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Original Article

The Effect of the Magnetic Field of High Intensities on Velocity Profiles of Slip Driven Non-Newtonian Fluid Flow through the Circular, Straight Microchannel

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Abstract - *In micro-devices, inertial forces tend to decrease, while surface effects dominate the flow and viscous effects. the numerical approach is becoming popular to describe fluid flow characteristics of slip flow through microchannels by Navier-Stokes equations in conjunction with the slip boundary. A study has been made numerically to understand the effect of magnetic induction on the degree of slip which depends upon the flow behaviour index, slip length, Reynolds no. In the present study, various observations have been made on the magnetohydrodynamic effect on non-Newtonian slip flow velocity profiles through microchannels to reveal the effect of flow behaviour index and slip coefficient. This work reveals that the slip coefficient plays a major role in the flow, and the externally applied magnetic field affects both centerline and slip velocity.*

Keywords *- Non-Newtonian fluids, Slip flow, Microchannels, Magnetohydrodynamics, Xanthan.*

1. Introduction

Biomedical devices and mechanical systems of micro sizes have become more predominant in scientific experiments and industrial and commercial uses. the last few decades have witnessed the development of machining and production of micro-devices that have developed the application of micro-electro-mechanical systems (MEMS) in various engineering fields. Micro-devices contain microchannels of different geometry and cross-sections. These micro-devices are now being widely used to analyze biological substances in liquid form. the experiment and numerical verification of liquid flow in microchannels showed that the conventional fluid mechanics theories could successfully predict flow behaviours in microchannels [1]. the experimental analysis of velocity profiles of the liquid flow of single-phase through microchannels depicted the existence of slip flow near the vicinity of the microchannel walls [2]. an expression of a general boundary condition is deduced to explain the behaviour of liquid flow near the solid boundary [3]. A deviation from the classical no-slip boundary condition toward the concept of slip phenomena has been proven [4]. the validity of the application of this theory has been explained for the micro to nanoscales [5]. Reports have been made to determine the effects of the slip boundary condition on viscosity [6], friction factor, Reynolds no. [7] etc. the Power law model can be applied to analyze non-Newtonian fluids passing through microchannels of various cross-sections [8]. When biological fluids like proteins, cells, DNA, embryos, blood, different chemical reagents and other similar substances are allowed to pass through the channels of microsystems, they mostly show the non-Newtonian fluid flow of single-phase liquid nature. Microchannels have a high surface-to-volume ratio with a very small volume. This plays an advantageous role in having a high heat and mass transfer rate. With the very low value of the characteristics length scale of the microchannels and surface tension, the viscous and electric force becomes predominant. Due to this high surface to volume ratio, microchannels offer the advantages of a high rate of heat and mass transfer. As compared to macro channel flow with the declining value of the dimensions of microdevices, inertial forces tend to fall off, and surface effects play a significant role in microchannels [9]. the amount of slip through the walls of microchannels also depends upon the f.Revalue, which predicts the possibility of slip in microchannels. In these devices, the values of roughness and charges at the surfaces of microchannels have a significant effect on the flow. the amount of slip is expressed by a boundary condition in general form was first proposed by Navier. This expression relates the slip velocity or, in other words, the velocity of the fluid at the solid wall with the shear rate at the wall or local shear rate ($\dot{\gamma}$). Solving the Navier Stokes equation with the help of the boundary condition is the foundation of solving problems associated with the slip flow. This analytical approach is becoming popular to relate different slip flow associated fluid characteristics like Reynold's Number, Knudsen number, Poiseuille number etc. [10]. the flow of non-Newtonian fluids through microchannels is an area where still there is a need for research to reveal the flow characteristics. the amount of slip on the microchannel wall also depends upon the Poiseuille Number, which predicts the extent of the possibility of the slip. A dimensionless quantity slip coefficient (β) is often used to describe the amount of slip on the walls [7] which is nothing but a ratio of slip velocity (v_s) the average or mean velocity (v_m). Works depicting the analytical modelling [11] and numerical simulation [12] of slip driven Newtonian [13] and non-Newtonian [14] fluids is an area of modern research that shows the application of slip induced fluid flow through microchannels of various geometries. Pressure drop is one of the fluid flow parameters for microchannels containing power-law non-Newtonian fluid in a microchannel [15]. Besides analytical and numerical simulation, few experimental works have reported the slip flow through microchannels of various cross-sections [16] in low Reynolds no. [17]. Similar works reveal the liquid flow phenomena of biological substances like a blood [18] through microchannels. the liquid flow of fluids mixed with nanoparticles with slip flow [19] is now an emerging area. the theoretical study by power-law for slip-flow of a non-Newtonian nanofluid shows dependence on the flow behaviour index on nanoparticle concentration in volume fraction [20]. the application of magnetic field on microchannels carrying fluids that are sensitive to the externally applied magnetic field has created an area of research that can be applied for better control of fluid flow through microchannels. It is observed that the application of a magnetic field of high intensities up to 15 T [21] can be achieved, which can be used for fluid flow through microchannels. Further work on steady and fully developed laminar flow of electrically conducting non-Newtonian fluid in square microchannels shows Power-Law [22] model can be applied, and MHD can affect fluid characteristics. Few works [23] [24] have shown the effect of MHD on slip flow. the present study has allowed a non-Newtonian fluid with electrical conductivity through the horizontal circular microchannel of 200 μm to observe the effect of magnetic induction (MHD) on the slip flow of non-Newtonian fluid. This work has considered xanthene in a formic acid solution, a non- Newtonian fluid with the property of electrical conductivity. the property of xanthene solution in formic acid is described in Table I. Xanthomonas campestris is a bacterium that produces extracellular hetero-polysaccharide. This polysaccharide is the main ingredient of xanthan gum. the main components of this extracellular hetero-polysaccharide are glucose, mannose and glucuronic acid, which remain in the solution at the molar ratio of 2:2:1. the solution exhibits electrical conductivity when xanthan gum is mixed with formic acid [25].

