

Original Article

A Measurement Application to Surfaces using Non-Linear Interpolation and Numerical Integration Methods

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Abstract - In this article, the area of a two-dimensional surface of the exteriors of the Universidad Nacional Mayor de San Marcos was calculated. It was done using the curve equation integration method, which will be obtained from the perimeter of the sector to be worked on by dividing it into sections, for which Google Maps was used to calculate the reference parameters. In general, not all curve equations are integrable; for this, the approximation methods such as Simpson 3/8, 1/3, and trapezoid were used. In addition, we will apply the Lagrange interpolation to find the equation of the curve; with the results obtained, we make a comparison between the methods, and so we see that the methods differ. Of them, the one with the least margin of error will be taken; these methods can be used in future works where the surface is irregular.

Keywords - Numerical integration, Lagrange interpolation, Simpson's method, Trapezoid.

1. Introduction

Numerical integration is the application of the study as the numerical value of a defined integral, which can be found through previously established procedures. This problem has a lot of significance since this mathematical discipline continues to have full force and a lot of interest. One of the most used methods is the calculations of the integrals of approximation of the area under a curve since architects and civil engineers use this to calculate the terrain where they are located. And hence his interest; therefore, all these scenarios poured into this discipline will be presented in the next report. Which consists of five chapters; the first is a small introduction to the subject. In the second chapter, the literary references were touched which helped us to have clearer concepts; in the third chapter, the methodology helped us to handle the concepts and definitions of the methods that we will use; in the fourth, results and discussions and finally, in the fifth chapter the conclusions were addressed.

2. Literary Review

Numerical integration methods have numerous applications in engineering. Accuracy in earthworks is important for many such as erosion studies, mining activity, and earth removal in construction projects. Generally, civil infrastructure works require modifications of the geometry of the terrain in the execution of their projects; it is the case of urban processes that involve the estimation of their locations in the volumes of surface to refine the control of costs and

budgets of the construction process, differences in your estimate between owner and contractor often cause problems that can even end up in legal issues.

The most used method on roads is that of cross-sections where concepts of integral calculation and discrete functions are applied for the identification of simple figures that make it practical to obtain the total areas in cut and fill, sometimes the rule of Trapezium, Simpson 1/3 and Simpson 3/8 is adopted to calculate the volume and obtain its average value, being the accuracy of the calculations proportional to the quality of representation of the terrain and depends on the density of the three-dimensional points, their distribution and interpolation method

In this way, engineering as a knowledge-transforming entity becomes a useful instrument to find intelligent and viable solutions, intending to contribute to the development of sustainable urban planning projects, which obey environmental preservation criteria, as a significant contribution to the process of location of artificial earth movements, defining the algorithm of less altimetric distortion and the calculation model of the area, that best suits surface conditions. Experimental development is proposed that allows, based on the construction and iconic scale model, the areas to be calculated using three mathematical models: Trapezoid rule, Simpson's 1/3 rule, and Simpson's rule 3/8.[1]



The transit or lamination of avenues by reservoirs is one of the fundamental problems of surface hydrology that is applied in the design and review of the excess works of a dam. In this type of work, engineers from different areas (civil, mechanical, electricians, geophysicists, geologists, among others) work. With the development of computer equipment, numerical methods for its calculation, all based on the continuity equation, have gained momentum as a tool to obtain results for this problem. With the comparison of the results obtained by using Newton's numerical integration methods: Trapezium, Simpson 1/3, and Simpson 3/8, to calculate the storage volume from the concept of the area enclosed between two curves (in this problem, the curves are the input and output hydropgrams). [2]

In the analysis of complex systems, it is widely used to determine synergy for the development of cities. However, sustainable cities need an effective energy diligence strategy based on management policies. Its objective is to encompass the synergy of localities by establishing a relationship between complexity and urban energy consumption. Demonstrating a technique for assessing physical complexity centered on a built environment in an urban region. Consequently, the study estimated urban complexity and energy consumption in cities using the Simpson manifold index and the passive volume ratio, which allowed physical complexity to be assessed using a sophisticated set. of tools to build more lasting cities. [3]

In the field of the topological project whose categorical structures have a high performance in thermal insulation, a design strategy is externalized in three layers, including macro and micro layers. Macroscopic structures are mesoscopic structures arranged periodically, whose topological configurations are subject to the material microstructures arranged at the micro-level. Microstructures and mesoscopic structures are independent of optimization, ensuring a hierarchical design synergy. The next step is four classical microstructures that provide the equivalent normalized thermal conductivities with arbitrary densities between 0 and 1. Then, it uses the floating projection technique to find the optimal topologies of thermoplastic structures. The objective function is defined by maximizing thermal conformity, subject to volume restriction that is added to the target function through the Lagrange multiplier. [4]

Also, when using Lagrange interpolation, it is so diverse that it can even be used not only to calculate the equation of the curve but can even be applied in the performance to measure the aliasing of the images. [5] This methodology will be used as a complement in the approximation to be able to estimate with a margin of accuracy, and this will be applied can be applied in the equations obtained, for example, when using a first and second-order calculation

when applying it in the precision of the non-linear frequency estimate. [6]

Consolidating the soil with stone columns is a distinguished method for improving the soil. This reinforcement by installing columns modifies the properties of the surrounding soft soil; this is known as a "smear." Despite the research, field studies on the characterization of the proper smear area are limited. Field research was developed to learn about the effects of the facility by collecting a series of intact soil samples from a soft soil site reinforced with newly installed stone columns, as well as conducting laboratory tests followed by non-linear regression analysis. He showed that the installation of stone columns induced a considerable alteration in the soft properties of the soil when they were closely spaced. An explicit model of the Lagrange fast finite-difference was carried out using the unit cell analogy, simulating the non-linear variation of the permeability and compressibility of soft clays. The parameters of the interrupted soft soil that they obtained were successfully used to make numerical models. In contrast, another study performed 2D finite element modeling. The results from the finite difference model coincided with the calculated field data compared to the finite element results. [7]

To calculate the fractal dimension, the standard method uses a compass that, through a specific radius, draws an arc that allows finding the points that intercept the curve. It is essential that when measuring with this method, the impact of the observer is considered. Since the rules, when read by the user, are very susceptible to reading or visual errors. The main contribution lies in promoting an algorithm to calculate the fractal dimension of fluctuating continuous functions on a fractally divided space, which also allows obtaining the integral of the function quite accurately by using the trapezoidal rule. [8]

It can be said that higher-order uncertain differential equations are used to capture differentiable uncertain systems with higher-order differentials. Generally, higher-order differential equations have no analytic solutions, such as the design of numerical methods for solving the uncertain higher-order differential equation. Using Simpson's method, a numerical method is designed to solve uncertain differential equations of higher order. It consists of a procedure through numerical experiments to demonstrate the usefulness and validity of the Simpson method. As well as calculating the integral of the uncertain differential equation of higher-order with the help of the same method. [9]

3. Methodology

Numerical integration is a tool of great importance because it allows us to find closer results of defined integrals that, when solved analytically, will take the time or because

the integrand is only known in tabulated form; an example would be the data obtained from an experiment. [10]

Approximating that integral by the integral of a simpler function approximating the original function.[11]

The numerical integration is framed within the theory of the approximation, within the area of Applied Mathematics, and is passing from time to time to have had growth during the last years; this is due to the appearance of computers and advanced programs that allow their calculations to be simpler and before this, referring them to different scientific areas. Consequently, this approximates the area under a curve, which at first glance, is impossible to perform its calculations. Still, many researchers have discovered how to solve it.

