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Original Article

Sensitivity Analysis of RC Beam under Flexure and Shear by FOSM Method

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Abstract *- Two types of problems are usually observed in the field of structural design. The complete cross-section dimensions of the member, including details about the reinforcing steel and its material qualities, are known in the first type of problem, known as analysis problems. The second type is design problems. Since the load effects, in this case, are known, it is necessary to choose the proper grades of materials, the size of the members, and reinforcement details. In this study design of an RC fixed beam has been undertaken to study the variations in the strength of reinforced concrete members in flexure and shear. The study investigates the sensitivity of design parameters, including concrete compressive strength, steel yield strength, area of steel reinforcement, width and effective depth of the beam, and external moment to the RC fixed beam. A sensitivity analysis of an RC beam is performed using the first-order second-moment method (FOSM). 20 percentage variations in members for flexure and shear of RC fixed beam have been analyzed by the MATLAB program. The FOSM method was used in arriving at the reliability index. The compressive strength of concrete, area of vertical stirrups and spacing of vertical stirrups have the least influence on flexure and shear strength. The strength of the RC fixed beam is significantly affected by the remaining variables.*

Keywords *- Area of steel, Effective depth, FOSM method, MATLAB, Reliability index.*

1. Introduction

A structure can be defined as an assembly of members proportioned to resist the design loads, and it should satisfy the requirements of safety and serviceability. Safety can be defined as an acceptable level of security against total failure, which can occur in concrete structures through various modes such as axial compression, flexure, shear, torsion, etc. Structural safety is always relative; it is not absolute and is usually measured in terms of probabilistic values. Structural problems are non-deterministic. There is always some uncertainty involved in the design and planning of an engineering system. Sensitivity analysis is a method for determining how different characteristics, like material constants, damping properties, geometrical parameters, and boundary conditions, affect structural response [13].

In reliability engineering, the probability of failure is measured using sensitivity analysis, and the reliability index helps determine the uncertainties that may significantly impact the structure's performance [2].

2. Codal Requirements for Flexural Reinforcement

Beam design requirements are based on the IS 456-2000. The important point to be noted is that partial safety factors should be eliminated because the sensitive analysis of the beam is based on the reliability theory. Two partial safety factors are prescribed in the present Indian standard [16].

2.1. Partial Safety Factor for Material

1.5 is a partial safety factor for concrete strength, which accounts for differences in the strength of concrete and the difference between in-situ and cube strength. The partial safety factor for steel is taken as 1.15, and it takes care of variations in the yield strength of steel, bar diameter, and bar positioning. A higher value of partial safety factor is used on concrete than on steel because the variations in steel tend to be less when compared to that of concrete, which takes place in the site [16].

2.2. Partial Safety Factor for Load

For dead and live loads, the partial safety factor proposed in the Indian standard [16] is 1.5. This factor takes care of variations in permanent and superimposed load. Flexure and shear are two important parameters used in this study for RC fixed beams.

2.3. Moment of Resistance

According to Indian standards, the theoretical equation in flexure is the rectangular section moment of resistance. The equation so used, without the partial safety factors, arrived as follows. All assumptions made in [16] are valid here. The stress block is assumed to consist of two parts such as rectangular and parabolic parts.

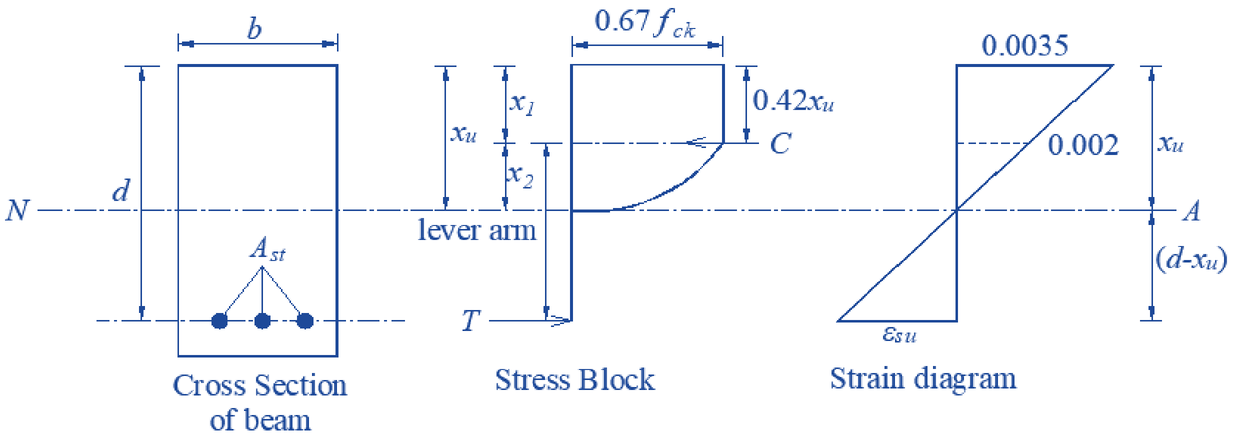

Fig. 1 Stress block parameters for singly reinforced rectangular section [16]

Let x_u be the neutral axis depth from the top fibre and x_1 be the rectangular portion depth.

The arm distance of the resisting couple is known as the lever arm of the section.

i.e., lever arm (z) = $(d - 0.42\,x_u)$

Area of stress block = (area of rectangular + parabolic portion)

Here, x_1 is the height of the stress block's rectangular part, and x_2 is the height of the stress block's parabolic part.

By a similar triangle, they are properly applied to the strain diagram (Figure 1).

$$\frac{\mathbf{0.0035}}{\boldsymbol{x_u}} = \frac{\mathbf{0.002}}{\boldsymbol{x_2}} \quad (1)$$

$$\Rightarrow x_2 = \left(\frac{0.002}{0.0035}\right) \times x_u \quad (2)$$

Parabolic height,

$$x_2 = \left(\frac{0.002}{0.0035}\right) \times x_u = \left(\frac{4}{7}\right) \times x_u = 0.5714x_u \approx 0.58x_u \quad (3)$$

$$\text{Rectangular height, } x_1 = x_u - 0.58x_u = 0.42x_u \quad (4)$$

Concrete is given a value of 0.67 because its apparent strength in compression at failure is 0.85 times that of a cylinder or 0.67 times that of a cube. The factor is 0.67 [17] when the cylinder strength is assumed to be 0.8 x cube strength.

