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Original Article

Applying Model Order Reduction Algorithm for Control Design of the Digital Filter

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Abstract *- Digital filters are increasingly used in the field of digital signal processing. Designing a digital filter is to determine the transfer function of a digital filter that meets the specified requirements. However, the design of digital filters often leads to high-order digital filters. The article has applied model reduction methods to reduce the high-order digital filter. Through comparison, evaluation of low-order filters according to different order reduction algorithms shows that low-order filters can completely replace high-order digital filters. Using a low-order digital filter will help reduce the calculation time, and increase the response speed of the filter, but still ensure the quality of the filter.*

Keywords - *Digital filter, Model order reduction, IIR digital filter.*

1. Introduction

Digital filtering techniques are increasingly used in the field of digital signal processing, as they can be used to speed up computations in digital filters by reducing the number of multiplications that must be performed per second. We know that during digital signal processing, the bandwidth of the frequency band can be varied as filters will suppress unwanted frequency components, such that the bandwidth of the processed signal will decrease and we can reduce the sampling frequency to match the signal's spectral width, which will reduce the number of computations in the digital filter. Due to its superior properties, the synthesis of digital filters and multi-span digital filtering techniques have been studied a lot in recent years and have obtained positive results in theory as well as application. engineering in telecommunications, speech processing, image processing, antenna systems, and digital audio technology. TDM multiplexing system, FDM multiplexing system, multi-span digital filtering. The achievements of FIR, IIR digital filters and multi-span digital filtering techniques over the years have been demonstrated in the study of multi-span digital systems, such as the synthesis of division and interpolation filters, analysis and synthesis filter banks, multi-phase representation and multi-phase structure. There are two ways to synthesize digital filters as follows: the first way is to design an analog filter and then use the equivalent conversion method to obtain a digital filter; the second way is to determine the numerical filter by optimal methods with the participation of the computer.

The design of the digital learning set is to determine a realizable transfer function G(z) that approximates the given frequency response specifications. If you want to use an IIR filter, you also need to make sure that G(z) is stable. After obtaining the digital filter G(z), the next step is to realize it in the form of a suitable filter structure [1]. However, the design of digital filters often leads to high-order digital filters. The order reduction technique is often used in filter design to obtain an equivalent order reduction linear IIR filter [1].

To achieve high accuracy, the mathematical model of the object or controller is usually a high-order model [2-5]. High-order models are difficult to analyze, and therefore too difficult to use. As such, there is a need for low-order models. Low-order models are those that describe the behavior of the system in a relatively precise manner, minimizing the disadvantage of unnecessary details [2-5]. Model order reduction is a method of simplifying high-order complex models. In the past years, many groups of model order reduction algorithms have been proposed, with many different development directions. Among them, Moore's balanced truncation algorithm [6-7] is the most popular one. The balanced truncation algorithm is determined to be able to preserve the stability of the original system during order reduction in [8] and it is possible to determine a formula for calculating the error limit when decreasing order in [9-10]. The balanced truncation algorithm performs the process of diagonalizing simultaneously the two control Gramian matrices and observed Gramian matrices of the original system. From there, a state transition matrix is determined, allowing the conversion of the original model represented in any basis system into an equivalent system representing the coordinate system in internal equilibrium space. Moore's balanced truncation algorithm [6-7] has the advantage of giving order reduction results with small order reduction errors.

Based on the balanced truncation algorithm, many order reduction algorithms have been proposed, such as the stochastic balancing method [11-12], frequency weighting [13], and the approximation method. Hankel-Norm Approximation [14], Schur's algorithm [15-16],…. Initially, the balanced truncation algorithm was mainly applied to stable linear systems. However, in practice, high-order linear models can also be unstable, so the balanced truncation algorithms are also extended to apply to unstable systems [11-16], or improved to apply to unstable systems such as the LQG balancing algorithm [17], Zhou's algorithm [18], Zilochian's algorithm [19], Boess's algorithm [20], the balanced truncation method is based on continuous-discrete mapping [21-22]. The second group of popular algorithms is a group of algorithms based on preserving the important eigenvalues of the high-order original model to determine the order of the low-order model, such as the algorithm that preserves the dominant point [23-24], ... The third group of reduction algorithms is proposed on the basis of applying the optimization criteria without regard to the important eigenvalues of the original model [29]. The fourth group of reduction algorithms is proposed based on matching some properties other than those of response [26]. In addition to us, other methods of order reduction do not belong to the above groups [27].

To be able to choose an appropriate low-order model, we need to compare and evaluate low-order models according to many different algorithms.

In the content of this paper, we are most interested in Moore's algorithm [6], Schur's algorithm [15-16], and Zilochian's algorithm [19], so we will use those methods to simplify the high-order filter.

