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Original Article

Performability and Sensitivity Analysis of the Three Pumps of a Desalination Water Pumping Station

Syed Mohd Rizwan[1], Kajal Sachdeva[2], Senguttuvan Alagiriswamy[3], Yaqoob Al Rahbi[4]

[1,3]Department of Applied Mathematics and Science, National University of Science and Technology, Sultanate of Oman.
[2]Department of Mathematics, M. D. University, India.
[4]Department of Process Engineering, International Maritime College, Sultanate of Oman

[1]Corresponding Author : syedrizwan@nu.edu.om

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Abstract *- The individual performance of the three pumps of a system distributing the desalinated water is analyzed. Three pumps are being used to supply the potable water to the end users after getting treated at the desalination plant. To ensure the continuous operation of the pumps to avoid water supply disruption, it is of utmost importance to monitor and maintain the operational capabilities of all three pumps. Maintenance data for five years on different failure reasons have been collected, which also includes the restoration and waiting times to bring the pumps back into operation. The main objective is to compare the operational capabilities of each of the three pumps to establish which pump is the least performing and needs attention to improve upon the entire system. Markov and regenerative processes have been used in the analysis to obtain the performance indicators of the three pumps in terms of reliability & availability. Sensitivity analysis has also been performed to establish the significance of different parameters on the reliability outcomes.*

Keywords *- Reliability, Desalination, Pumping station, Markov processes, Regenerative processes, Sensitivity analysis.*

1. Nomenclature

$\mathcal{S}_0, \mathcal{S}_1, \mathcal{S}_2, \mathcal{S}_3$	Operating state, Wear Ring Damaged partial failure state, Mechanical Seal Leaks partial failure state, and Impeller Broken complete failure state of Pump 1
$\mathcal{S}_0', \mathcal{S}_1', \mathcal{S}_2'$	Operating state, Mechanical Seal Leaks partial failure state, and Shaft Broken complete failure state of Pump 2
$\mathcal{S}_0'', \mathcal{S}_1'', \mathcal{S}_2'', \mathcal{S}_3''$	Operating state, Wear Ring Damaged partial failure state, Mechanical Seal Leaks partial failure state, and Impeller Broken complete failure state of Pump 3
λ_{11}	Rate of failure of WRD of pump 1
λ_{12}	Rate of failure of MSL of pump 1
λ_{13}	Rate of failure of IB of pump 1
λ_{21}	Rate of failure of MSL of pump 2
λ_{22}	Rate of failure of SB of pump 2
λ_{31}	Rate of failure of WRD of pump 3
λ_{32}	Rate of failure of MSL of pump 3
λ_{33}	Rate of failure of IB of pump 3
•	Regenerative states of pumps
◇	Partially failed states of pumps
○	Operational states of pumps
□	Completely failed states
©	Laplace convolution symbol

(s)	Stieltje's convolution symbol
*	Symbol for Laplace Transforms
**	Laplace Stieltje's transforms symbol
Λ_0	System's steady-state availability
B_0	Expected busy period for the maintenance
$\zeta_i^k(t)$	Density function of first passage time from state i to a failed state j of pump k, k =1,2,3.
$p_{ij}^k(t)$, $Q_{ij}^k(t)$	Density functions of first passage time from a regenerative state i to the regenerative state j or to a failed state j of pump k in (0, t].
$g_1(t)$, $G_1(t)$	Density functions of repair rate of the failures due to WRD in Pump 1
$g_2(t)$, $G_2(t)$	Density functions of repair rate of the failures due to MSL in Pump 1
$g_3(t)$, $G_3(t)$	Density functions of repair rate of the failures due to IB in Pump 1
$g_4(t)$, $G_4(t)$	Density functions of repair rate of the failures due to MSL in Pump 2
$g_5(t)$, $G_5(t)$	Density functions of repair rate of the failures due to SB in Pump 2
$g_6(t)$, $G_6(t)$	Density functions of repair rate of the failures due to WRD in Pump 3

$g_7(t)$, $G_7(t)$	Density functions of repair rate of the failures due to MSL in Pump 3
$g_8(t)$, $G_8(t)$	Density functions of repair rate of the failures due to IB in Pump 3
WRD	Wear Ring Damaged
MSL	Mechanical Seal Leaks
IB	Impeller Broken
SB	Shaft Broken

2. Introduction

Technological systems were examined in the past from the standpoint of reliability to comprehend better the impact of malfunctions and repairs on the system's overall performance under various operating situations and assumptions. Researchers have contributed to this field and addressed issues related to industrial system maintenance strategies, and specifically focused on the performability aspects of the system pertaining to the reliability and the cost-benefit analysis.