Now, the stress block area equals the sum of the rectangular and parabolic parts.

$$= (0.67 \times f_{\mathrm{ck}} \times 0.42x_u) + \left(\frac{2}{3} \times 0.67 \times f_{\mathrm{ck}} \times 0.58x_u\right)$$

$$= (0.2814 f_{\mathrm{ck}} x_u) + (0.2590 f_{\mathrm{ck}} x_u) = 0.5404 f_{\mathrm{ck}} x_u \quad (5)$$

The compressive and tensile force on the section is given by,

Compressive force,

$$C = b \times 0.5404 f_{\mathrm{ck}} x_u = 0.5404 f_{\mathrm{ck}} \mathrm{b} x_u \quad (6)$$

$$\text{Tensile force, } T = f_y A_{\mathrm{st}} \quad (7)$$

For equilibrium,
Compressive force = Tensile force

$$0.5404 f_{\mathrm{ck}} \mathrm{b} x_u = f_y A_{\mathrm{st}}$$

$$\therefore x_u = \frac{f_y A_{\mathrm{st}}}{0.5404 f_{\mathrm{ck}} b} \quad (8)$$

For tension failure, the moment of resistance of the beam's cross-section is determined by taking moments about the centre of compression.

$$M = T \times z = f_y A_{\mathrm{st}} \times (d - 0.42x_u) \quad (9)$$

Substituting for x_u from Equation 9, we get

$$M = f_y A_{\mathrm{st}} \times \left[d - 0.42 \times \left(\frac{f_y A_{\mathrm{st}}}{0.5404 f_{\mathrm{ck}} b}\right)\right]$$

$$\therefore M = f_y A_{\mathrm{st}} d - \left(\frac{0.77 f_y^2 A_{\mathrm{st}}^2}{f_{\mathrm{ck}} b}\right) \quad (10)$$

If x_u/d equals the limiting value, the limiting moment is calculated using the compressive force moment at the tension steel level.

$$M_{\mathrm{lim}} = C \times z = 0.5404 f_{\mathrm{ck}} \mathrm{b} x_{u_{\mathrm{max.}}} \times \left(d - 0.42 x_{u_{\mathrm{max.}}}\right)$$

Now divide 'd^2' on both sides

$$\frac{M_{\mathrm{lim}}}{d^2} = \left(\frac{0.5404 f_{\mathrm{ck}} x_{u_{\mathrm{max.}}} \mathrm{bd}}{d^2}\right) - \left(\frac{0.5404 \times 0.42 f_{\mathrm{ck}} x_{\mathrm{umax.}}^2 b}{d^2}\right)$$

$$\frac{M_{\mathrm{lim}}}{d^2} = 0.5404 f_{\mathrm{ck}} b \frac{x_{u_{\mathrm{max.}}}}{d}\left[1 - 0.42\left(\frac{x_{u_{\mathrm{max.}}}}{d}\right)\right]$$

$$M_{\mathrm{lim}} = \left\{0.5404 f_{\mathrm{ck}} b \left(\frac{x_{u_{\mathrm{max.}}}}{d}\right) \times \left[1 - 0.42\left(\frac{x_{u_{\mathrm{max.}}}}{d}\right)\right]\right\} d^2$$

$$M_{\mathrm{lim}} = 0.5404 f_{\mathrm{ck}} \mathrm{bd}^2 \left(\frac{x_{u_{\mathrm{max.}}}}{d}\right) \times \left[1 - 0.42\left(\frac{x_{u_{\mathrm{max.}}}}{d}\right)\right] \quad (11)$$

From a similar triangle properly applied to the strain diagram, we get

$$\frac{\varepsilon_{su}}{(d-x_u)}=\frac{\varepsilon_{cu}}{x_u}$$

$$x_u=\frac{\varepsilon_{cu}}{\varepsilon_{su}}\times(d-x_u)=\frac{0.0035}{\varepsilon_{su}}\times(d-x_u)$$

$$=\left(\frac{0.0035}{\varepsilon_{su}}\times d\right)-\left(\frac{0.0035}{\varepsilon_{su}}\times x_u\right)$$

$$\Rightarrow\frac{x_u}{d}=\left(\frac{0.0035}{\varepsilon_{su}+0.0035}\right) \quad (12)$$

Where $\left(\frac{0.0035}{\varepsilon_{su}+0.0035}\right)$ is called the neutral axis factor

The tension reinforcement strain must not be less after removing the safety factor for material 1.15 for steel.

$$\varepsilon_{su}=0.002+\frac{f_y}{E_s}$$

Substituting ε_{su} in above x_u/d Equation 12, we get

$$\frac{x_u}{d}=\left(\frac{0.0035}{0.002+\frac{f_y}{E_s}+0.0035}\right)=\left(\frac{0.0035}{0.0055+\frac{f_y}{E_s}}\right)$$

Where, E_s= = 2 x 10^5 N/mm^2

The steel grade determines the x_u/d value, which is equal to the limiting value. Table 1 shows the rectangular section's moment of resistance.

Equation 8 gives, $x_u=\frac{f_y A_{st}}{0.5404 f_{ck} b}$

$$\Rightarrow\frac{A_{st} f_y}{f_{ck}}=0.5404\mathrm{bx}_u \quad (13)$$

As per SP-16 or IS 456:2000

$$p_t=\frac{100\times A_{st}}{\mathrm{bd}}\ \ or\ \mathrm{A_{st}}=\frac{p_t\mathrm{bd}}{100} \quad (14)$$

Substituting the above Equation 14 in 13

$$\frac{p_t\mathrm{bdf}_y}{100 f_{ck}}=0.5404\mathrm{bx}_u$$

$$\Rightarrow p_t\left(\frac{f_y}{f_{ck}}\right)=0.5404\times100\times\frac{x_u}{d}=54.04\left(\frac{x_u}{d}\right)$$

If x_u/d equals the limiting value, then

$$p_t=54.04\left(\frac{f_{ck}}{f_y}\right)\left(\frac{x_{u\ max.}}{d}\right) \quad (15)$$

Table 2 depicts the maximum percentage of reinforcing steel for singly reinforced rectangular sections with different concrete and steel grades.

2.4. Shear Reinforcement

IS 456:2000, the most recent version of the Indian code, proposes an empirical formula based on the experimental studies of [13], which takes into account the grade of concrete and longitudinal reinforcement ratio and is described in the form of a table (Table 19 of the [16]). The relevant equation is only given in [17] as follows:

$$\tau_c=\left(0.85\sqrt{0.8\mathrm{f_{ck}}}\right)\times\left\{\frac{\left(\sqrt{(1+5\beta_s)}-1\right)}{6\beta_s}\right\} \quad (16)$$

Where $\beta_s=\begin{cases}\frac{(0.8\mathrm{f_{ck}})}{(6.89\mathrm{p}_t)}\\ 1.0\end{cases}$, whichever is greater

Table 1. M lim for different steel grades

Grade of steel	With partial safety factor (IS 456:2000)		Without a partial safety factor	
	$\frac{x_{u\ max.}}{d}$	Expression for $M_{u\ lim}$	$\frac{x_{u\ max.}}{d}$	Expression for M_{lim}
Fe-250	0.5313	$0.148 f_{ck}bd^2$	0.5185	$0.220 f_{ck}bd^2$
Fe-415	0.4791	$0.138 f_{ck}bd^2$	0.4620	$0.201 f_{ck}bd^2$
Fe-500	0.4560	$0.133 f_{ck}bd^2$	0.4375	$0.193 f_{ck}bd^2$