2. High-Order IRR Filter Model

Consider a digital IIR filter with a transfer function in the form H(z) in equation (1) and the general structure of a laddered IIR filter as follows [1]

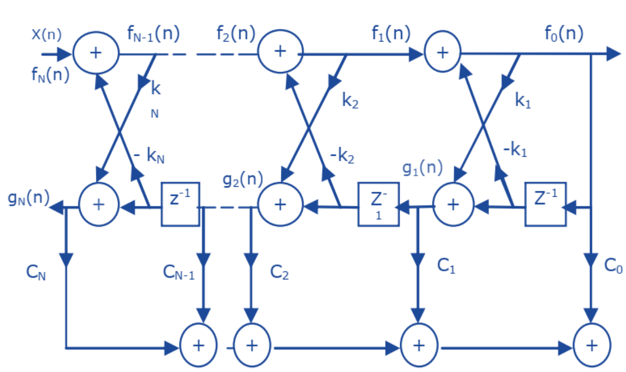

Fig. 1 Structure of Lattice-ladder IIR filter oh N^th order. [1]

In [28], the authors determined the transfer function model of the 30^th^-order IRR digital filter as follows:

$$H(z) = \frac{b_0 z^N + b_1 z^{N-1} + \ldots + b_{N-1} z + b_N}{a_0 z^N + a_1 z^{N-1} + \ldots + a_{N-1} z + a_N} \quad (1)$$

$$H(z) = \frac{A(z)}{B(z)}$$

with

$$A(z) = 0.0187z^{30} - 0.0566z^{29} + 0.0811z^{28} - 0.0816z^{27}$$
$$+0.0670z^{26} - 0.0485z^{25} + 0.0313z^{24} - 0.0163z^{23} + 0.0044z^{22}$$
$$+0.0052z^{21} - 0.0123z^{20} + 0.0173a^{19} - 0.0202z^{18} + 0.0221z^{17}$$
$$-0.0254z^{16} + 0.0299z^{15} - 0.0310z^{14} + 0.0272z^{13} - 0.0204z^{12}$$
$$+0.0123z^{11} - 0.0053z^{10} - 0.0002z^{9} + 0.0041z^{8}$$
$$-0.0064z^{7} + 0.0071z^{6} - 0.0056z^{5} + 0.0026z^{4}$$
$$-0.0001z^{2} - 0.0004z^{2}$$

$$B(z) = z^{30} - 4.0365z^{29} + 7.5547z^{28} - 9.2123z^{27} + 8.6181z^{26}$$
$$-6.7989z^{25} + 4.7364z^{24} - 2.8452z^{23} + 1.265z^{22} - 0.0026z^{21}$$
$$-0.9745z^{20} + 1.6829z^{19} - 2.1471z^{18} + 2.4311z^{17} - 2.7209z^{16}$$
$$+3.1473z^{15} - 3.4718z^{14} + 3.3593z^{13} - 2.7903z^{12} + 1.9504z^{11}$$
$$-1.0840z^{10} + 0.3537z^{9} + 0.2019z^{8} - 0.5694z^{7} + 0.7502z^{6}$$
$$-0.7201z^{5} + 0.4839z^{4} - 0.1735z^{3} - 0.0114z^{2} + 0.0304z - 0.0065$$

The 30^th^-order digital filter (original digital filter) when used in practice will have many disadvantages such as complex filter structure, and slow response time. In order to keep the filter structure simple, fast response time, and still maintain the important characteristics of a good filter, it is necessary to simplify the original digital filter.

3. Reducing the Order of High-Order IIR Digital Filter

To simplify the original digital filter, we convert the digital filter to linear form through the transformation $z = s + 1$, the results are obtained as follows

$$H(s) = \frac{C(s)}{D(s)}$$

with

$$C(s) = 0.0187s^{30} + 0.5044s^{29} + 6.574s^{28} + 55.13s^{27}$$
$$+334.2s^{26} + 1559s^{25} + 5824s^{24} + 1.788.10^{4}s^{23}$$
$$+4.597.10^{4}s^{22} + 1.003.10^{5}s^{21} + 1.878.10^{5}s^{20}$$
$$+3.035.10^{5}s^{19} + 4.261.10^{5}s^{18} + 5.216.10^{5}s^{17}$$
$$+5.578.10^{5}s^{16} + 5.219.10^{5}s^{15} + 4.269.10^{5}s^{14}$$
$$+3.05.10^{5}s^{13} + 1.897.10^{5}s^{12} + 1.023.10^{5}s^{11}$$
$$+4.753.10^{4}s^{10} + 1.888.10^{4}s^{9} + 6343s^{8}$$
$$+1777s^{7} + 407.4s^{6} + 74.54s^{5} + 10.52s^{4}$$
$$+1.088s^{3} + 0.076s^{2} + 0.0031s + 7.80610^{-18}$$