Table 1. Properties of xanthan solution in formic acid (25°c)

Xanthan [wt/vol%]	ρ (kg/m^3)	n	K (kg s^{n-2}/m)	Electrical conductivity (Ωm)$^{-1}$
0.5	1225	0.761	0.214	0.01343
1.0	1230	0.473	2.274	0.02657

2. Materials & Methods

In this study, the fluid flow of non-Newtonian nature through circular, straight microchannels using the power-law model at the entrance region as well as the developed section has been considered to understand the effect of slip velocity and a fluid flow parameter commonly known as slip coefficient β, defined as the ratio of slip velocity to the average velocity on the fluid field when externally applied magnetic field (B_0) is applied.

This numerical study covers the simultaneous development of flow for power-law fluids along with the effect of an externally applied magnetic field. the total procedure has been conducted using a commercial software package. This work is concerned with the effect of slip coefficients for different power-law indexes. the present problem is depicted schematically in Figure 1. the solution to the problem is restricted to only half the circular microchannel on either side of the microchannel, along with the symmetry of the computational domain. All the fluid properties are kept constant. the Ostwald–de Waele power law is considered to model the shear stress. the fluid at the circular microchannel entry is considered uniform velocity (u_e).

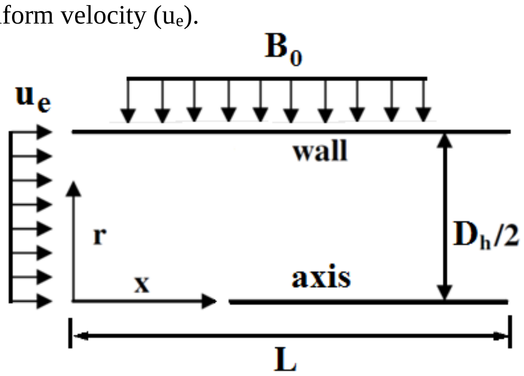

Fig. 1 Fluid domain of circular microchannel with the externally applied magnetic field (B_0)

2.1 Mathematical Modelling

the Navier Stokes equation in cylindrical co-ordinates along the r-z direction can be expressed as:

$$\rho\left(v_r \frac{\partial v_z}{\partial r} + v_z \frac{\partial v_z}{\partial z}\right) = -\frac{\partial P}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)\right] \tag{1}$$