Table 2. Maximum % of $p_{t,lim.}$ for singly reinforced rectangular sections for different concrete and steel grades

Grade of concrete	With partial safety factor (SP 16)			Without a partial safety factor		
	Fe- 250	Fe- 415	Fe- 500	Fe- 250	Fe- 415	Fe- 500
20	1.759	0.955	0.755	2.242	1.203	0.946
25	2.198	1.194	0.943	2.802	1.504	1.182
30	2.638	1.433	1.132	3.362	1.805	1.419
35	3.078	1.672	1.321	3.923	2.106	1.655
40	3.518	1.911	1.510	4.483	2.406	1.891

Table 3. Uncertainties in basic variables of the beam subjected to flexure

Variables	Characteristic values	Coefficient of variation	Mean value	Standard deviation	Distribution
A_{st}	601.083 mm^2	5%	654.953	32.7476	Normal
b	300 mm	5%	326.886	16.344	
d	500 mm	5%	544.811	27.240	
f_{ck}	25 N/mm^2	5%	27.240	1.362	
f_y	415 N/mm^2	5%	452.193	22.609	
M_{ext}	118.33 $\times10^6$ N.mm	25%	83.847$\times10^6$	20.961$\times10^6$	

The factor 0.8 in the formula, according to [17], converts cylinder strength to cube strength, and the factor 0.85 is a reducing factor related to the partial safety factor ($1/\gamma_m$),

The equation may also be used to approximate the values in Table 19 of the [16]:

$$V_u = V_{uc} + V_{us} = \tau_c \text{bd} + \frac{0.87 f_y A_{sv} d}{S_v} \tag{17}$$

After removing the safety factors for materials of 1.15 for steel and 1.5 for concrete, the proposed shear strength equation can be rewritten,

$$\tau_c = 1.5 \times \left[\left(0.85\sqrt{0.8 f_{ck}}\right) \times \left\{ \frac{\left(\left(\sqrt{(1+5\beta_s)}-1\right)\right)}{6\beta_s} \right\} \right] \tag{18}$$

$$V = V_c + V_s = 1.5 \times \tau_c \text{bd} + 1.15 \times \left(\frac{0.87 f_y A_{sv} d}{S_v}\right) = 1.5 \times \tau_c \text{bd} + \left(\frac{f_y A_{sv} d}{S_v}\right) \tag{19}$$

3. Formulation and Analysis

The present study conducts sensitivity analysis on a fixed RC beam with a 6m span subjected to gravity and earthquake loads. It observes that the beam performs its function (resist against flexural moment and shear) without failure for the intended lifetime. Strength and shear prediction equations and statistical distributions for the variables are involved in this computation of the reliability index β. MATLAB program is generated to calculate the same. Formulations and analysis for the beam are as follows.

3.1. Sectional Properties of Beam Selected

The following parameters are considered based on design criteria for the sensitive analysis of the RC fixed beam. The beam width is 300mm, the overall depth is 550mm, the effective depth is 500mm, and the effective cover is 50mm; the concrete and steel characteristic strengths are 25 and 415 N/mm^2, respectively.

For convenience, this problem divided into two parts
Case (1): Member subjected to flexure
Case (2): Member subjected to shear
(Note: The formulations for two case equations are linear equations. So, no needs to go for Taylor's expansion. The partial derivatives method is suitable for the calculation of the standard deviation of the Margin of safety).

4. FOSM Method

The First Order Second Moment Method (FOSM) helps to simplify the functional relationship and reduces the complexities of calculating the probability of failure.

Case (1): Member subjected to flexure

Table 3 presents the uncertainties in the basic variables of the beam subjected to flexure.

Reliability computations for normally distributed load and resistance

The concept of no-failure exits when the resistance *(R)* of a structural member is not exceeded by the loads *(S)*. This forms the basis for determining the reliability of the system.
Reliability = *P(R>S) = P(R-S>0)*

Here *R* represents resistive moment; *S* represents moment due to external loads. After removing partial safety factors, the moment of resistance is,

$$R = A_{st} \times f_y \times d - \left(\frac{0.77 \times {A_{st}}^2 \times {f_y}^2}{b \times f_{ck}}\right) \tag{20}$$

$$S = M_{ext} \tag{21}$$

The mean values for *R* and *S* are given by

$$\mu_R = \overline{R} = \overline{A_{st}} \times \overline{f_y} \times \overline{d} - \left(\frac{0.77 \times \overline{A_{st}}^2 \times \overline{f_y}^2}{\overline{b} \times \overline{f_{ck}}}\right) \tag{22}$$

$$\mu_S = \overline{S} = \overline{M_{ext}} \tag{23}$$

Now, partially differentiating $\overline{R}$and $\overline{S}$ w.r.t variables, we get

$$\frac{\partial \overline{R}}{\partial \overline{A_{st}}} = \overline{f_y} \times \overline{d} - \left(\frac{1.54 \times \overline{A_{st}} \times \overline{f_y}^2}{\overline{b} \times \overline{f_{ck}}}\right)$$

$$\frac{\partial \overline{R}}{\partial \overline{f_y}} = \overline{A_{st}} \times \overline{d} - \left(\frac{1.54 \times \overline{A_{st}}^2 \times \overline{f_y}}{\overline{b} \times \overline{f_{ck}}}\right); \frac{\partial \overline{R}}{\partial \overline{d}} = \overline{A_{st}} \times \overline{f_y}$$

$$\frac{\partial \overline{R}}{\partial \overline{b}} = \frac{0.77 \times \overline{A_{st}}^2 \times \overline{f_y}^2}{\overline{b}^2 \times \overline{f_{ck}}}; \frac{\partial \overline{R}}{\partial \overline{f_{ck}}} = \frac{0.77 \times \overline{A_{st}}^2 \times \overline{f_y}^2}{\overline{b} \times \overline{f_{ck}^2}}$$

$$\frac{\partial \overline{S}}{\partial \overline{M_{ext}}} = -1$$

The standard deviation values for *R* and *S* is given by,

$$\sigma_R^2 = \left(\frac{\partial \overline{R}}{\partial \overline{A_{st}}} \times \sigma_{A_{st}}\right)^2 + \left(\frac{\partial \overline{R}}{\partial \overline{f_y}} \times \sigma_{f_y}\right)^2 + \left(\frac{\partial \overline{R}}{\partial \overline{d}} \times \sigma_d\right)^2 + \left(\frac{\partial \overline{R}}{\partial \overline{b}} \times \sigma_b\right)^2 + \left(\frac{\partial \overline{R}}{\partial \overline{f_{ck}}} \times \sigma_{f_{ck}}\right)^2 \tag{24}$$

$$\sigma_S^2 = \left(\frac{\partial \overline{V}}{\partial \overline{M_{ext}}} \times \sigma_{M_{ext}}\right)^2 \tag{25}$$

Now the reliability index for the beam under flexure is given by

$$\beta = \frac{\mu_R - \mu_S}{\sqrt{{\sigma_R}^2 + {\sigma_S}^2}} \tag{26}$$

Case (2): Member subjected to shear

The uncertainties in basic variables of the beam subjected to shear are tabulated in Table 4.