$$D(z) = s^{30} + 25.96s^{29} + 325.5s^{28} + 2624s^{27} + 1.527.10^{4}s^{26}$$
$$+6.837.10^{4}s^{25} + 2.448.10^{5}s^{24} + 7.197.10^{5}s^{23} + 1.77.10^{6}s^{22}$$
$$+3.69.10^{6}s^{21} + 6.588.10^{6}s^{20} + 1.015.10^{7}s^{19} + 1.355.10^{7}s^{18}$$
$$+1.575.10^{7}s^{17} + 1.597.10^{7}s^{16} + 1.413.10^{7}s^{15} + 1.091.10^{7}s^{14}$$
$$+7.338.10^{6}s^{13} + 4.284.10^{6}s^{12} + 2.161.10^{6}s^{11} + 9.356.10^{5}s^{10}$$
$$+3.447.10^{5}s^{9} + 1.068.10^{5}s^{8} + 2.743.10^{4}s^{7} + 5719s^{6}$$
$$+941.3s^{5} + 117.6s^{4} + 10.49s^{3} + 0.6116s^{2} + 0.0222s + 0.0003$$

In this paper, we apply Moore's algorithm [6], Schur's algorithm [15-16], and Zilochian's algorithm [19] to reduce the original digital filter. The results of the order reduction of the original digital filter are shown in the following tables

Table 1. The result of simplifying the original digital filter according to Moore's algorithm [6]

Order	$H_r(s)$
9	$\dfrac{0.0187s^9 + 0.04967s^8 + 0.05561s^7 + 0.03852s^6 + 0.0184s^5 + 0.005202s^4 + 0.00107s^3 + 0.0001051s^2 + 8.796.10^{-6}s - 3.275.10^{-12}}{s^9 + 1.646s^8 + 1.462s^7 + 0.8564s^6 + 0.3202s^5 + 0.07699s^4 + 0.01264s^3 + 0.001011s^2 + 5.23.10^{-5}s + 8.511.10^{-7}}$
8	$\dfrac{0.0187s^8 + 0.04942s^7 + 0.04528s^6 + 0.01811s^5 + 0.005011s^4 + 0.0007887s^3 + 7.787.10^{-5}s^2 + 4.641.10^{-6}s - 2.804.10^{-11}}{s^8 + 1.633s^7 + 0.9228s^6 + 0.3102s^5 + 0.06789s^4 + 0.009002s^3 + 0.0006617s^2 + 2.978.10^{-5}s + 4.487.10^{-7}}$
7	$\dfrac{0.0187s^7 + 0.04024s^6 + 0.02872s^5 + 0.009503s^4 + 0.002202s^3 + 0.0002223s^2 + 1.989.10^{-5}s + 2.118.10^{-11}}{s^7 + 1.14s^6 + 0.5416s^5 + 0.1484s^4 + 0.02659s^3 + 0.002192s^2 + 0.0001168s + 1.925.10^{-6}}$
6	$\dfrac{0.0187s^6 + 0.02776s^5 + 0.007638s^4 + 0.002003s^3 + 0.0001852s^2 + 1.81.10^{-5}s + 6.41.10^{-10}}{s^6 + 0.4603s^5 + 0.1301s^4 + 0.02303s^3 + 0.001926s^2 + 0.0001042s + 1.763.10^{-6}}$

Table 2. The result of simplifying the original digital filter according to Schur's algorithm [15-16]

Order	$H_r(s)$
9	$\dfrac{0.0187s^9 + 0.04967s^8 + 0.05561s^7 + 0.03852s^6 + 0.0184s^5 + 0.005202s^4 + 0.00107s^3 + 0.0001051s^2 + 8.796.10^{-6}s - 3.275.10^{-12}}{s^9 + 1.646s^8 + 1.462s^7 + 0.8564s^6 + 0.3202s^5 + 0.07699s^4 + 0.01264s^3 + 0.001011s^2 + 5.23.10^{-5}s + 8.511.10^{-7}}$
8	$\dfrac{0.0187s^8 + 0.04942s^7 + 0.04528s^6 + 0.01811s^5 + 0.005011s^4 + 0.0007887s^3 + 7.787.10^{-5}s^2 + 4.641.10^{-6}s - 2.804.10^{-11}}{s^8 + 1.633s^7 + 0.9228s^6 + 0.3102s^5 + 0.06789s^4 + 0.009002s^3 + 0.0006617s^2 + 2.978.10^{-5}s + 4.487.10^{-7}}$
7	$\dfrac{0.0187s^7 + 0.04024s^6 + 0.02872s^5 + 0.009503s^4 + 0.002202s^3 + 0.0002223s^2 + 1.989.10^{-5}s + 2.118.10^{-11}}{s^7 + 1.14s^6 + 0.5416s^5 + 0.1484s^4 + 0.02659s^3 + 0.002192s^2 + 0.0001168s + 1.925.10^{-6}}$
6	$\dfrac{0.0187s^6 + 0.02776s^5 + 0.007638s^4 + 0.002003s^3 + 0.0001852s^2 + 1.81.10^{-5}s + 6.41.10^{-10}}{s^6 + 0.4603s^5 + 0.1301s^4 + 0.02303s^3 + 0.001926s^2 + 0.0001042s + 1.763.10^{-6}}$