Keep in mind that this scenario presents a shape or curve defined as $f(x)$ and is defined between an interval within the space $[a, b]$ and is presented as follows:[12]

$$I(f) = \int_a^b f(x)dx$$

With this equation posed, many people have been using it for geometrically impossible spaces to conventional calculation. In the field of construction, many architects and civil engineers are seen with scenarios as areas that are not defined by the naked eye, and their calculation would be impossible to do conventionally. Then here, an approximation method is used. When fractionating part of the area is calculated with an established limit and a high interval, the area would be calculated with an approximation such high that the margin of error would be almost zero. Then with the above, it will be possible to use the formulas already demonstrated, such as the trapezoid or the Simpson method.

The limitation to deal with in this research is to find a subjectivity in the results since, throughout this research, we can verify that using both the calculations and the Simpson Method or the Trapezium present calculations so precise that their margin of error is almost insignificant. However, you also must know when to use it and see which of the two applications with the information provided. Additionally, being hypothetical calculations of an area that has not yet been determined, it is not possible to acquire it from a database since the areas are subjective in any purchase, so calculations would be made assuming the margin of reality at the time of presenting the research. Thus, when performing the calculations, it was impossible to find the surface's topographic information or the measurements of the UNMSM, so another application had to be used to extract the calculations manually with an accuracy level of 87% approx.

3.1. Trapezium Method

The evaluation of the integrals is called quadrature, from an old problem in geometry known as the Greek squaring of the circle utilizing regular inscribed and circumscribed polygons, a process that earned Archimedes to limit the value of pi. [13]

It is one of the methods that is most used in the approximation of the area under the curve where it is emphasized that this method is used as long as the integral does not have a solution in a conventional way where the approximation of the method is expected to be continuous and positive within the interval to make the approximation with this rule a first-order polynomial will be used, and this is represented as follows:[14]

$$P_1(x) = f(a) + \frac{f(b) - f(a)}{b - a}(x - a)$$

Then the integral is replaced:

$$\begin{aligned} I &= \int_a^b f(x)dx \approx \int_a^b P_1(x)dx \\ &\approx \int_a^b \left[f(a) + \frac{f(b) - f(a)}{b - a}(x - a) \right] dx \end{aligned}$$

And in the end, the integral has:

$$\int_a^b f(x)dx \approx (b - a) \frac{f(a) + f(b)}{2}$$

The error term corresponds to:

$$E = -\frac{1}{12} f''(\epsilon)(b - a)^3$$

Being ϵ a number belonging to the interval $[a, b]$.

3.2. Simpson rules

Unlike the trapezoid method, which uses trapezoid forms to approximate the calculation of the area under the curve, Simpson's method uses the approximation of equally spaced subdivisions. It adjusts them by a parabola, and by adding all the subdivisions, you can have the almost exact approximation of that area.

For example, if there is another point in the middle between $f(a)$ and $f(b)$ The three points can be joined with a parabola. If there are two points equally spaced between $f(a)$ and $f(b)$ A third-degree polynomial can join the four points. The formulas that result from taking the integrals under those polynomials are known as Simpson's rules.[15]

To calculate the area under the curve by applying Simpson's rule, the following formula is used:

$$I = \int_a^b P_2(x)dx + \text{error term} = \frac{b-a}{6}[f(a) + 4f(m) + f(b)] + E(f),$$

Where $E(f)$ is the error term:

$$\int_a^b f(x)dx \approx \frac{b-a}{6}[f(a) + 4f(m) + f(b)].$$

3.2.1. Simpson 1/3

This method generates a more precise approximation; it consists of connecting successive groups of three points on the curve utilizing second-degree parabolas and adding the areas under the parabolas to obtain the approximate area. [16]

And this representation has the following form:

$$\int_a^b f(x)dx \approx \frac{h}{3}[f(a) + 4f(x_m) + f(b)] , \text{ with } h = \frac{(b-a)}{2}$$

The estimated error is given by:

$$E(f) = -\frac{h^5}{90}f^{(4)}(\varepsilon) ,$$

$E(f)$ Involves the fourth derivative and $\varepsilon \in [a, b]$.

3.2.2. Simpson 3/8

The Simpson 3/8 integration scheme considers third-degree passing polynomials between every four points of the function $y(x)$ To apply this method, n must be a multiple of 3. [16]

The following equation gives it:

$$\int_a^b f(x)dx \approx \frac{3h}{8} \left[f(a) + 3f\left(\frac{2a+b}{3}\right) + 3f\left(\frac{a+2b}{3}\right) + f(b) \right] \text{ with } h = \frac{(b-a)}{2},$$

The estimated error is given by:

$$E(f) = -\frac{3h^5}{80}f^{(4)}(\varepsilon),$$

$E(f)$ Involves the fourth derivative and $\varepsilon \in [a, b]$.

You must consider the advantages presented by these rules to have a more accurate calculation. The rule of the Trapezium and that of Simpson 1/3 are simpler, but that of the Trapezium is more optimal for improper integrals. Still, in terms of precision, the rule of Simpson 3/8 is more

approximate to the real values. As for the disadvantages, it should be taken into account that the trapezoid should take large intervals in case it gets closer. In Simpson 1/3, this will work with even intervals, and Simpson 3/8 would have to be a multiple of three. [17]

3.3. Lagrange Interpolation

This method consists of constructing the degree interpolator polynomial n that goes through $n + 1$ points (x_i, y_i) of the form:

$$P_n(x) = \sum_{i=0}^n L_i(x)y_i$$

Where the functions $L_i(x)$ meet $L_i(x_k) = 0$ if $i \neq k$ and $L_i(x_i) = 1$. This property guarantees $P_n(x_k) = y_k$. Functions $L_i(x)$ are constructed as:

$$L_i(x) = \frac{(x - x_1)(x - x_2)(x - x_3) \cdots \cdots (x - x_n)}{(x_i - x_1)(x_i - x_2)(x_i - x_3) \cdots \cdots (x_i - x_n)}$$

This method allows us to construct the interpolator polynomial explicitly easily. Two more interpolations are linear and parabolic. [18]

To find the area of the Universidad Nacional Mayor de San Marcos and in general of any area, first, we must delimit it and locate it in space, where we will use the GPS. We can approximate the measurements obtained from Google Maps and thus take significant calculations; with the points found, we can find the equation using Lagrange interpolation. Finally, by applying the integral, we will find the total area, and of the result obtained, we can compare it with the approximation methods of Simpson and Trapezium.

3.4. Location of the Space

To calculate the area of any two-dimensional surface, it must first be located, and then the GPS will be used; in this way, it will be possible to find it without difficulties.

Using this tool, we know that the Universidad Nacional Mayor de San Marcos (UNMSM) is located exactly at the points -12.057182 and -77.081794 of latitude and longitude, respectively.