The hydro-dynamically developed flows and power-law non-Newtonian fluids having constant properties throughout the computational domain in a circular microchannel (Figure 1) can be simplified. the steady-state fluid flow in the fully-developed condition of a power-law fluid has been considered in the circular microchannel. From the relation (1), it can be written:

$$\frac{\partial}{\partial r}(r\, \tau_{rz}) = -r\,\frac{\partial P}{\partial z} \quad (2)$$

the expression of shear stress (τ_{rz}) in the above equation (2) can be written as

$$\tau_{rz} = K\left(-\frac{\partial v_z}{\partial r}\right)^n \quad (3)$$

the Navier's slip boundary conditions can be summarized as follows:

at, $r=R_0$ $v_z=v_s$

at, $r=0$ $dv_s/dr = 0$

the non-dimensional form of the above boundary conditions can be stated as:

at, $R=1$ $v=\beta$

at, $R=0$ $dV/dR = 0$

Using the expressions (2) & (3) when boundary conditions are applied, we get the expression of velocity along the z-direction (v_z).

$$v_z = v_s + \left(\frac{3n+1}{n+1}\right)\left\{1-\left(\frac{r}{R_0}\right)^{\frac{n+1}{n}}\right\}(v_m - v_s) \quad (4)$$

Here mean velocity (v_m) can be expressed as,

$$v_m = v_s + \left(\frac{n}{n+1}\right)\left(-\frac{\Delta P}{L}\frac{R_0}{2m}\right)^n R_0 \quad (5)$$

Following are the parameters in their non-dimensional form, which are useful to find out the non-dimensional form of velocity V.

$$\frac{V_z}{V_m} = V, \frac{r}{R_0} = R, \frac{l}{R_0} = L, \frac{v_s}{v_m} = \beta, \frac{z}{R_0} = Z, \frac{p}{\rho v_m^2} = P,$$

$$Re = \frac{\rho\, v_m^{2-n} r_0^n}{m}$$

Using dimensionless quantities, the above equation can be expressed as

$$V = \beta + \left(\frac{3n+1}{n+1}\right)\left\{1-(R)^{\frac{n+1}{n}}\right\}(1-\beta) \quad (6)$$

at the centreline, where $r/R_0 = R = 0$ the velocity reaches its maximum value of v_{max}, which can be formulated as

$$V_{max} = \beta + \left(\frac{3n+1}{n+1}\right)(1-\beta) \quad (7)$$

A. Numerical Simulation

the governing equations are used in the numerical analysis of the externally applied magnetic field of strength B_0 and electrical conductivity σ, summarized as follows.

Continuity equation:

$$\frac{\partial v_r}{\partial r} + \frac{\partial v_z}{\partial z} = 0 \quad (8)$$

In non-dimensional form, the above equation can be written as:

$$\frac{\partial V_R}{\partial R} + \frac{\partial V_Z}{\partial Z} = 0 \quad (9)$$

Momentum equation:

$$\frac{\partial^2 v_z}{\partial r^2} + \frac{1}{r}\frac{\partial v_z}{\partial r} = \frac{1}{\mu}\frac{\partial p}{\partial z} + \frac{\sigma}{\mu}B_0^2 v_z \quad (10)$$

In non-dimensional form, the above equation can be written as:

$$\frac{\partial^2 V_z}{\partial R^2} + \frac{1}{R}\frac{\partial V_z}{\partial R} = Re.G + Ha.V_z \quad (11)$$

A mesh sensitivity test has been performed on the final set of numerical simulations. the actual number of meshes is selected based on the mesh independence result of the final solution. the present study involves 20000 elements and 22011 nodes to conduct the numerical tests. the governing equations were implemented on 10 × 2000 grids. These elements are arrayed in the dimension of a 20 mm long microtube of 200µm radius. the total computational domain is discretized into non-overlapping rectangular mesh elements. the convergence criterion is set to 10^{-6}. When the finite difference value reaches the value of 10^{-6}, the simulation terminates, and the solution of the numerical procedure is captured for the mesh result.

3. Result & Discussion

The present work has been done to understand the slip induced non-Newtonian fluid flow for the different values of the flow behaviour index. This work also extends its scope to compare velocity profiles of the flow when the externally applied magnetic field is applied on the microchannel. the accuracy of the total numerical procedure is achieved through a comparative study of results with specific cases which are already available and acceptable [7]. A comparison of velocity profiles in the fully developed section of the microchannel is depicted in figure 2, which shows an excellent agreement between the present work and already available results. A thorough study of different values of flow behaviour index and slip coefficients has been done. Table II shows a typical comparative result for validation of the present work.