Table 3. The result of simplifying the original digital filter according to Zilochian's algorithm [19]

Order	$H_r(s)$
9	$\dfrac{0.0187s^9+0.05015s^8+0.05555s^7+0.03725s^6+0.0176s^5+0.004918s^4+0.001008s^3+9.866e-05s^2+8.242.10^{-6}s-6.246.10^{-14}}{s^9+1.672s^8+1.433s^7+0.8238s^6+0.3043s^5+0.07266s^4+0.01187s^3+0.0009486s^2+4.902.10^{-5}s+7.976.10^{-7}}$
8	$\dfrac{0.0187s^8+0.03956s^7+0.02785s^6+0.009551s^5+0.002251s^4+0.0002525s^3+2.298.10^{-5}s^2+3.955.10^{-7}s+7.334.10^{-15}}{s^8+1.103s^7+0.5342s^6+0.1503s^5+0.02787s^4+0.00258s^3+0.0001526s^2+4.117.10^{-6}s+3.827.10^{-8}}$
7	$\dfrac{0.0187s^7+0.03921s^6+0.02709s^5+0.008996s^4+0.002066s^3+0.0002091s^2+1.858.10^{-5}s-4.551.10^{-13}}{s^7+1.085s^6+0.5124s^5+0.1398s^4+0.02497s^3+0.002055s^2+0.0001092s+1.798.10^{-6}}$
6	$\dfrac{0.0187s^6+0.02933s^5+0.009583s^4+0.002388s^3+0.0002381s^2+2.179.10^{-5}s+1.76.10^{-11}}{s^6+0.5405s^5+0.1566s^4+0.02864s^3+0.002373s^2+0.0001273s+2.108.10^{-6}}$

4. Results and Discussion

To evaluate the order reduction filters, we compare the transient (step) and frequency (bode) response of the reduced filters with the original filter, the results are shown in Figure 2-Figure 7 as follows:

The step response of the 9th, 8th, 7th and 6th-order filters is exactly the same as that of the original filter. The frequency (bode) response of the 9th-order filters according to the order reduction algorithms completely coincides with the frequency response of the original filter.