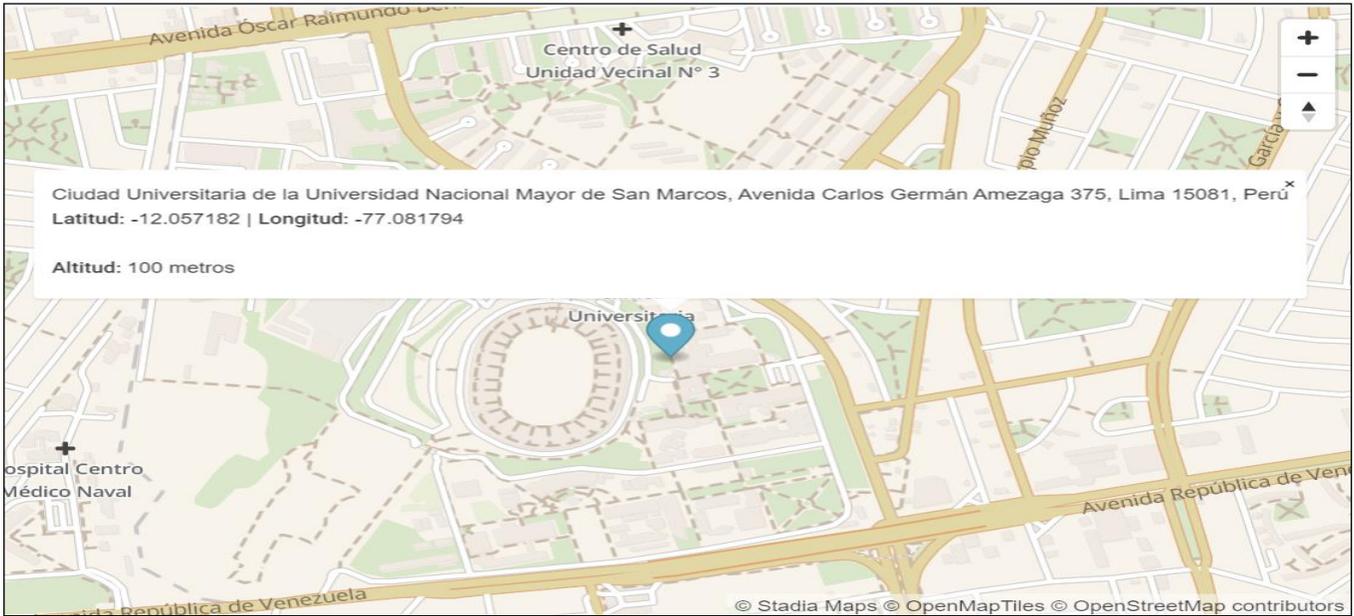


Fig. 1 Latitude and longitude of UNMSM

3.5. Delimit Sections

Having the surface located will be known by its shape if the calculation can be direct, but if its figure is complex, it is recommended to divide it into sections.

The form of the UNMSM will be delimited into four sections because its calculation will be simpler and more precise. For this calculation, Av. República de Venezuela will be taken as Axis X.

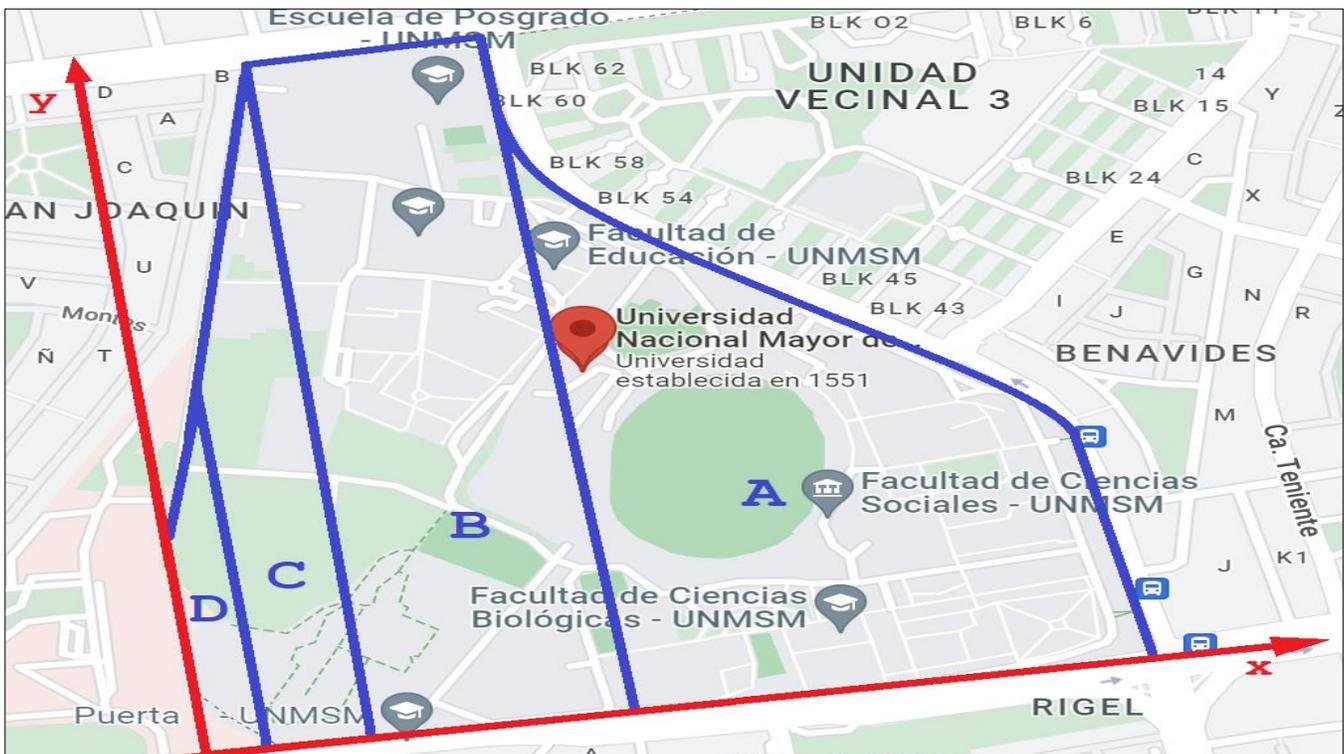


Fig. 2 Delimitation of the UNMSM

3.6. Calculate Distance Points

Considering the sections our surface will have, the distance from one point to another will be found; taking a reference axis for this, Google Maps will be used using the option to measure distance.

It should be noted that the localized distances must be exact, considering all decimals. Still, in this research, an approximation will be made, taking integer values so that the equation and calculations are understandable.

When finding the distances and the extreme points of them, considering the X axis and the sections to be calculated, they will have to be put in tables to find their equation.