Table 2. Comparison of Developed Centerline Velocity Obtained from the Present Study and Available Results

n=1 .25 & Re=100	β=0.0	β=0.1	β=0.2
	V_{max}	V_{max}	V_{max}
Barkhordari & Etemad	2.11	2.00	1.88
Present Study	2.07	1.96	1.88

to compare the centerline velocities along the length of the straight microchannel for the entrance zone and developed section, figure 3 may be referred which depicts the values for different slip coefficients. This figure denotes the development of fluid flow in the straight microchannel and how the flow achieves the maximum velocity (V_{max}) in the centerline for the different values of β that can be seen in this diagram. A comparison of velocity profiles for a constant slip coefficient β=0.1 has been

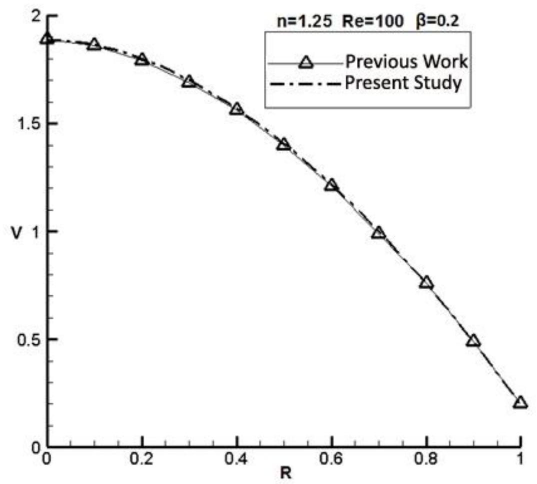

Fig. 2 Comparison of velocity profile obtained analytically and numerically

done for pseudoplastic (n=0.5) and dilatant (n=1.5) with Newtonian fluid (n=1.0) and the result is depicted in figure 4. the extent of slip in the vicinity of the wall is the same for three types of non-Newtonian fluids. Still, by the nature of the pseudoplastic fluid, it attends the least centerline velocity at the developed section of the channel. It is revealed that the maximum velocity is the highest for dilatant fluids. the least effect of inertia forces on dilatant fluid causes the higher values of centerline velocity. to understand the effect of β on the fluid flow patterns, figure 5 can be referred to.

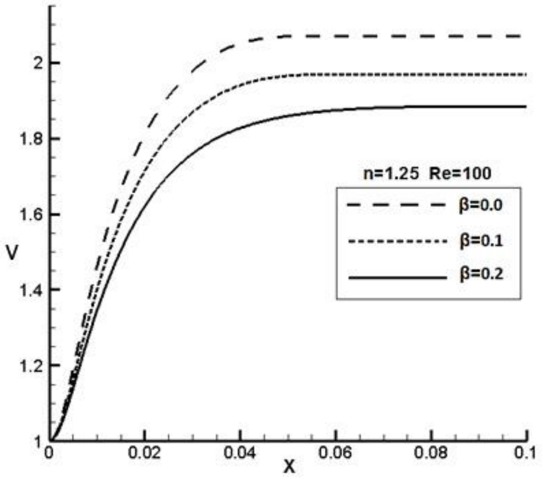

Fig. 3 Comparison of centerline velocities for different Slip Co-efficient (β)

for a given fluid n=0.5, the effect of β is less significant for the centerline velocities, but this is achieved due to the variance of slip velocity near the wall. When slip velocity varies, the fluid maintains a steady V_{max} at the centerline. This happens due to the balance of inertia force and the fluid's continuous shear rate. the slip length near the wall changes when β varies. A comparative study has been made and reported to observe this effect. Figure 6 shows when β increases, then slip length increases for non-Newtonian fluids to increase the value of slip velocity near the wall.

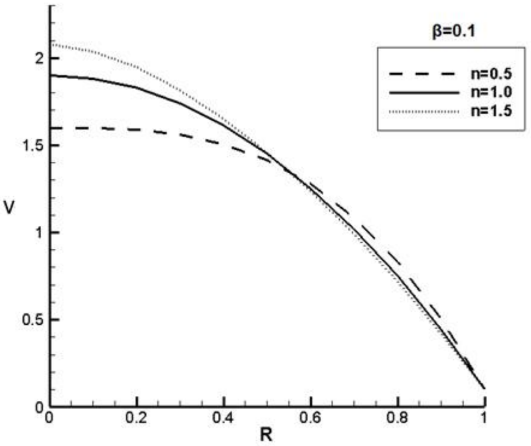

Fig. 4 Velocity profiles of slip flow through microchannel for the different flow behaviour index