Reliability computations for normally distributed load and resistance

The concept of no-failure exits when the resistance *(R)* of a structural member is not exceeded by the loads (S).
Reliability = *P(R>S) = P(R-S>0)*

Here R represents shear resistance *(V_R)*; S represents shear due to external loads. After removing partial safety factors, the shear resistance is,

$$R = V_c + V_s = 1.5\tau_c \text{bd} + \frac{f_y A_{sv} d}{S_v} \quad (27)$$

$$S = V \quad (28)$$

If $V<V_R$, This member is reliable against shearing forces.

This forms the basis of determining the reliability index of the system given by,

$$\beta = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R{}^2 + \sigma_S{}^2}}$$

The mean values for R and S are given by

$$\mu_R = \overline{R} = 1.5\overline{\tau_c}\overline{bd} + \frac{\overline{f_y A_{sv} d}}{\overline{S_v}} \quad (29)$$

$$\mu_S = \overline{S} = \overline{V} \quad (30)$$

Now, partially differentiating $\overline{R}$and $\overline{S}$ w.r.t variables, we get,

$$\frac{\partial \overline{R}}{\partial \overline{\tau_c}} = 1.5 \times \overline{b} \times \overline{d};\ \frac{\partial \overline{R}}{\partial \overline{b}} = 1.5 \times \overline{\tau_c} \times \overline{d}$$

$$\frac{\partial \overline{R}}{\partial \overline{d}} = 1.5 \times \overline{\tau_c} \times \overline{b} + \left(\frac{\overline{f_y} \times \overline{A_{sv}}}{\overline{S_v}}\right)$$

$$\frac{\partial \overline{R}}{\partial \overline{f_y}} = \left(\frac{\overline{A_{sv}} \times \overline{d}}{\overline{S_v}}\right);\ \frac{\partial \overline{R}}{\partial \overline{A_{sv}}} = \left(\frac{\overline{f_y} \times \overline{d}}{\overline{S_v}}\right)$$

$$\frac{\partial \overline{R}}{\partial \overline{S_v}} = -\left(\frac{\overline{f_y} \times \overline{A_{sv}} \times \overline{d}}{\overline{S_v^2}}\right);\ \frac{\partial \overline{S}}{\partial \overline{V}} = -1$$

The standard deviation values for R and S is given by

$$\sigma_R^2 = \left(\frac{\partial \overline{R}}{\partial \overline{\tau_c}} \times \sigma_{\tau_c}\right)^2 + \left(\frac{\partial \overline{R}}{\partial \overline{b}} \times \sigma_b\right)^2 + \left(\frac{\partial \overline{R}}{\partial \overline{d}} \times \sigma_d\right)^2 + \left(\frac{\partial \overline{R}}{\partial \overline{f_y}} \times \sigma_{f_y}\right)^2 + \left(\frac{\partial \overline{R}}{\partial \overline{A_{sv}}} \times \sigma_{A_{sv}}\right)^2 + \left(\frac{\partial \overline{R}}{\partial \overline{S_v}} \times \sigma_{S_v}\right)^2 \quad (31)$$

$$\sigma_S^2 = \left(\frac{\partial \overline{S}}{\partial \overline{V}} \times \sigma_V\right)^2 \quad (32)$$

5. Results and Discussion

The sensitivity analysis for RC fixed beam is performed using the first-order second moment (FOSM) method, and the results for flexure and shear are tabulated.

5.1. Beam under Flexure

Results of the sensitivity analysis FOSM (Partial derivative method) method for the beam under flexure are tabulated in the following Tables from 5 to 10. These Tables show that the reliability index value varies with respect to variations in design variables, namely A_{st}, f_y, d, f_{ck}, b & M_{ext}. Table 5 shows that β the value increases from 1.71807 to 3.78738 due to 20% variations in the mean value of A_{st} and other parameters (f_y, d, f_{ck}, b & M_{ext}) kept constant. Table 6 shows that β the value increases from 2.76261 to 2.86986 due to 20% variations in the mean value of b and other parameters (A_{st}, f_y, f_{ck}, d & M_{ext}) kept constant. Table 7 shows that β the value increases from 1.61096 to 3.88295 due to 20% variations in the mean value of d and other parameters (A_{st}, f_y, f_{ck}, b & M_{ext}) kept constant. Table 8 shows that β the value increases from 2.76261 to 2.86986 due to 20% variations in the mean value of f_{ck} and other parameters (A_{st}, f_y, d, b & M_{ext}) kept constant. Table 9 shows that β the value increases from 1.71807 to 3.78738 due to 20% variations in the mean value of f_y and other parameters (A_{st}, f_{ck}, d, b & M_{ext}) kept constant. Table 10 shows that β the value decreases from 4.07111 to 1.8734 due to 20% variations in the mean value of M_{ext} and other parameters (A_{st}, f_y, d, b, f_{ck}) kept constant. The required amount of steel for -20% and +20% variation is 523.962 and 785.943 $mm^{2,}$ respectively. But the maximum area of steel for a singly reinforced rectangular section is 2256 mm^2.

Using Equation 14 and Table 2, the maximum area of steel for a singly reinforced rectangular section for f_{ck} =25 N/mm^2 and f_y =415 N/mm^2 is given by,

Table 4. Uncertainties in basic variables of a beam subjected to shear

Variables	Characteristic values	Coefficient of variation	Mean value	Standard deviation	Distribution
τ_c	0.4194 N/mm^2	5%	0.4569	0.02284	Normal
b	300 mm	5%	326.886	16.344	
d	500 mm	5%	544.811	27.240	
f_y	415 N/mm^2	5%	452.193	22.609	
A_{sv}	100.53 mm^2	5%	109.539	5.476	
S_v	300 mm	5%	326.886	16.344	
V	77473 N	25%	54896.70	13724.2	

$$A_{\text{st, max.}} = \frac{p_t \text{bd}}{100} = \frac{1.504 \times 300 \times 500}{100} = 2256\, mm^2$$

This indicates the above range of steel quantity is within 2256 mm^2 representing the beam is still under reinforced section.