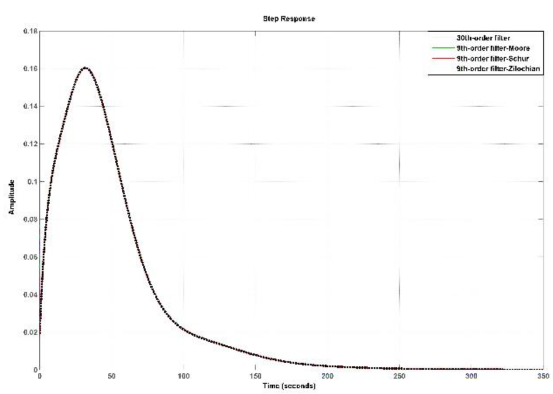

Fig. 2 Step response of the 9th-order and 30th-order filter

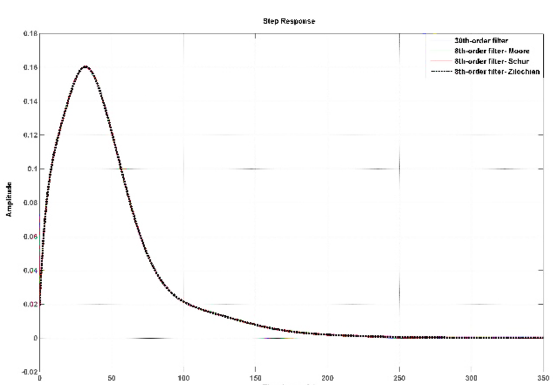

Fig. 3 Step response of the 8th-order and 30th-order filter

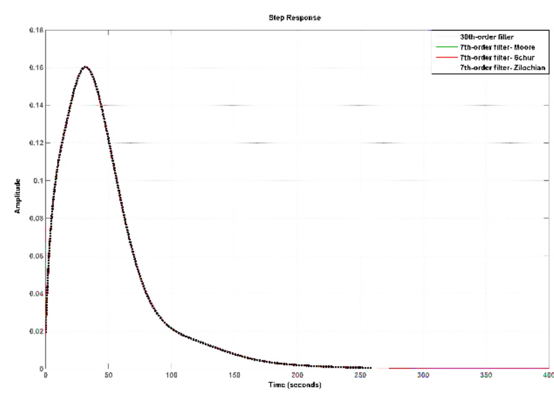

Fig. 4 Step response of the 7th-order and 30th-order filter

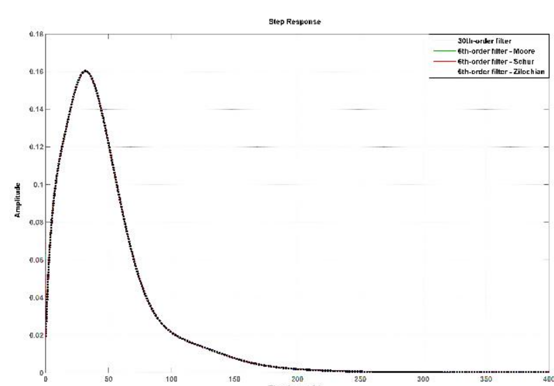

Fig. 5 Step response of the 6th-order and 30th-order filter

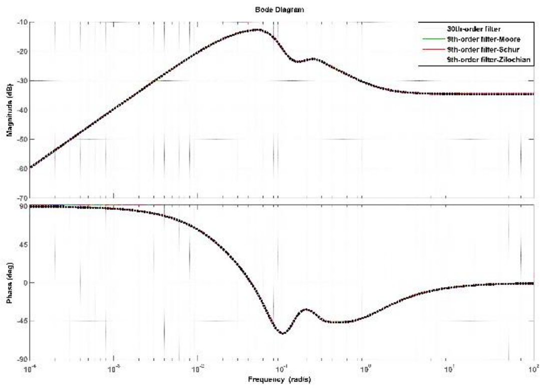

Fig. 6 Bode response of the 9th-order and original filter

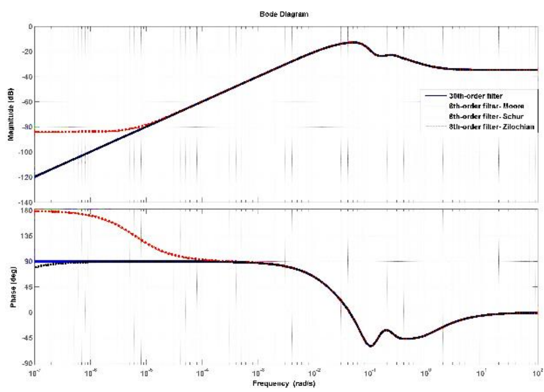

Fig. 7 Bode response of the 8th-order and original filter

The frequency response of the 8th-order filter according to Moore's algorithm coincides with the frequency response of the 8th-order filter according to the Schur algorithm. In the frequency range $\omega < 1.34.10^{-5}$ rad/s, the frequency magnitude response of the 8th-order filter according to Moore's algorithm is different from the frequency magnitude response of the original filter. In the frequency range $\omega > 1.34.10^{-5}$ rad/s, the frequency magnitude response of the 8th-order filter according to Moore's algorithm and the frequency magnitude response of the original filter are identical.

In the frequency range $\omega < 0.000132$ rad/s, the frequency phase response of the 8th-order filter according to Moore's algorithm is different from the frequency response of the original filter. In the frequency range $\omega > 0.000132$ rad/s, the frequency phase response of the 8th-order filter according to Moore's algorithm and the frequency phase response of the original filter are identical.

The frequency magnitude response of the 8th-order filter according to Zilochian's algorithm and the frequency magnitude response of the original filter is identical.

In the frequency range $\omega <1,92.10^{-6}$ rad/s, the frequency phase response of the 8th-order filter according to Zilochian's algorithm is different from the frequency phase response of the original filter. In the frequency range $\omega >1,92.10^{-6}$ rad/s, the frequency phase response of the 8th-order filter according to Zilochian's algorithm and the frequency phase response of the original filter are identical.

