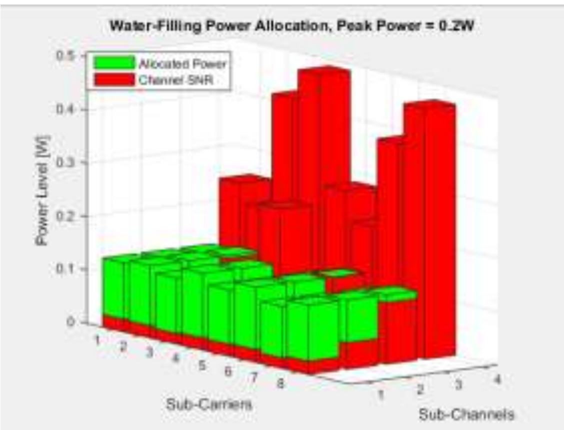

Fig. 5 : Water Filling Power Allocation

Fig. 6 shows the allocated power as per “2D” water filling algorithm.

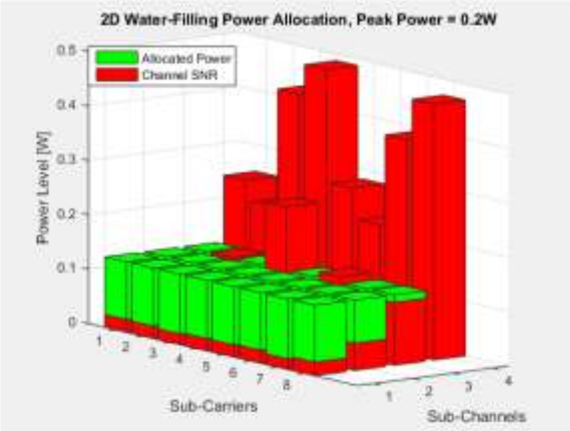

Fig. 6 : "2D" Water Filling Power Allocation

A comparison of effective capacity with peak power constraint for the different power allocation methods discussed is shown in Fig. 7.

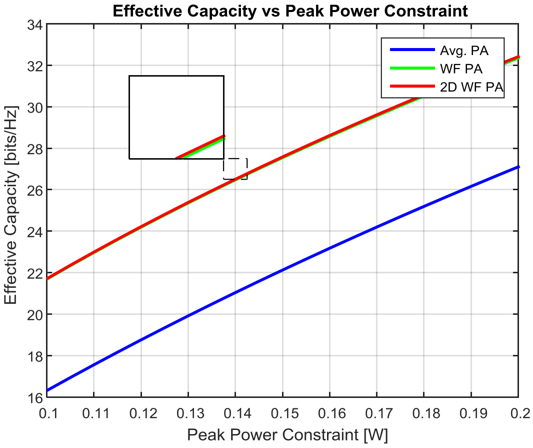

Fig. 7 : Effective Capacity vs Peak Power Constraint

The comparison of energy efficiency with the peak power constraint is shown in Fig. 8.

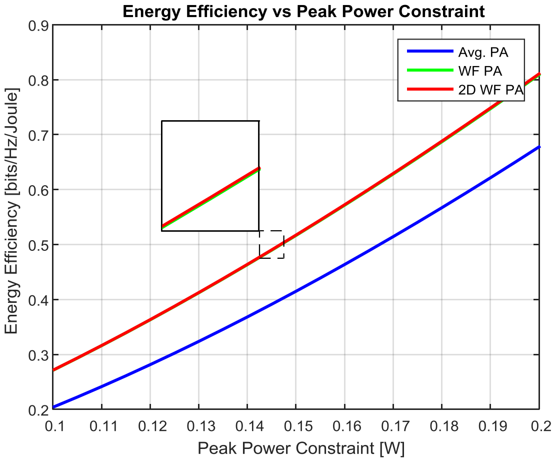

Fig. 8 : Energy Efficiency vs Peak Power Constraint

The difference in effective capacity and energy efficiency between water filling and “2D” water filling techniques might look very small, but the multiplication with the system bandwidth makes the performance improvement quite significant in applications.

IX. CONCLUSIONS & FUTURE WORK

Energy Efficient Optimal Power Allocation (EEOPA) technique proposed by Xiaohu Ge, et. al in [1] is analysed in this development and found to have some limitations. To overcome these limitations alternate water filling based power allocation techniques are proposed. From the simulations results it can be observed that the proposed methods shows improvements in the channel capacity and energy efficiency compared to traditional average power allocation. The effects of imperfect CSI at the transmitter need to be further analysed as a future development. The possibilities to introduce QoS constraints in water filling methods also need to be explored.

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