Table 5. Reliability index values for variations in A_{st}

$\overline{A_{\text{st}}}$	$\overline{f_y}$	$\overline{d}$	$\overline{f_{\text{ck}}}$	$\overline{b}$	$\overline{M_{\text{ext}}}$	β
523.962	452.193	544.811	27.240	326.886	83.847 $\times 10^6$	1.71807
589.458	452.193	544.811	27.240	326.886	83.847 $\times 10^6$	2.29187
654.953	452.193	544.811	27.240	326.886	83.847 $\times 10^6$	2.82727
720.448	452.193	544.811	27.240	326.886	83.847 $\times 10^6$	3.32526
785.943	452.193	544.811	27.240	326.886	83.847 $\times 10^6$	3.78738

Table 6. Reliability index values for variations in *b*

$\overline{A_{\text{st}}}$	$\overline{f_y}$	$\overline{d}$	$\overline{f_{\text{ck}}}$	$\overline{b}$	$\overline{M_{\text{ext}}}$	β
654.953	452.193	544.811	27.240	261.509	83.847 $\times 10^6$	2.76261
654.953	452.193	544.811	27.240	294.198	83.847 $\times 10^6$	2.79865
654.953	452.193	544.811	27.240	326.886	83.847 $\times 10^6$	2.82727
654.953	452.193	544.811	27.240	359.575	83.847 $\times 10^6$	2.85055
654.953	452.193	544.811	27.240	392.264	83.847 $\times 10^6$	2.86986

Table 7. Reliability index values for variations in *d*

$\overline{A_{\text{st}}}$	$\overline{f_y}$	$\overline{d}$	$\overline{f_{\text{ck}}}$	$\overline{b}$	$\overline{M_{\text{ext}}}$	β
654.953	452.193	435.849	27.240	326.886	83.847 $\times 10^6$	1.61096
654.953	452.193	490.33	27.240	326.886	83.847 $\times 10^6$	2.23919
654.953	452.193	544.811	27.240	326.886	83.847 $\times 10^6$	2.82727
654.953	452.193	599.292	27.240	326.886	83.847 $\times 10^6$	3.37495
654.953	452.193	653.773	27.240	326.886	83.847 $\times 10^6$	3.88295

Table 8. Reliability index values for variations in f_{ck}

$\overline{A_{\text{st}}}$	$\overline{f_y}$	$\overline{d}$	$\overline{f_{\text{ck}}}$	$\overline{b}$	$\overline{M_{\text{ext}}}$	β
654.953	452.193	544.811	21.7924	326.886	83.847 $\times 10^6$	2.76261
654.953	452.193	544.811	24.5165	326.886	83.847 $\times 10^6$	2.79865
654.953	452.193	544.811	27.240	326.886	83.847 $\times 10^6$	2.82727
654.953	452.193	544.811	29.9646	326.886	83.847 $\times 10^6$	2.85055
654.953	452.193	544.811	32.6886	326.886	83.847 $\times 10^6$	2.86986

Table 9. Reliability index values for variations in f_y

$\overline{A_{\text{st}}}$	$\overline{f_y}$	$\overline{d}$	$\overline{f_{\text{ck}}}$	$\overline{b}$	$\overline{M_{\text{ext}}}$	β
654.953	361.754	544.811	27.240	326.886	83.847 $\times 10^6$	1.71807
654.953	406.974	544.811	27.240	326.886	83.847 $\times 10^6$	2.29187
654.953	452.193	544.811	27.240	326.886	83.847 $\times 10^6$	2.82727
654.953	497.412	544.811	27.240	326.886	83.847 $\times 10^6$	3.32526
654.953	542.631	544.811	27.240	326.886	83.847 $\times 10^6$	3.78738

Table 10. Reliability index values for variations in M_{ext}

$\overline{A_{\text{st}}}$	$\overline{f_y}$	$\overline{d}$	$\overline{f_{\text{ck}}}$	$\overline{b}$	$\overline{M_{\text{ext}}}$	β
654.953	452.193	544.811	27.240	326.886	67.078$\times 10^6$	4.07111
654.953	452.193	544.811	27.240	326.886	75.462 $\times 10^6$	3.40739
654.953	452.193	544.811	27.240	326.886	83.847 $\times 10^6$	2.82727
654.953	452.193	544.811	27.240	326.886	92.232 $\times 10^6$	2.31941
654.953	452.193	544.811	27.240	326.886	100.617 $\times 10^6$	1.8734

In figure 2 to 6, it is shown that β increases with an increase in A_{st}, b, d, f_{ck}, f_y and β decrease with an increase in an external moment, as shown in figure 7 by the FOSM method.