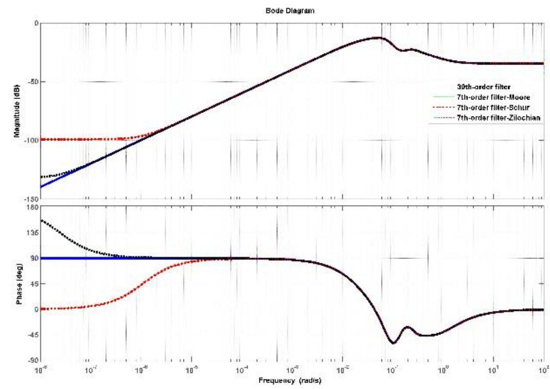

Fig. 8 Bode response of the 7th-order and original filter

The frequency response of the 7th-order filter according to Moore's algorithm and the bode response of the 7th-order filter according to the Schur algorithms are identical.

In the frequency range $\omega < 2.56.10^{-6}$ rad/s, the frequency magnitude response of the 7th-order filter according to Moore's algorithm is different from that of the original filter. In the frequency range $\omega >2.56.10^{-6}$ rad/s, the frequency magnitude response of the 7th-order filter according to Moore's algorithm and the frequency magnitude response of the original filter are identical.

In the frequency range < 0.000108 rad/s, the frequency phase response of the 7th-order filter by Moore's algorithm is different from that of the original filter. In the frequency range < 0.000108 rad/s, the frequency phase response of the 7th-order filter according to Moore's algorithm and the frequency response of the original filter are identical.

In the frequency range $\omega < 7.04.10^{-8}$ rad/s, the frequency magnitude response of the 7th-order filter according to Zilochian's algorithm is different from that of the original filter. In the frequency range $\omega > 7.04.10^{-8}$ rad/s, the frequency magnitude response of the 7th-order filter according to Zilochian's algorithm and the frequency magnitude response of the original filter are identical.

In the frequency range $\omega < 8.24.10^{-7}$ rad/s, the frequency phase response of the 7th-order filter according to Zilochian's algorithm is different from the frequency response of the original filter. In the frequency range $\omega >8.24.10^{-7}$ rad/s, the frequency phase response of the 7th-order filter according to Zilochian's algorithm and the frequency phase response of the original filter are identical.

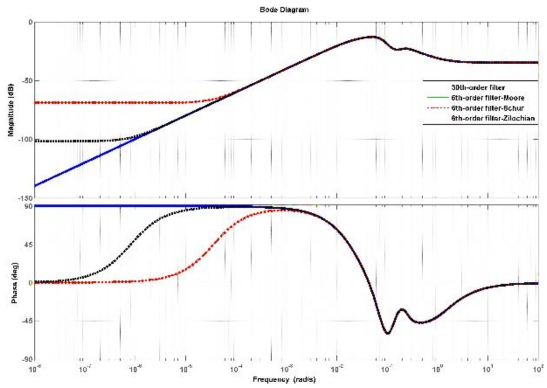

Fig. 9 Bode response of the 6^{th}-order and original filter

The bode response of the 6^{th}-order filter according to Moore's algorithm and the bode response of the 6^{th}-order filter according to the Schur algorithms are identical.

In the frequency range $\omega < 8.04.10^{-5}$ rad/s, the frequency magnitude response of the 6^{th}-order filter according to Moore's algorithm is different from that of the original filter. In the frequency range $\omega > 8.04.10^{-5}$ rad/s the frequency magnitude response of the 6^{th}-order filter according to Moore's algorithm and the frequency magnitude response of the original filter are identical.

In the frequency range $\omega < 0.00276$, the frequency phase response of the 6^{th}-order filter by Moore's balanced truncation algorithm is different from that of the original filter. In the frequency range $\omega > 0.00276$ rad/s rad/s, the frequency phase response of the 6^{th}-order filter according to Moore's algorithm and the frequency response of the original filter are identical.

In the frequency range $\omega < 1.71.10^{-6}$ rad/s, the frequency magnitude response of the 6^{th}-order filter according to Zilochian's algorithm is different from that of the original filter. In the frequency range $\omega > 1.71.10^{-6}$ rad/s the frequency magnitude response of the 6^{th}-order filter according to Zilochian's algorithm and the frequency magnitude response of the original filter are identical.

In the frequency range $\omega < 8.2.10^{-5}$ rad/s, the frequency phase response of the 6^{th}-order filter according to Zilochian's algorithm is different from the frequency response of the original filter. In the frequency range $\omega > 8.2.10^{-5}$ rad/s, the frequency phase response of the 6^{th}-order filter according to Zilochian's algorithm and the frequency phase response of the original filter are identical.

Comment: If the response quality requirements of the order reduction filter are not different from the original filter, we can choose 9^{th}-order filters to replace the original filter. It is possible to use the 8^{th}-order filter according to Zilochian's algorithm instead of the original filter if the frequency phase characteristic deviation is allowed in the low-frequency region ($\omega < 1{,}92.10^{-6}$ rad/s). If the amplitude characteristic deviation in the low-frequency region is acceptable ($\omega < 7.04.10^{-8}$ rad/s), the 7^{th}-order filter according to Zilochian's algorithm can be used instead of the original filter.