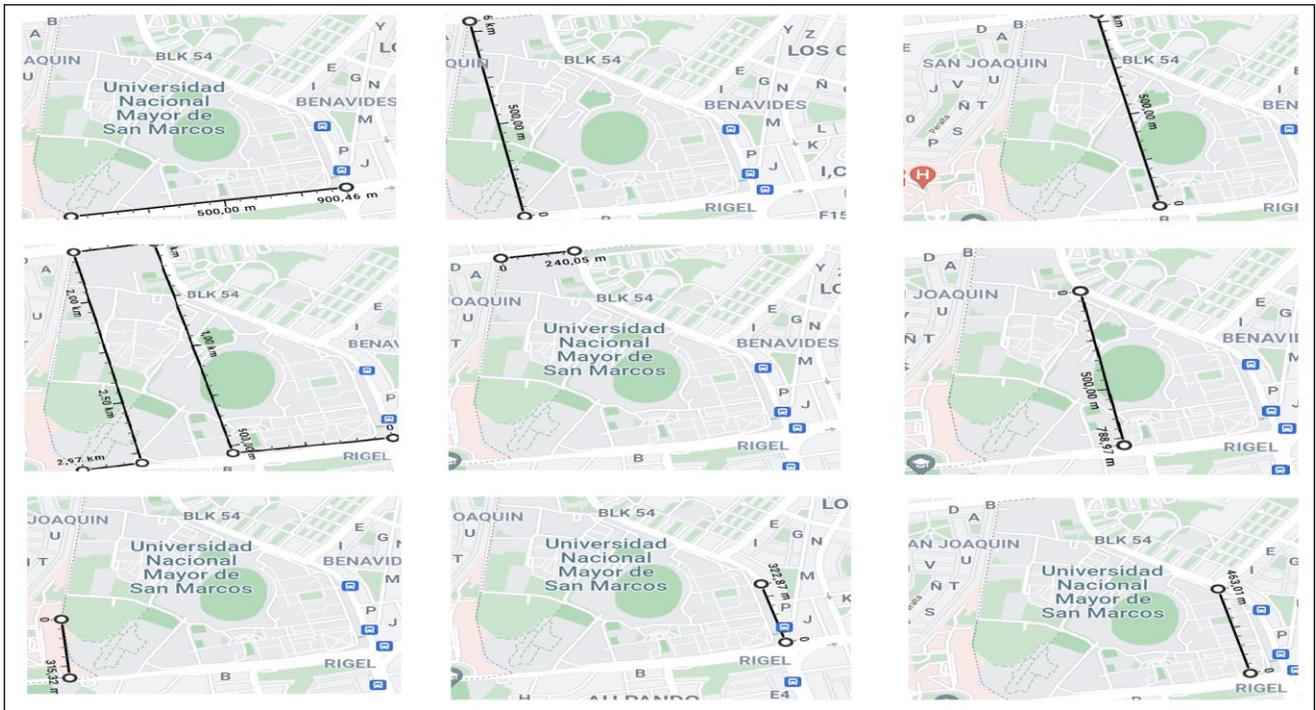


Fig. 3 Calculations of Point Distances in the UNMSM

3.7. Measurement Tables

To find the equation of a curve, you must have the points that represent it, and these on the x-axis will have values that start from the zero point to the extreme point of the distance that is the Y axis, and these will be recorded in segments to know the interpolation points.

Table 1. Area A Values

| AREA A | | |
|--------|-----|-----|
| Points | x | y |
| 1 | 0 | 910 |
| 2 | 34 | 870 |
| 3 | 260 | 535 |
| 4 | 490 | 320 |

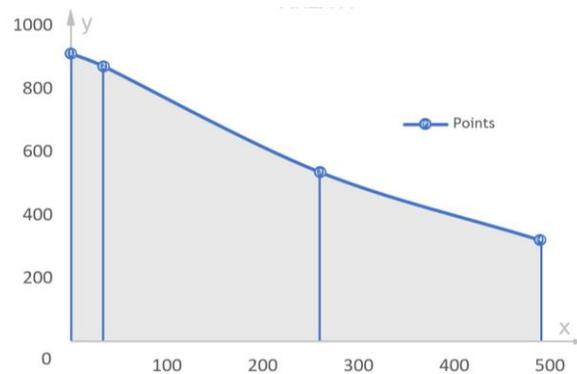


Fig. 4 Area A Curve

Table 2. Area B Values

| AREA B | | |
|--------|-----|------|
| Points | x | y |
| 1 | 0 | 1060 |
| 2 | 140 | 1055 |
| 3 | 260 | 1050 |

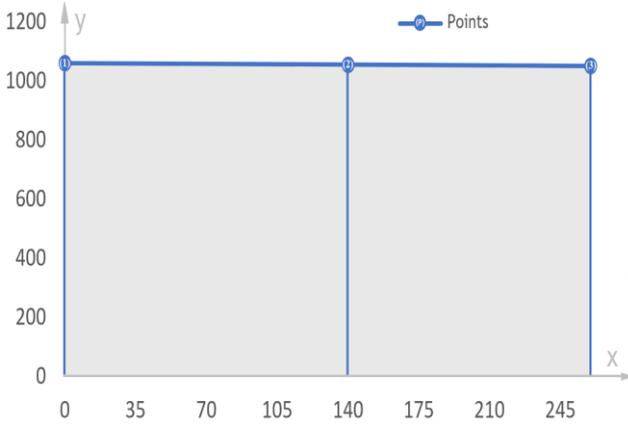


Fig. 5 Area B Curve

Table 3. Area C Values

| AREA C | | |
|--------|-----|------|
| Points | x | y |
| 1 | 0 | 420 |
| 2 | 100 | 740 |
| 3 | 210 | 1060 |

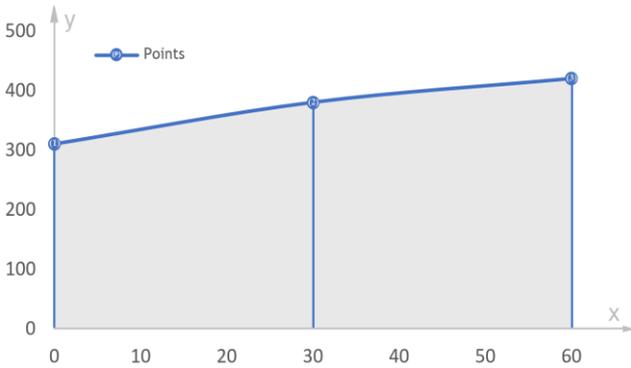


Fig. 6 Area C Curve

Table 4. Area D Values

| AREA D | | |
|--------|----|-----|
| Points | x | y |
| 1 | 0 | 310 |
| 2 | 30 | 380 |
| 3 | 60 | 420 |

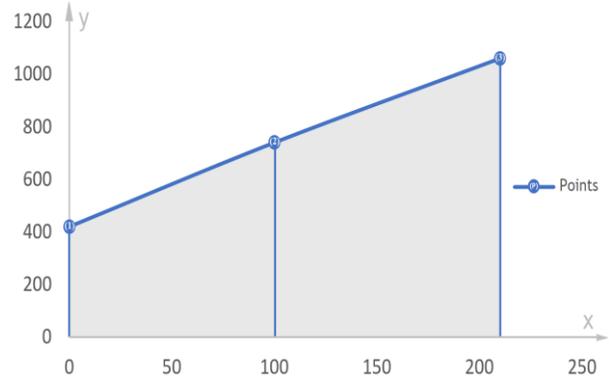


Fig. 7 Area D Curve

3.8. Calculate The Equation

Having the points of both the X Axis and the Y Axis you can apply any program that allows you to obtain the equation of the curve, however, it is recommended to use the Lagrange interpolation for the precision it provides. Then in this way, the calculation will be made to each of the sections:

3.8.1. Area A: Lagrange Interpolation

$$L(x) = Y_1L_1(x) + Y_2L_2(x) + Y_3L_3(x) + Y_4L_4(x)$$

$$L(x) = 910 \frac{(x-34).(x-260).(x-490)}{(0-34).(0-260).(0-490)} + 870 \frac{(x-0).(x-260).(x-490)}{(34-0).(34-260).(34-490)} + 535 \frac{(x-0).(x-34).(x-490)}{(260-0).(260-34).(260-490)} + 320 \frac{(x-0).(x-34).(x-260)}{(490-0).(490-34).(490-260)}$$

$$L(x) = 910 \frac{(x-34).(x-260).(x-490)}{(-34).(-260).(-490)} + 870 \frac{x.(x-260).(x-490)}{34.(-226).(-456)} + 535 \frac{x.(x-34).(x-490)}{260.226.(-230)} + 320 \frac{x.(x-34).(x-260)}{490.456.230}$$

$$L(x) = 910 \frac{(x^2-294x+8840).(x-490)}{8840.(-490)} + 870 \frac{(x^2-260x).(x-490)}{(-7684).(-456)} + 535 \frac{(x^2-34x).(x-490)}{58760.(-230)} + 320 \frac{(x^2-34x).(x-260)}{223440.230}$$

$$L(x) = 910 \frac{(x^3-784x^2+152900x-4331600)}{(-4331600)} + 870 \frac{(x^3-750x^2+127400x)}{3503904} + 535 \frac{(x^3-524x^2+16660x)}{(-13514800)} + 320 \frac{(x^3-294x^2+8840x)}{51391200}$$

$$L(x) = -\frac{(x^3-784x^2+152900x-4331600)}{4331600} + \frac{145(x^3-750x^2+127400x)}{583984} - \frac{107(x^3-524x^2+16660x)}{2702960} + \frac{2(x^3-294x^2+8840x)}{321195}$$

$$L(x) = \frac{77821}{16042405470}x^3 - \frac{1136043}{436528040}x^2 - \frac{7017557419}{6416962188}x + 910$$

$$L(x) = 0.000005x^3 - 0.0026x^2 - 1.0936x + 910$$

3.8.2. Area B: Lagrange Interpolation

$$L(x) = Y_1L_1(x) + Y_2L_2(x) + Y_3L_3(x)$$

$$L(x) = 1060 \frac{(x-140).(x-260)}{(0-140).(0-260)} + 1055 \frac{(x-0).(x-260)}{(140-0).(140-260)} + 1050 \frac{(x-0).(x-140)}{(260-0).(260-140)}$$

$$L(x) = 1060 \frac{(x-140).(x-260)}{(-140).(-260)} + 1055 \frac{x.(x-260)}{140.(-120)} + 1050 \frac{x(x-140)}{260.120}$$

$$L(x) = 1060 \frac{(x^2 - 400x + 36400)}{36400} + 1055 \frac{(x^2 - 260x)}{(-16800)} + 1050 \frac{(x^2 - 140x)}{31200}$$

$$L(x) = \frac{53}{1820}(x^2 - 400x + 36400) - \frac{211}{3360}(x^2 - 260x) + \frac{7}{208}(x^2 - 140x)$$

$$L(x) = -\frac{1}{43680}x^2 - \frac{71}{2184}x + 1060$$

$$L(x) = -0.00002x^2 - 0.0325x + 1060$$

3.8.3. Area C: Lagrange Interpolation

$$L(x) = Y_1L_1(x) + Y_2L_2(x) + Y_3L_3(x)$$

$$L(x) = 420 \frac{(x-100).(x-210)}{(0-100).(0-210)} + 740 \frac{(x-0).(x-210)}{(100-0).(100-210)} + 1060 \frac{(x-0).(x-100)}{(210-0).(210-100)}$$

$$L(x) = 420 \frac{(x-100).(x-210)}{(-100).(-210)} + 740 \frac{x.(x-210)}{100.(-110)} + 1060 \frac{x.(x-100)}{210.110}$$

$$L(x) = 420 \frac{(x^2 - 310x + 21000)}{21000} + 740 \frac{(x^2 - 210x)}{(-11000)} + 1060 \frac{(x^2 - 100x)}{23100}$$

$$L(x) = \frac{1}{50}(x^2 - 310x + 21000) - \frac{37}{550}(x^2 - 210x) + \frac{53}{1155}(x^2 - 100x)$$

$$L(x) = -\frac{8}{5775}x^2 + \frac{3856}{1155}x + 420$$

$$L(x) = -0.0014x^2 + 3.3385x + 420$$

3.8.4. Area D: Lagrange Interpolation

$$L(x) = Y_1L_1(x) + Y_2L_2(x) + Y_3L_3(x)$$

$$L(x) = 310 \frac{(x-30).(x-60)}{(0-30).(0-60)} + 380 \frac{(x-0).(x-60)}{(30-0).(30-60)} + 420 \frac{(x-0).(x-30)}{(60-0).(60-30)}$$

$$L(x) = 310 \frac{(x-30).(x-60)}{(-30).(-60)} + 380 \frac{x.(x-60)}{30.(-30)} + 420 \frac{x.(x-30)}{60.30}$$

$$L(x) = 310 \frac{(x^2 - 90x + 1800)}{1800} + 380 \frac{(x^2 - 60x)}{(-900)} + 420 \frac{(x^2 - 30x)}{1800}$$

$$L(x) = \frac{31}{180}(x^2 - 90x + 1800) - \frac{19}{45}(x^2 - 60x) + \frac{7}{30}(x^2 - 30x)$$

$$L(x) = -\frac{1}{60}x^2 + \frac{17}{6}x + 310$$

$$L(x) = -0.0167x^2 + 2.8333x + 310$$

3.9. Calculate the Area

With the equation obtained from each section, the area under the curve will have to be found using integrals.

3.9.1. Area A: Integral calculation

$$S_A(x) = \int_0^{490} 0.000005x^3 - 0.0026x^2 - 1.0936x + 910 dx$$

$$S_A(x) = 284710.8658$$

3.9.2 Area B: Integral calculation

$$S_B(x) = \int_0^{260} -0.00002x^2 - 0.0325x + 1060 dx$$

$$S_B(x) = 274384.32666$$

3.9.3. Area C: Integral calculation

$$S_C(x) = \int_0^{210} -0.0014x^2 + 3.3385x + 420 dx$$

$$S_C(x) = 157492.125$$

3.9.4 Area D: Integral calculation

$$S_D(x) = \int_0^{60} -0.0167x^2 + 2.8333x + 310 dx$$

$$S_D(x) = 22497.54$$

4. Results and Discussions

By having the area of each of the sections, the approximation methods of Simpson 3/8, Simpson 1/3, and Trapezium will be applied to compare with the area obtained, obtain the margin of error, and thus choose the most optimal for the calculation.

4.1. Calculation of Simpson's rule 3/8

To perform the approximation calculation of the Simpson method 3/8, presented in tables 5, 6, 7, and 8, it is being considered that "n" has a unique value of 6 because this, in turn, is a multiple of 3 and pair, thus maintaining a homogeneous result so that it can be compared, remembering that "n" can take any value. To calculate the iterations of the tables, we had to subtract the integration interval (b-a), and this was divided by "n," resulting in the value "h" that would help us complete the table from iteration 0 to iteration 6 that will be completed with the successive sum of that value.

Finally, the results of each of the iterations (I) it was multiplied by the characteristic coefficient of Simpson 3/8, and the sum of the products obtained will be multiplied by the value "h" and 3/8, giving us the result of the approximate area.