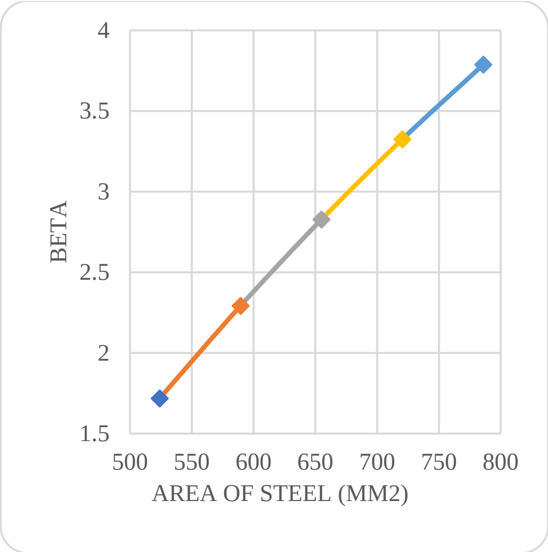

Fig. 2 Sensitivity of area of steel *(A_{st})* versus β

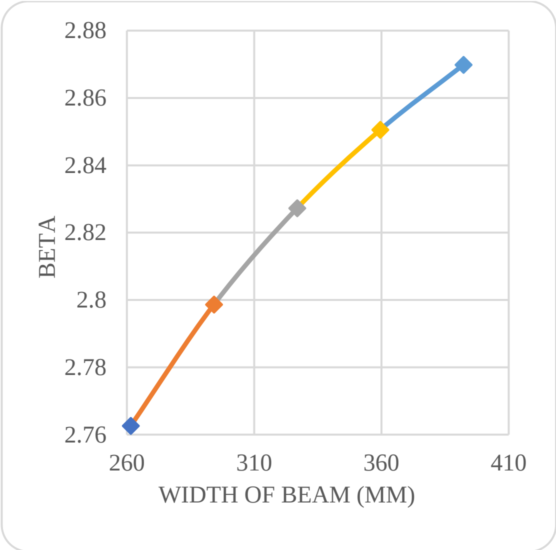

Fig. 3 Sensitivity of width of beam *(b)* versus β

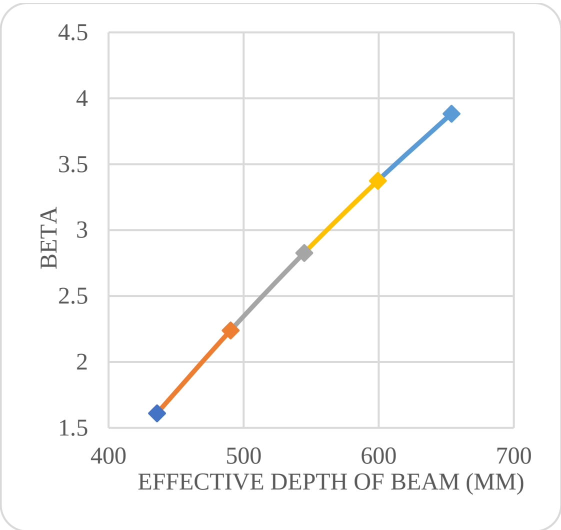

Fig. 4 Sensitivity of effective depth of beam *(d)* versus β

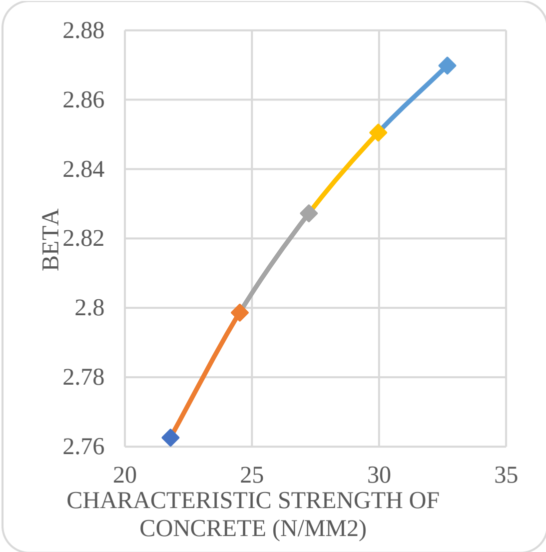

Fig. 5 Sensitivity of characteristic strength of concrete *(f_{ck})* versus β

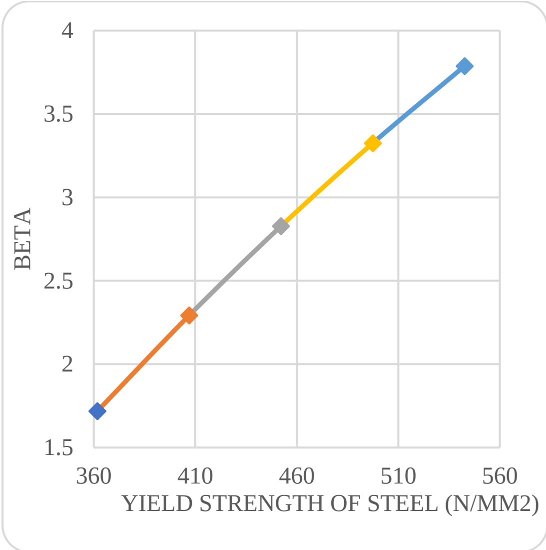

Fig. 6 Sensitivity of yield strength of steel *(f_y)* versus β

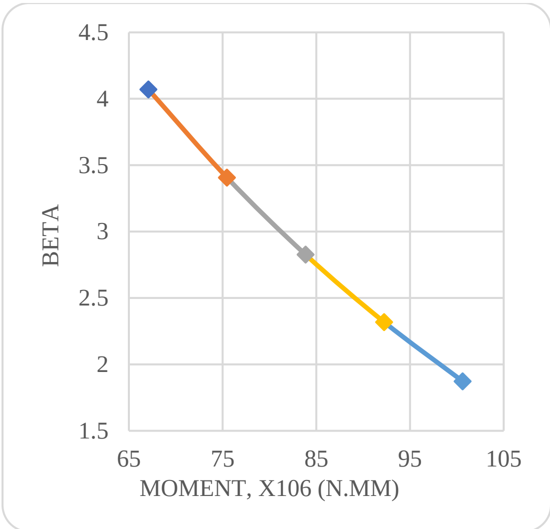

Fig. 7 Sensitivity of external moment *(M_{ext})* versus β

5.2. Beam under Shear

Results of the sensitive analysis FOSM method for the beam under shear are tabulated in the following Tables from 11 to 17. These tables show that the reliability index value varies with respect to variations in design variables, namely τ_c, b, d, f_y, A_{sv}, S_v & V. Table 11 shows that β value increases from 6.52966 to 7.96613 due to 20% variations in the mean value of τ_c and other parameters (b, d, f_y, A_{sv}, S_v & V) kept constant. Table 12 shows that β the value increases from 6.52966 to 7.96613 due to 20% variations in the mean value of b and other parameters (τ_c, d, f_y, A_{sv}, S_v & V) kept constant. Table 13 shows that β the value increases from 5.93822 to 8.36105 due to 20% variations in the mean value of d and other parameters (τ_c, b, d, A_{sv}, S_v & V) kept constant. Table 14 shows that β the value increases from 6.79813 to 7.7620 due to 20% variations in the mean value of f_y and other parameters (τ_c, b, d, A_{sv}, S_v & V) kept constant. Table 15 shows that β the value increases from 6.79813 to 7.7620 due to 20% variations in the mean value of A_{sv} and other parameters (τ_c, b, d, f_y, S_v & V) kept constant. Table 16 shows that β the value increases from 7.86213 to 6.89036 due to 20% variations in the mean value of S_v and other parameters (τ_c, b, d, f_y, A_{sv} & V) kept constant. Table 17 shows that β the value increases from 8.58005 to 6.19584 due to 20% variations in the mean value of V and other parameters (τ_c, b, d, f_y, A_{sv} &) kept constant.

Table 11. Reliability index values for variations in τ_c

$\overline{\tau_c}$	$\overline{b}$	$\overline{d}$	$\overline{f_y}$	$\overline{A_{sv}}$	$\overline{S_v}$	$\overline{V}$	β
0.3655	326.886	544.811	452.193	109.54	326.886	54896.7	6.52966
0.4112	326.886	544.811	452.193	109.54	326.886	54896.7	6.94275
0.4569	326.886	544.811	452.193	109.54	326.886	54896.7	7.31798
0.5026	326.886	544.811	452.193	109.54	326.886	54896.7	7.65814
0.5483	326.886	544.811	452.193	109.54	326.886	54896.7	7.96613

Table 12. Reliability index values for variations in b

$\overline{\tau_c}$	$\overline{b}$	$\overline{d}$	$\overline{f_y}$	$\overline{A_{sv}}$	$\overline{S_v}$	$\overline{V}$	β
0.4569	261.509	544.811	452.193	109.54	326.886	54896.7	6.52966
0.4569	294.198	544.811	452.193	109.54	326.886	54896.7	6.94275
0.4569	326.886	544.811	452.193	109.54	326.886	54896.7	7.31798
0.4569	359.575	544.811	452.193	109.54	326.886	54896.7	7.65814
0.4569	392.264	544.811	452.193	109.54	326.886	54896.7	7.96613