The order reduction filters according to Zilochian's algorithm give better response results (more coincident with the original response) than filters according to Moore's algorithm and Schur's algorithm.

5. Conclusion

The article has applied different order reduction algorithms to reduce the order of 30^{th}-order digital filters. The results of the comparison and evaluation of low-order filters by different order reduction methods show that: Zilochian's balanced truncation algorithm gives the best order reduction results compared to low-order filters according to other algorithms. The response of the 9^{th}-order filters completely coincides with the response of the 30^{th}-order filters, so a 9^{th}-order filter can be used instead of the 30^{th}-order filter. The 8^{th}-order filter according to the Zilochian balanced truncation algorithm also can be instead of the 30^{th}-order filter if small errors in the low-frequency region are acceptable. The simulation results show that using order reduction algorithms to reduce the order of higher-order filters is correct and effective.

References

[1] Komarasamy, Dr Ramesh and A. Nirmalkumar, "Design of Digital IIR Filters with the Advantages of Model Order Reduction Technique". *International Journal of Electronic Engineering Research*, vol. 1, no. 2, pp. 89–100, 2009.

[2] Ngoc Kien Vu, Hong Quang Nguyen, "Balancing Control of Two-Wheel Bicycle Problems", *Mathematical Problems In Engineering*, pp. 1-12, 2020. Crossref, https://doi.org/10.1155/2020/6724382

[3] Ngoc Kien Vu and Hong Quang Nguyen, "Design Low-Order Robust Controller for Self-Balancing Two-Wheel Vehicle", *Mathematical Problems in Engineering*, pp. 22, 2021. Crossref, https://doi.org//10.1155/2021/6693807

[4] Vu Ngoc Kien, Nguyen Hien Trung and Nguyen Hong Quang, "Design Low Order Robust Controller for the Generator's Rotor Angle Stabilization PSS System", *Emerging Science Journal*, vol. 5, no. 5, pp. 598-618, 2021. Crossref, https://doi.org/0.28991/esj-2021-01299

[5] Vu Ngoc Kien, Nguyen Hong Quang and Ngo Kien Trung, "Application of Model Reduction for Robust Control of Self-Balancing Two-Wheeled Bicycle," *TELKOMNIKA Telecommunication, Computing, Electronics and Control*, vol. 19, no. 1, pp. 252-264, 2021. https://doi.org/10.12928/telkomnika.v19i1.16298

[6] B. Moore, "Principal Component Analysis in Linear Systems: Controllability, Observability, and Model Reduction," *IEEE Transactions on Automatic Control*, vol. 26, no. 1, pp. 17–32, 1981, Crossref, https://doi.org/10.1109/Tac.1981.1102568

[7] Vu Ngoc Kien and Nguyen Hong Quang, "Model Reduction of Unstable Systems Based on Balanced Truncation Algorithm," *International Journal of Electrical and Computer Engineering,* vol. 11, no. 3, pp. 2045-2053, 2021. Crossref, https://doi.org//10.11591/ijece.v11i3.pp2045-2053

[8] I. Perenbo and L.M. Silverman, "Model Reduction Via Balanced State Space Representation", *IEEE Transactions on Automatic Control*, vol. 27, no. 2, pp. 328-387, 1982. Crossref, https://doi.org/10.1109/TAC.1982.1102945

[9] K. Glover, "All Optimal Hankel-Norm Approximations of Linear Multivariable Systems and Their L, ∞ -Error Bounds†," *International Journal of Control,* vol. 39, no. 6, pp. 1115-1193, 1984. Crossref, https://doi.org/10.1080/00207178408933239

[10] D. Enns, "Model Reduction with Balanced Realizations: An Error Bound and A Frequency Weighted Generalization," *The 23rd IEEE Conference on Decision and Control*, 1984. Crossref, https://doi.org/10.1109/CDC.1984.272286

[11] U.B. Desai and D. Pal, "A Transformation Approach to Stochastic Model Reduction," *IEEE Transactions on Automatic Control,* vol. 29, no. 12, pp. 1097–1100, 1984. Crossref, https://doi.org/10.1109/TAC.1984.1103438

[12] M. Green, "A Relative Error Bound for Balanced Stochastic Truncation," *IEEE Transactions on Automatic Control*, vol. 33, no. 10, pp. 961 – 965, 1998. Crossref, https://doi.org/10.1109/9.7255