Table 5. Calculation of area A with Simpson's rule 3/8

| ANALYSIS OF VALUES | |
|--------------------|----------|
| a | 0 |
| b | 490 |
| n | 6 |
| h | 81.66667 |
| Sum | 9296.681 |

Table 6. Calculation of area B with Simpson's rule 3/8

| ZZZAREA B | | | | |
|----------------------------|-------------|-------------|--------|-------------|
| I | x | f(x) | Coeff. | Product |
| 0 | 0 | 1060 | 1 | 1060 |
| 1 | 43.3333333 | 1058.554111 | 3 | 3175.662333 |
| 2 | 86.6666667 | 1057.033111 | 3 | 3171.099333 |
| 3 | 130 | 1055.437 | 2 | 2110.874 |
| 4 | 173.3333333 | 1053.765778 | 3 | 3161.297333 |
| 5 | 216.6666667 | 1052.019444 | 3 | 3156.058333 |
| 6 | 260 | 1050.198 | 1 | 1050.198 |
| Area by Simpson's Rule 3/8 | | | | 274384.3267 |

Table 7. Calculation of area C with Simpson's rule 3/8

| ANALYSIS OF VALUES | |
|--------------------|---------|
| a | 0 |
| b | 210 |
| n | 6 |
| h | 35 |
| Sum | 11999.4 |

| AREA C | | | | |
|----------------------------|-----|----------|--------|------------|
| I | x | f(x) | Coeff. | Product |
| 0 | 0 | 420 | 1 | 420 |
| 1 | 35 | 535.1325 | 3 | 1605.3975 |
| 2 | 70 | 646.835 | 3 | 1940.505 |
| 3 | 105 | 755.1075 | 2 | 1510.215 |
| 4 | 140 | 859.95 | 3 | 2579.85 |
| 5 | 175 | 961.3625 | 3 | 2884.0875 |
| 6 | 210 | 1059.345 | 1 | 1059.345 |
| Area by Simpson's Rule 3/8 | | | | 157492.125 |

| AREA A | | | | |
|----------------------------|-------------|-------------|--------|-------------|
| I | x | f(x) | Coeff. | Product |
| 0 | 0 | 910 | 1 | 910 |
| 1 | 81.6666667 | 806.0721343 | 3 | 2418.216403 |
| 2 | 163.3333333 | 683.8032963 | 3 | 2051.409889 |
| 3 | 245 | 559.533625 | 2 | 1119.06725 |
| 4 | 326.6666667 | 449.6032593 | 3 | 1348.809778 |
| 5 | 408.3333333 | 370.352338 | 3 | 1111.057014 |
| 6 | 490 | 338.121 | 1 | 338.121 |
| Area by Simpson's Rule 3/8 | | | | 284710.8658 |

| ANALYSIS OF VALUES | |
|--------------------|----------|
| a | 0 |
| b | 60 |
| n | 6 |
| h | 10 |
| Sum | 5999.344 |

| AREA D | | | | |
|----------------------------|----|---------|--------|----------|
| I | x | f(x) | Coeff. | Product |
| 0 | 0 | 310 | 1 | 310 |
| 1 | 10 | 336.663 | 3 | 1009.989 |
| 2 | 20 | 359.986 | 3 | 1079.958 |
| 3 | 30 | 379.969 | 2 | 759.938 |
| 4 | 40 | 396.612 | 3 | 1189.836 |
| 5 | 50 | 409.915 | 3 | 1229.745 |
| 6 | 60 | 419.878 | 1 | 419.878 |
| Area by Simpson's Rule 3/8 | | | | 22497.54 |

4.2. Calculation of Simpson's rule 1/3

Similarly, to perform the approximation calculation of the Simpson method 1/3 presented in Tables 9, 10, 11, and 12, the "n" will also be considered to have a value of 6. To calculate the iterations of the tables, the integration interval (b-a) had to be subtracted, and this was divided by "n," resulting in the value "h" that would help us complete the table from iteration 0 to iteration 6, which will be completed with the successive sum of that value.

Finally, the results of each of the iterations (I) it was multiplied by the characteristic coefficient of Simpson 1/3, and the sum of the products obtained will be multiplied by the value "h" and 1/3, giving us the result of the approximate area.

Table 9. Calculation of area A with Simpson's rule 1/3

| ANALYSIS OF VALUES | |
|--------------------|------------|
| a | 0 |
| b | 490 |
| n | 6 |
| h | 81.6666667 |
| Sum | 10458.7665 |

| AREA A | | | | |
|----------------------------|------------|------------|--------|-------------|
| I | x | f(x) | Coeff. | Product |
| 0 | 0 | 910 | 1 | 910 |
| 1 | 81.6666667 | 806.072134 | 4 | 3224.288537 |
| 2 | 163.333333 | 683.803296 | 2 | 1367.606593 |
| 3 | 245 | 559.533625 | 4 | 2238.1345 |
| 4 | 326.666667 | 449.603259 | 2 | 899.2065185 |
| 5 | 408.333333 | 370.352338 | 4 | 1481.409352 |
| 6 | 490 | 338.121 | 1 | 338.121 |
| Area by Simpson's Rule 1/3 | | | | 284710.8658 |

Table 10. Calculation of area B with Simpson's rule 1/3

| ANALYSIS OF VALUES | |
|--------------------|------------|
| a | 0 |
| b | 260 |
| n | 6 |
| h | 43.3333333 |
| Sum | 18995.838 |

Table 11. Calculation of area C with Simpson's rule 1/3

| ANALYSIS OF VALUES | |
|--------------------|-----------|
| a | 0 |
| b | 210 |
| n | 6 |
| h | 35 |
| Sum | 13499.325 |

| AREA C | | | | |
|----------------------------|-----|----------|--------|------------|
| I | x | f(x) | Coeff. | Product |
| 0 | 0 | 420 | 1 | 420 |
| 1 | 35 | 535.1325 | 4 | 2140.53 |
| 2 | 70 | 646.835 | 2 | 1293.67 |
| 3 | 105 | 755.1075 | 4 | 3020.43 |
| 4 | 140 | 859.95 | 2 | 1719.9 |
| 5 | 175 | 961.3625 | 4 | 3845.45 |
| 6 | 210 | 1059.345 | 1 | 1059.345 |
| Area by Simpson's Rule 1/3 | | | | 157492.125 |

Table 12. Calculation of area D with Simpson's rule 1/3

| ANALYSIS OF VALUES | |
|--------------------|----------|
| a | 0 |
| b | 60 |
| n | 6 |
| h | 10 |
| Sum | 6749.262 |

| AREA D | | | | |
|----------------------------|----|---------|-------|----------|
| I | x | f(x) | Coeff | Product |
| 0 | 0 | 310 | 1 | 310 |
| 1 | 10 | 336.663 | 4 | 1346.652 |
| 2 | 20 | 359.986 | 2 | 719.972 |
| 3 | 30 | 379.969 | 4 | 1519.876 |
| 4 | 40 | 396.612 | 2 | 793.224 |
| 5 | 50 | 409.915 | 4 | 1639.66 |
| 6 | 60 | 419.878 | 1 | 419.878 |
| Area by Simpson's Rule 1/3 | | | | 22497.54 |

4.3. Calculation of the trapezium method

To perform the approximation calculation with the trapezoid method, presented in tables 13, 14, 15, and 16, it will also be considered that "n" is 6. The same procedure as in the Simpson method tables will be used because to calculate the iterations, the integration interval (b-a) will have to be subtracted and divided by "n." As a result, the value "h" and this table will be completed with the successive sum of said value from iteration 0 to iteration 6. The values obtained will be multiplied by the trapezoid's characteristic coefficient, where the products' sum will be multiplied by the value "h" and 1/2, giving us the result of the approximate area.