It is observed that when β changes, the variation of maximum velocity is observed. Figure 7 shows that this variation is more predominant in pseudoplastic fluids than in dilatant fluids. But for all the cases, V_{max} decreases when β increases due to the increment of slip near the wall where the fluid flow decreases their centerline value of velocity to maintain force and mass balance according to the law of conservation. the extent of slip depends upon the Poiseuille no. (C_f. Re), which denotes the friction that fluid possesses when it passes over the wall. for slip flow, these values changes with the increase of hydraulic diameter (D_h).

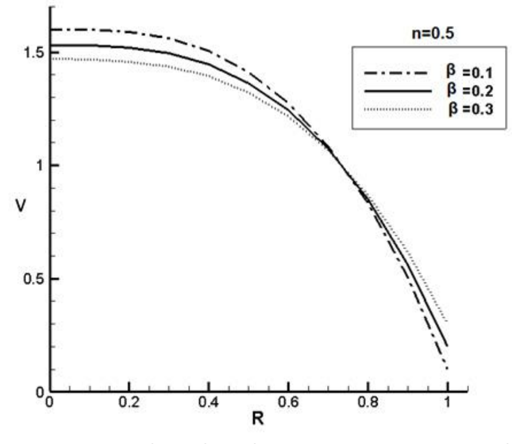

Fig. 5 Velocity profiles of slip flow through microchannel for different slip co-efficient

A comparative study has been made in support of this observation. Figure 8 shows the variation of the ratio of the Poiseuille no. between slip and no-slip flow for Newtonian and non-Newtonian fluids.

the present work finds the effect of the externally applied magnetic field of high intensities on a typical bio-fluid that is non-Newtonian. It is observed that an externally applied magnetic field has a

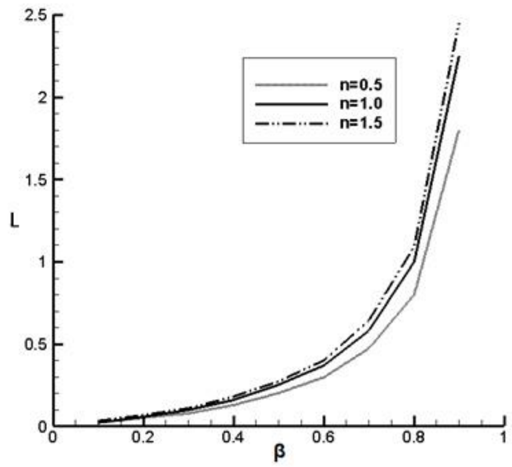

Fig. 6 Relation between Slip Length and Slip coefficients

Notable effect on the characteristics of fluid flow. This can be observed through the changes in the shapes of velocity profiles, as shown in figure 9.

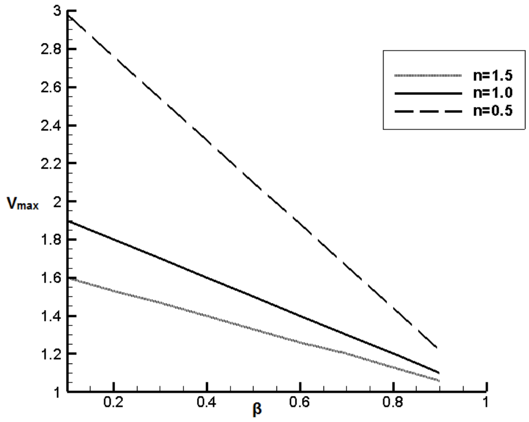

Fig. 7 Comparison of developed maximum velocity (V_{max}) and Slip Coefficients (β)

When a magnetic field is applied to the fluid flow having β=0.2, it is observed that the velocity profile becomes flat in shape with the increasing intensities.
With the increasing value of magnetic intensities, V_{max} decreases, and the corresponding slip velocity near-wall increases.