[13] W. Gawronski and J.N. Juang, "Model Reduction In Limited Time and Frequency Intervals," *International Journal of Systems Science*, vol. 21, no. 2, pp. 349-376, 1990. Crossref, https://doi.org//10.1080/00207729008910366

[14] A. C. Antoulas, D. C. Sorensen and S. Gugercin, "A Survey of Model Reduction Methods for Large-Scale Systems", *Structured Matrices In Mathematics, Computer Science, and Engineering*, vol. 280, 2001. Crossref, https://doi.org/10.1090/conm/280

[15] M. G. Safonov and R. Y. Chiang, "A Schur Method for Balanced-Truncation Model Reduction," *In IEEE Transactions on Automatic Control*, vol. 34, no. 7, pp. 729-733, 1989, Crossref, https://doi.org/10.1109/9.29399.

[16] N.K. Vu, H.Q. Nguyen, K.T. Ngo and P. N. Dao, "Study on Model Reduction Algorithm Based on Schur Analysis," *Advances in Intelligent Systems and Computing*, vol. 1254, 2021. Crossref, https://doi.org/10.1007/978-981-15-7527-3_18

[17] E. A. Jonckheere and L. M. Silverman, "A New Set of Invariants for Linear System – Application to Reduced Order Compensator Design", *IEEE Transactions on Automatic Control,* vol. 28, no. 10, pp. 953-964, 1983. Crossref, https://doi.org/10.1109/TAC.1983.1103159

[18] K. Zhou, G. Salomon and E. Wu, "Balanced Realization and Model Reduction for Unstable Systems," *International Journal of Robust and Nonlinear Control*, vol. 9, no. 3, pp. 183-198 https://doi.org//10.1002/(SICI)1099-1239(199903)9:3<183::AID-RNC399>3.0.CO;2-E

[19] A. Zilouchian, "Balanced Structures and Model Reduction of Unstable Systems," *IEEE Proceedings of the SOUTHEASTCON '91,* 1991. https://doi.org/10.1109/SECON.1991.147956

[20] C. Boess, N. Nichols and A. Bunse-Gerstner, "Model Reduction for Discrete Unstable Control Systems Using a Balanced Truncation Approach," *Preprint MPS, University of Reading*, 2010.

[21] Ngoc Kien Vu, Hong Quang Nguyen, "Reduced Order of the Unstable System Using Balanced Truncation Algorithm Based on Continuous-Discrete Mapping and Its Application to Control," *Journal of Mechanical Engineering Research and Developments*, vol. 43 no.6, pp. 299-307, 2020

[22] Kien Vu Ngoc and Quang Nguyen Hong, "Combined Balanced Truncation Algorithm and Linear Differential Mapping for Controller Order Reduction Problem," *Journal of Mechanical Engineering Research and Developments*, vol. 43, no. 4, pp. 310-319, 2020.

[23] Ngoc Kien Vu and Hong Quang Nguyen, "Model Order Reduction Algorithm Based on Preserving Dominant Poles," *International Journal of Control, Automation, and Systems,* vol. 19, pp. 2047-2058, 2021, Crossref, https://doi.org/10.1007/S12555-019-0990-8

[24] Vu Ngoc Kien and Nguyen Hong Quang, "Model Reduction in Schur Basic with Pole Retention and H2 Norm Error Bound," *Journal of Mechanical Engineering Research and Developments*, vol. 44, no. 1, pp. 279-293, 2021.

[25] Er. Nitin Yadav and Er. Dharmendra Singh, "Model Order Reduction Using Routh Approximation Method, Factor Division Method and Genetic Algorithm," *SSRG International Journal of Electrical and Electronics Engineering*, vol. 5, no. 9, pp. 9-15, 2018. Crossref, https://doi.org/10.14445/23488379/IJEEE-V5I9P102

[26] N. K. Sinha, I. El-Nahas and R. T. H. Alden, "Routh Approximation of Multivariable Systems," *Problems of Control and Information Theory*, vol. 11, no. 3, pp. 420-425, 1982.

[27] S. Panda, J. S. Yadav, N. P. Patidar and C. Ardil, "Evolutionary Techniques for Model Order Reduction of Large Scale Linear Systems", *International Journal of Electrical and Computer Engineering,* vol. 6, No. 9, 2012.

[28] V. R. Dehkordi and A. G. Aghdam, "A Computationally Efficient Algorithm for Order-Reduction of IIR Filters Using Control Techniques," *Proceedings of 2005 IEEE Conference on Control Applications,* pp. 898-903, 2005. https://doi.org/10.1109/CCA.2005.1507243

[29] N. G. Nath and N. N. San, "An Approach to Linear Model Reduction," *Control Cybersecurity*, vol. 20 , no. 2, pp. 69-89, 1991.