Table 13. Reliability index values for variations in d

$\overline{\tau_c}$	$\overline{b}$	$\overline{d}$	$\overline{f_y}$	$\overline{A_{sv}}$	$\overline{S_v}$	$\overline{V}$	β
0.4569	326.886	435.849	452.193	109.54	326.886	54896.7	5.93822
0.4569	326.886	490.33	452.193	109.54	326.886	54896.7	6.67644
0.4569	326.886	544.811	452.193	109.54	326.886	54896.7	7.31798
0.4569	326.886	599.292	452.193	109.54	326.886	54896.7	7.87559
0.4569	326.886	653.773	452.193	109.54	326.886	54896.7	8.36105

Table 14. Reliability index values for variations in f_y

$\overline{\tau_c}$	$\overline{b}$	$\overline{d}$	$\overline{f_y}$	$\overline{A_{sv}}$	$\overline{S_v}$	$\overline{V}$	β
0.4569	326.886	544.811	361.754	109.54	326.886	54896.7	6.79813
0.4569	326.886	544.811	406.974	109.54	326.886	54896.7	7.06802
0.4569	326.886	544.811	452.193	109.54	326.886	54896.7	7.31798
0.4569	326.886	544.811	497.412	109.54	326.886	54896.7	7.54896
0.4569	326.886	544.811	542.631	109.54	326.886	54896.7	7.7620

Table 15. Reliability index values for variations in A_{sv}

$\overline{\tau_c}$	$\overline{b}$	$\overline{d}$	$\overline{f_y}$	$\overline{A_{sv}}$	$\overline{S_v}$	$\overline{V}$	β
0.4569	326.886	544.811	452.193	87.6317	326.886	54896.7	6.79813
0.4569	326.886	544.811	452.193	98.5857	326.886	54896.7	7.06802
0.4569	326.886	544.811	452.193	109.54	326.886	54896.7	7.31798
0.4569	326.886	544.811	452.193	120.494	326.886	54896.7	7.54896
0.4569	326.886	544.811	452.193	131.448	326.886	54896.7	7.7620

Table 16. Reliability index values for variations in S_v

$\overline{\tau_c}$	$\overline{b}$	$\overline{d}$	$\overline{f_y}$	$\overline{A_{sv}}$	$\overline{S_v}$	$\overline{V}$	β
0.4569	326.886	544.811	452.193	109.54	261.509	54896.7	7.86213
0.4569	326.886	544.811	452.193	109.54	294.198	54896.7	7.5735
0.4569	326.886	544.811	452.193	109.54	326.886	54896.7	7.31798
0.4569	326.886	544.811	452.193	109.54	359.575	54896.7	7.09156
0.4569	326.886	544.811	452.193	109.54	392.264	54896.7	6.89036

Table 17. Reliability index values for variations in V

$\overline{\tau_c}$	$\overline{b}$	$\overline{d}$	$\overline{f_y}$	$\overline{A_{sv}}$	$\overline{S_v}$	$\overline{V}$	β
0.4569	326.886	544.811	452.193	109.54	326.886	43917.4	8.58005
0.4569	326.886	544.811	452.193	109.54	326.886	49407.1	7.93289
0.4569	326.886	544.811	452.193	109.54	326.886	54896.7	7.31798
0.4569	326.886	544.811	452.193	109.54	326.886	60386.4	6.73856
0.4569	326.886	544.811	452.193	109.54	326.886	65876.1	6.19584