Table 13. Calculation of area A with the trapezoid

| ANALYSIS OF VALUES | |
|--------------------|------------|
| a | 0 |
| b | 490 |
| n | 6 |
| h | 81.6666667 |
| Sum | 6986.85031 |

| AREA A | | | | |
|------------------------------|------------|------------|--------|-------------|
| I | x | f(x) | Coeff. | Product |
| 0 | 0 | 910 | 1 | 910 |
| 1 | 81.6666667 | 806.072134 | 2 | 1612.144269 |
| 2 | 163.333333 | 683.803296 | 2 | 1367.606593 |
| 3 | 245 | 559.533625 | 2 | 1119.06725 |
| 4 | 326.666667 | 449.603259 | 2 | 899.2065185 |
| 5 | 408.333333 | 370.352338 | 2 | 740.7046759 |
| 6 | 490 | 338.121 | 1 | 338.121 |
| Area by the trapezoid method | | | | 285296.3875 |

Table 14. Calculation of area B with the trapezoid method

| ANALYSIS OF VALUES | |
|--------------------|------------|
| a | 0 |
| b | 260 |
| n | 6 |
| h | 43.3333333 |
| Sum | 12663.8169 |

| AREA B | | | | |
|------------------------------|-------------|------------|--------|-------------|
| I | x | f(x) | Coeff. | Product |
| 0 | 0 | 1060 | 1 | 1060 |
| 1 | 43.3333333 | 1058.55411 | 2 | 2117.108222 |
| 2 | 86.6666667 | 1057.03311 | 2 | 2114.066222 |
| 3 | 130 | 1055.437 | 2 | 2110.874 |
| 4 | 173.3333333 | 1053.76578 | 2 | 2107.531556 |
| 5 | 216.666667 | 1052.01944 | 2 | 2104.038889 |
| 6 | 260 | 1050.198 | 1 | 1050.198 |
| Area by the trapezoid method | | | | 274382.6993 |

Table 15. Calculation of area C with the trapezoid

| ANALYSIS OF VALUES | |
|--------------------|---------|
| a | 0 |
| b | 210 |
| n | 6 |
| h | 35 |
| Sum | 8996.12 |

| AREA C | | | | |
|------------------------------|-----|----------|--------|------------|
| I | x | f(x) | Coeff. | Product |
| 0 | 0 | 420 | 1 | 420 |
| 1 | 35 | 535.1325 | 2 | 1070.265 |
| 2 | 70 | 646.835 | 2 | 1293.67 |
| 3 | 105 | 755.1075 | 2 | 1510.215 |
| 4 | 140 | 859.95 | 2 | 1719.9 |
| 5 | 175 | 961.3625 | 2 | 1922.725 |
| 6 | 210 | 1059.345 | 1 | 1059.345 |
| Area by the trapezoid method | | | | 157432.1.1 |

Table 16. Calculation of area D with the trapezoid

| ANALYSIS OF VALUES | |
|--------------------|----------|
| a | 0 |
| b | 60 |
| n | 6 |
| h | 10 |
| Sum | 4496.168 |

| AREA D | | | | |
|------------------------------|----|---------|--------|----------|
| I | x | f(x) | Coeff. | Product |
| 0 | 0 | 310 | 1 | 310 |
| 1 | 10 | 336.663 | 2 | 673.326 |
| 2 | 20 | 359.986 | 2 | 719.972 |
| 3 | 30 | 379.969 | 2 | 759.938 |
| 4 | 40 | 396.612 | 2 | 793.224 |
| 5 | 50 | 409.915 | 2 | 819.83 |
| 6 | 60 | 419.878 | 1 | 419.878 |
| Area by the trapezoid method | | | | 22480.84 |

4.4. Analysis with C++ software

To give solidity to the result obtained, the result obtained will be validated using the software "C++, 6.3".

Where the structure of the pseudocode will first be applied, and the respective modifications will be made for each integration method.

4.4.1. Implementation of the Simpson's rule 3/8 in C++

When carrying out this program, the objective is to validate the results through software that allows us to appreciate the same result more simplified way.

It will be considered that the variable "w" will be replaced in areas A, B, C, and D, then request to enter the integration limits of each of them, respectively. In each calculation, the modification must be made in the pseudocode when entering the variable "w."

```

1 #include <iostream>
2 #include <cmath>
3
4 using namespace std;
5
6 double f(double x){
7     double w="ENTER THE EQUATION OF THE AREA TO BE CALCULATED";
8     return w;
9 }
10
11 int main() {
12     cout.precision(4);
13     cout.setf(ios::fixed);
14     int n,i;
15     double a,b,c,h,sum=0,integral;
16     cout<< "\nThe integration limit \n\nBottom is a = ";
17     cin>>a;
18     cout<< "\nTop is b = ";
19     cin>>b;
20     cout<< "\nEnter the number of subintervals that is not odd: \nn= ";
21     cin>>n;
22
23     double x[n+1],y[n+1];
24     h=(b-a)/n;
25     for (i=0;i<n+1;i++){
26         x[i]=a+i*h;
27         y[i]=f(x[i]);
28     }
29
30     for (i=1;i<n;i++){
31         if(i%3==0)
32             sum=sum+2*y[i];
33         else
34             sum=sum+3*y[i];
35     }
36     integral=3*h/8*(y[0]+y[n]+sum);
37     cout<< "\nThe defined area is "<<integral<<"\n"<<endl;
38     return 0;
39 }
40

```

Fig. 8 Pseudocode of Simpson rule 3/8 implementation in C++

By executing the pseudocode replacing in the variable "w" the equations of the areas, the following results will be obtained

| | |
|--|--|
| <p style="text-align: center;">Area A</p> <p>The integration limit</p> <p>Bottom is a= 0</p> <p>Top is b= 490</p> <p>Enter the number of subintervals that is not odd: n= 6</p> <p>The defined area is 284710.8658</p> | <p style="text-align: center;">Area B</p> <p>The integration limit</p> <p>Bottom is a= 0</p> <p>Top is b= 260</p> <p>Enter the number of subintervals that is not odd: n= 6</p> <p>The defined area is 274384.3267</p> |
| <p style="text-align: center;">Area C</p> <p>The integration limit</p> <p>Bottom is a= 0</p> <p>Top is b= 210</p> <p>Enter the number of subintervals that is not odd: n= 6</p> <p>The defined area is 157492.1250</p> | <p style="text-align: center;">Area D</p> <p>The integration limit</p> <p>Bottom is a= 0</p> <p>Top is b= 60</p> <p>Enter the number of subintervals that is not odd: n= 6</p> <p>The defined area is 22497.5400</p> |