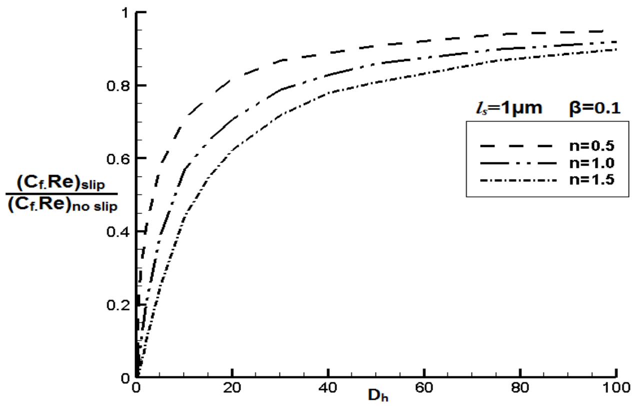

Fig. 8 Effect of hydraulic diameter (D_h) on Poiseuille No. (C_f.Re)

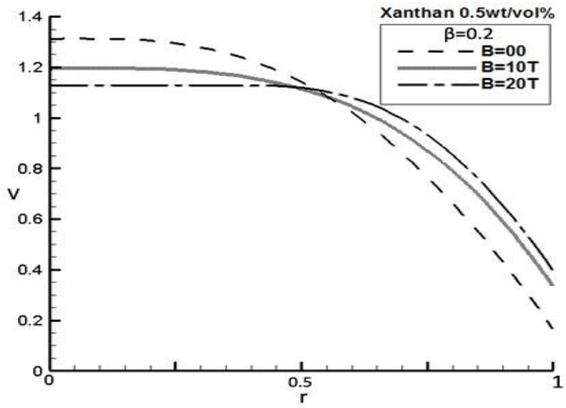

Fig. 9 the effect of the externally applied magnetic field on velocity profiles

This happens due to the effect of magnetohydrodynamics on the body force, and consequently, when intensity increases, the maximum velocity decreases. to encounter this situation for maintaining the force and mass balance of fluid flow, corresponding slip flow increases, a major notable observation.

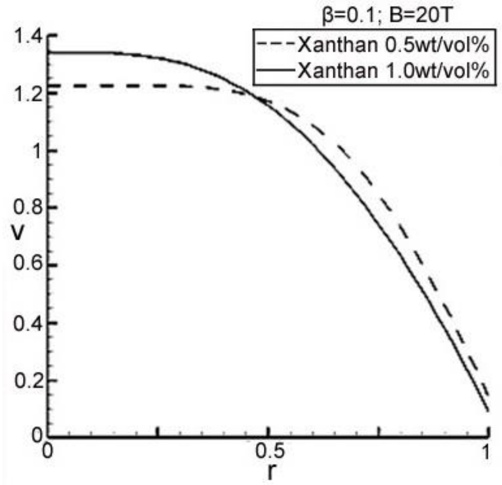

Fig. 10 Effect of concentration of Xanthan gum in Formic acid when B=20Tesla applied externally

With the change of the concentration of Xanthan in the solution of formic acid, the characteristics of non-Newtonian fluid flow change. Due to this reason, the consistency index and flow behaviour index change their values. to assess the effect of magnetic intensities on the changing properties of the fluid, figure 10 may be referred to. It is observed that when the flow behaviour index is decreased, then for a given externally applied magnetic field, the fluid flow shows an increased value of centerline velocity. This happens due to fluid properties like density, consistency index and electrical conductivity.

4. Conclusion

The velocity profile of the slip flow of non-Newtonian fluids shows that pseudoplastic fluids have higher centerline velocity and discharge than that dilatant fluids when exposed to the same slip coefficient β. If the slip coefficient varies for a constant flow behaviour index, centerline velocities increase and slip velocity decreases. Slip coefficient (β) affects the flow parameters like dimensionless slip length, centerline velocity and wall friction coefficient; with the increasing tendency in β dimensionless slip length increases, the centerline velocity decreases linearly. the maximum variation in these parameters can be seen for pseduoplastic fluids. for dilatant fluid, the nature of the effect remains similar but less predominant. When hydraulic diameter increases, then wall friction, the effect increases, and after a certain value, the same becomes asymptotic. the numerical analysis of the flow of Xanthan solution in Formic acid through a circular microchannel reveals the results of slip flow of non-Newtonian fluid. When the magnetic field is applied to the flow, slip becomes more predominant near walls, and centerline velocity decreases. When the electrical conductivity of the solution increases with Xanthan concentration, slip flow increases and consequently, centerline velocity and average velocity decrease. the results from the present study can be used for the better design of microchannel flow and MEMS.

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