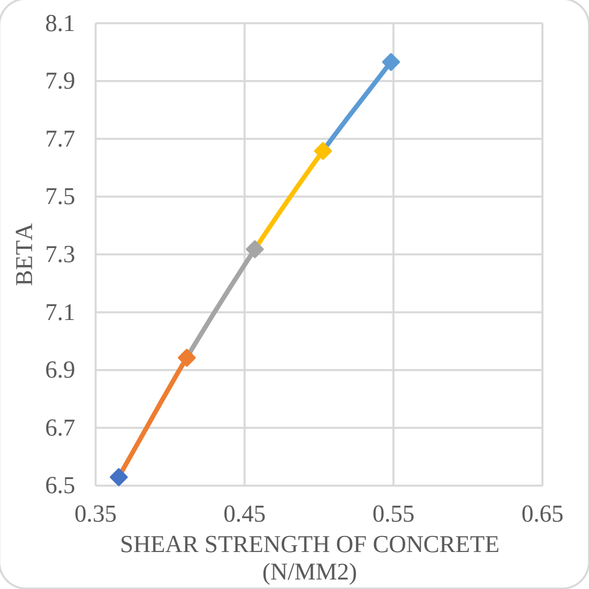

Fig. 8 Sensitivity of shear strength of concrete (τ_c) versus β

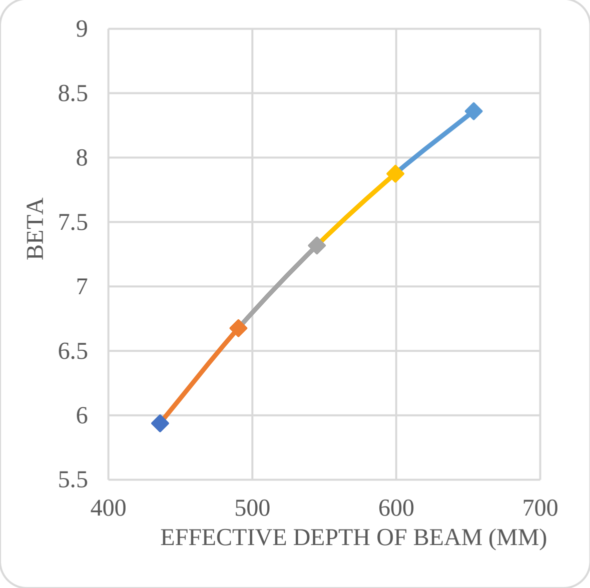

Fig. 10 Sensitivity of effective depth of beam *(d)* versus β

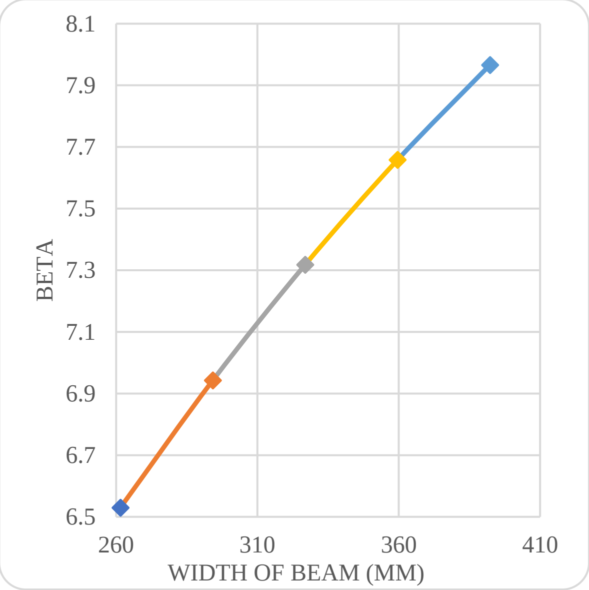

Fig. 9 Sensitivity of width of beam *(b)* versus β

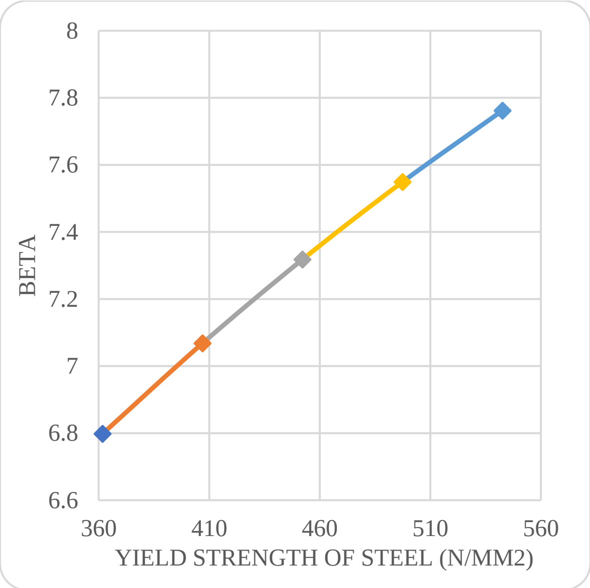

Fig. 11 Sensitivity of yield strength of steel (f_y) versus β

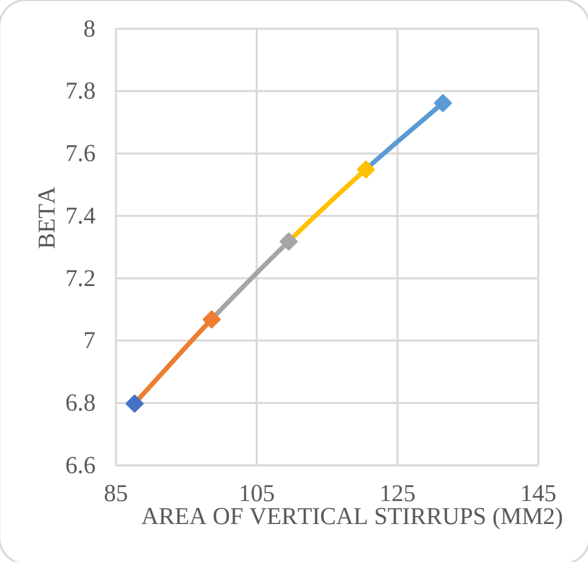

Fig. 12 Sensitivity of area of vertical stirrups (A_{sv}) versus β

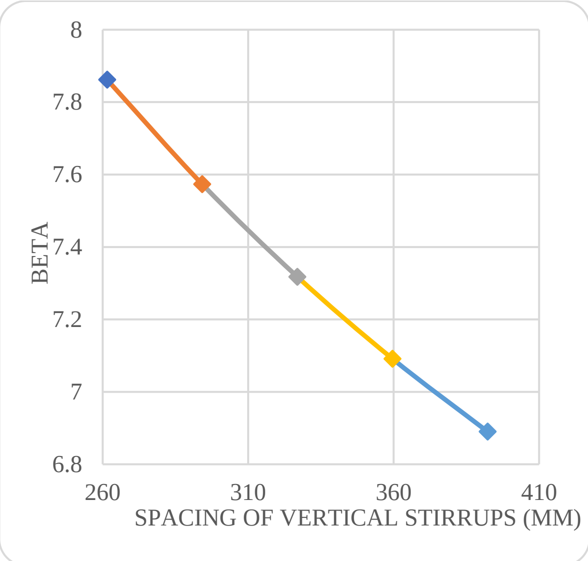

Fig. 13 Sensitivity of spacing of vertical stirrups (S_v) versus β

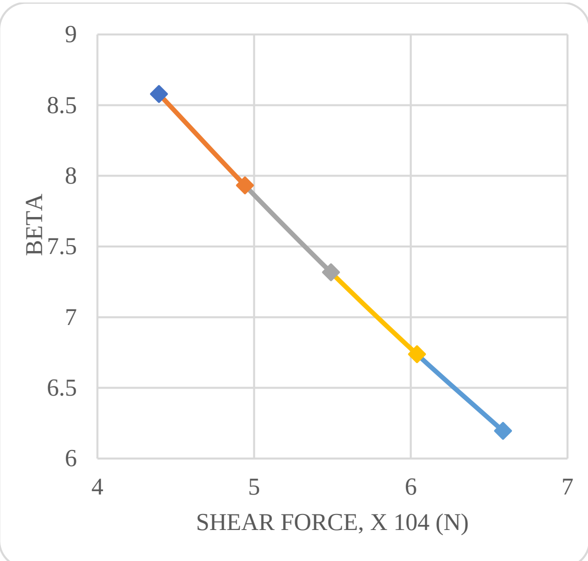

Fig. 14 Sensitivity of shear force (V) versus β

Table 18. Variations in β for the case of the beam under bending

40% variations in variables	Variations in β
A_{st}	2.06931
b	0.10725
d	2.27199
f_{ck}	0.10725
f_y	2.06931
M_{ext}	2.19771

Table 19. Variations in β for the case of the beam under shear

40% variations in variables	Variations in β
τ_c	1.43647
b	1.43647
d	2.42283
f_y	0.96387
A_{sv}	0.96387
S_v	0.97177
V	2.38421

Figures 8 to 12 show that β increases with the increase in (c, b, d, fy, Asv and β decreases with an increase in Sv, V as shown in figures 13 and 14, respectively, respectively, by the FOSM method. The variations β for the case of the beam under bending and shear are tabulated in tables 18 & 19, respectively. Table 18 shows that variables A_{st}, d, f_y and M_{ext} are more sensitive to the reliability index β by the FOSM method under flexure. Table 19 shows that variables τ_c, b, d, and V are more sensitive to the reliability index β by the AFOSM method under shear.

6. Conclusion

An attempt has been made to conduct a sensitivity analysis of a fixed beam by considering the FOSM method. Based on the analysis, the following conclusions can be made.

- The reliability index increases with an increase in steel area, beam width, the effective depth of the beam, and concrete and steel strength and decreases with an increase in the external moment for the beam under flexure using the FOSM method.
- The reliability index increases with an increase in the shear strength of concrete, beam width, effective depth, and strength of steel. But decreases with an increase in the spacing of stirrups and shear force for the beam under shear.
- The flexural strength of RC fixed beam is more sensitive to the following variables: effective depth of the beam, area of steel in tension reinforcement, yield strength of steel, and external moment and is insensitive to strength of concrete and breadth of the beam by FOSM method.
- The RC fixed beam's shear strength is more sensitive to the following variables: shear force, the effective depth of the beam and insensitivity to the strength of steel, area of vertical stirrups and spacing of stirrups by the FOSM method.
- The probability of failure for RC fixed beam is P_f =0.002353 or 1 in 425.

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