Fig. 9 Results of the areas applying Simpson's rule 3/8 in C++

4.4.2. Implementation of the Simpson's rule 1/3 in C++

In the same way, the variable "q" will be replaced in each of the areas A, B, C, and D and then request to enter the integration limits of each of them; respectively, it should be noted that in each calculation the modification must be made in the pseudocode when entering the variable "q."

```

1  #include <iostream>
2  #include <cmath>
3
4  using namespace std;
5
6  double f(double x){
7      double q="ENTER THE EQUATION OF THE AREA TO BE CALCULATED";
8      return q;
9  }
10 int main() {
11     cout.precision(4);
12     cout.setf(ios::fixed);
13     int n,i;
14     double a,b,c,h,sum=0,integral;
15     cout<< "\n\nThe integration limit \n\nBottom is a = ";
16     cin>>a;
17     cout<< "\n\nTop is b = ";
18     cin>>b;
19     cout<< "\n\nEnter the number of subintervals that is not odd: \nn= ";
20     cin>>n;
21
22     double x[n+1],y[n+1];
23     h=(b-a)/n;
24     for (i=0;i<n+1;i++){
25         x[i]=a+i*h;
26         y[i]=f(x[i]);
27     }
28
29     for (i=1;i<n;i++){
30         if(i%2==0)
31             sum=sum+2*y[i];
32         else
33             sum=sum+4*y[i];
34     }
35
36     integral=1*h/3*(y[0]+y[n]+sum);
37     cout<< "\n\nThe defined area is "<<integral<<"\n"<<endl;
38     return 0;
39 }

```

Fig. 10 Pseudocode of Simpson rule 1/3 implementation in C++

By executing the pseudocode replacing in the variable "q" the equations of the areas, the following results will be obtained

| | |
|--|--|
| <p style="text-align: center;">Area A</p> <p>The integration limit</p> <p>Bottom is a= 0</p> <p>Top is b= 490</p> <p>Enter the number of subintervals that is not odd: n= 6</p> <p>The defined area is 284710.8658</p> | <p style="text-align: center;">Area B</p> <p>The integration limit</p> <p>Bottom is a= 0</p> <p>Top is b= 260</p> <p>Enter the number of subintervals that is not odd: n= 6</p> <p>The defined area is 274384.3267</p> |
| <p style="text-align: center;">Area C</p> <p>The integration limit</p> <p>Bottom is a= 0</p> <p>Top is b= 210</p> <p>Enter the number of subintervals that is not odd: n= 6</p> <p>The defined area is 157492.1250</p> | <p style="text-align: center;">Area D</p> <p>The integration limit</p> <p>Bottom is a= 0</p> <p>Top is b= 60</p> <p>Enter the number of subintervals that is not odd: n= 6</p> <p>The defined area is 22497.5400</p> |

Fig. 11 Results of the areas applying Simpson's rule 1/3 in C++

4.4.3. Implementation of the trapezium method in C++

Finally, the variable "z" will be replaced in each of the areas A, B, C, and D and then request to enter the integration limits of each of them, respectively; it should be noted that in each calculation, the modification must be made in the pseudocode when entering the variable "z."

```

1  #include <iostream>
2  #include <cmath>
3
4  using namespace std;
5
6  double f(double x){
7      double z="ENTER THE EQUATION OF THE AREA TO BE CALCULATED";
8      return z;
9  }
10 }
11 int main() {
12     cout.precision(4);
13     cout.setf(ios::fixed);
14     int n,i;
15     double a,b,c,h,sum=0,integral;
16     cout<< "\nThe integration limit \n\nBottom is a= ";
17     cin>>a;
18     cout<< "\nTop is b= ";
19     cin>>b;
20     cout<< "\nEnter the number of subintervals that is not odd: \nn=";
21     cin>>n;
22
23     double x[n+1],y[n+1];
24     h=(b-a)/n;
25     for (i=0;i<n+1;i++){
26         x[i]=a+i*h;
27         y[i]=f(x[i]);
28     }
29
30     for (i=1;i<n;i++){
31         if(i%2==0)
32             sum=sum+2*y[i];
33         else
34             sum=sum+2*y[i];
35     }
36
37     integral=1*h/2*(y[0]+y[n]+sum);
38     cout<< "\nThe defined area is "<<integral<<"\n"<<endl;
39     return 0;
40 }

```

Fig. 12 Pseudocode of trapezium method implementation in C++

By executing the pseudocode replacing in the variable "z" the equations of the areas, the following results will be obtained

| | |
|--|--|
| <p style="text-align: center;">Area A</p> <p>The integration limit</p> <p>Bottom is a= 0</p> <p>Top is b= 490</p> <p>Enter the number of subintervals that is not odd: n= 6</p> <p>The defined area is 285296.3875</p> | <p style="text-align: center;">Area B</p> <p>The integration limit</p> <p>Bottom is a= 0</p> <p>Top is b= 260</p> <p>Enter the number of subintervals that is not odd: n= 6</p> <p>The defined area is 274382.6993</p> |
| <p style="text-align: center;">Area C</p> <p>The integration limit</p> <p>Bottom is a= 0</p> <p>Top is b= 210</p> <p>Enter the number of subintervals that is not odd: n= 6</p> <p>The defined area is 157432.1000</p> | <p style="text-align: center;">Area D</p> <p>The integration limit</p> <p>Bottom is a= 0</p> <p>Top is b= 60</p> <p>Enter the number of subintervals that is not odd: n= 6</p> <p>The defined area is 22480.8400</p> |

Fig. 13 Results of the areas applying the trapezoidal method in C++

4.5. Comparison of Results

Table 17. Comparison with actual Topographic Area method

| FINAL CALCULATIONS | |
|-----------------------|-------------|
| Area with integral | 739084.8575 |
| Real topographic area | 738844 |
| Difference | 240.85746 |
| Error % | 0.032599231 |

Table 17 shows that the actual topographic area provided by SUNARP allows us to appreciate the difference with the approximate calculation of this research and that the margin of error is only 0.033%.

Table 18. Comparison of Final Calculations

| SUMMARY | | | |
|---------|-------------|-------------|-------------|
| Areas | Simpson 3/8 | Simpson 1/3 | Trapeze |
| A | 284710.8658 | 284710.8658 | 285296.3875 |
| B | 274384.3267 | 274384.3267 | 274382.6993 |
| C | 157492.125 | 157492.125 | 157432.1 |
| D | 22497.54 | 22497.54 | 22480.84 |
| Total | 739084.8575 | 739084.8575 | 739592.0268 |

The results of the areas presented of the trapezoid method, Simpson 1/3 and Simpson 3/8, allow us to visualize, in table 18, that the difference is negligible, and using the Lagrange method gives us a perfect accuracy with the Simpson method 3/8 and 1/3.

5. Conclusion

The calculation using the integral of areas A, B, C, and D of the UNMSM are 284710.8658, 274384.3267, 157492.125, and 22497.54 square meters. That adds up to a total of 739084.8575. Additionally, from the topographic charts obtained from SUNARP, a document registered on March 24, 1961, it has an area of approximately 738844 square meters, giving a margin of error is 0.033%.

To find the area of any two-dimensional surface, and in this case of the UNMSM, it is advisable to use the Simpson Method of 1/3 or 3/8 because the accuracy is 100% in both when using the Lagrange method find the equation of the curve. A program such as C++ is also recommended so that the calculations entered are obtained more quickly.

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