Mokaddis et al. [1] analyzed the reliability and mean time to system recovery with one repairman, two-unit warm standby system that is susceptible to deterioration; Li and Chen [2] examined the residual life span of an independent, k-out-of-n system.; Rizwan [3] described a reliability modelling approach for programmable logic controllers, along with examples of how it may be utilized in the biscuit manufacturing industry. Zuhair & Rizwan [4] suggested a two-unit cold standby system with the idea of a repairman's rest and estimated system effectiveness indices; Parashar & Taneja [5] evaluated the profit of two PLCs were set up in a master-slave configuration, with the master unit was operative while the slave unit was in hot standby. The slave unit had a lower failure rate than the master unit.; Gupta & Tewari [6] analyzed a thermal power plant for the system analysis. Mathew et al. [7]-[10] developed various models for a casting plant to analyse loading crane operations as a two-unit system with full and reduced installed capacity. Rizwan et al. [11]-[13] calculated cost profit analysis of desalination and the wastewater treatment plant; Padmavathi et al. [14] analyzed a desalination plant and obtained the reliability indices for the system effectiveness; Sharma & Taneja [15] analyzed a two-unit standby oil delivery system with three sorts of failures: total failure, normal to partial failure, and partial to complete failure; Sharma & Kaur [16]-[17] assessed the availability of a compressor unit that was functioning in a milk factory when it failed owing to a variety of failure types that may be classified as serviceable, repairable, and replaced category. Kumar et al. [18] examined a thermal power plant's furnace drafting air cycle performance study. It includes three primary subsystems where both series and parallel arrangements of these subsystems are organized. Bhatia et al. [19] considered the reliability and economic analysis of a system using induced drafting (ID) fans installed in boilers used in thermal power plants. Three identical ID fans, two operational and one in cold standby are installed on the boiler under examination to provide backup. Parashar et al. [20] analyzed the system's reliability with induced drafting (ID) fans installed in thermal power plant boilers. The boiler pertinent to this study has three ID fans installed; two are continuously in use, while the third one serves as a warm standby. Ram & Singh [21], Bhardwaj & Singh [22], Gupta & Gupta [23], and Niwas et al. [24] worked on how to analyze the financial performance of a reliability model for a single-unit system that includes post-inspection, post-repair, preventative maintenance, and replacements. Rizwan et al. [25]-[27] studied desalination plants with winter closure and regarding the reliability of the residential treatment facility for wastewater is conducted. Padmavathi et al. [28] examined the models representing the two operational scenarios for a desalination plant to determine which model was superior. Al Rahbi et al. [29]-[36] focused on the rodding anode plant in the aluminum industry and discussed models portraying different situations of the plant for the reliability analysis where subsystems analysis, a system of butt thimble removal station analysis, single and multiple repairers with single and multiple units' analysis is carried out. Barak et al. [37]-[38] analyzed a two-unit cold standby system under various atmospheric conditions and a stochastic study of a redundant system with an emphasis on inspection. Wang et al. [39], Yusuf et al. [40], Goyal et al. [41], Gupta et al. [42], Singh et al. [43] have covered a warm standby repairable system with two different units and one technician, a linear sequential 2-out-of-4 system under both on- and offline preventative maintenance was modelled and evaluated for reliability, examined the sewage treatment plant's reliability metrics, examined the viability and accessibility of generators, which are essential components of steam turbine power plants, taking into account random repair time and exponential failures, considered two non-identical cold standby system. Taj et al. [44]-[52] produced a number of models for different operating scenarios of the cable plant's subsystems and main system for examination and model comparability. The models covered here are for subsystems where repairs are prioritized over-servicing, a situation of the plant with six maintenance categories is considered, rod breakdown system analysis, the situation of the cable plant with storage surplus produce is considered, winter operating strategy, and 3-unit system analysis. Sachdeva et al. [53] analyzed the sensitivity and profitability of a system covered under insurance and extended conditional warranty. Rizwan et al. [54] analyzed a three-unit pumping station as a single system. The pumping station operating with three pumps for pumping the desalinated water from the reservoirs is studied to understand the operational capability of the pumping station as a single system by getting the dependability metrics, such as mean time between failures, steady-state availability, and anticipated busy period for system recovery. For this investigation, maintenance information on the pumps over a five-year period was gathered from the station. The data are used to estimate the failure and restoration rates for each of the pumps. However, the gap noticed in [54] is the pumping station with three pumps was considered as a single system for the analysis which does not reveal that which pump, which parameter is the main contributing factor for the low system performance, and what are the most or the least influencing parameters those are affecting the reliabilities of the individual pumps. This possibly could have been better addressed if the entire analysis had been carried out for the individual pumps' performance, and the sensitivity analysis [53] for each of the three pumps could have been a valuable addition to the entire analysis.

Therefore, the novelty of the present work lies in its case-specific analysis of the individual pumps to compare the operational capabilities of each of the three pumps and to establish which pump is the least performing and need attention to improve upon the entire system. Further, a detailed sensitivity analysis is carried out to determine whether a parameter significantly impacts the reliability outcomes. This will help the maintenance team to focus on the preventive maintenance strategies pertaining to the specific pump and specific failure types rather than further identifying the reasons at the macro level. The rest of the model assumptions and descriptions about the system are retained as in [54].

3. Data Summary

Estimated Rates for all three Pumps

Pump 1:

Estimated rate of Pump 1 failure due to WRD failure:
$\lambda_{11} = 0.0012$
Estimated rate of Pump 1 failure due to MSL:
$\lambda_{12} = 0.00097$
Estimated rate of Pump 1 failure due to IB:
$\lambda_{13} = 0.00071$
Estimated restoration rate for Pump 1 after fixing WRD repair:
$\alpha_1 = 0.138$
Estimated restoration rate for Pump 1 after fixing MSL repair:
$\alpha_2 = 0.123$
Estimated restoration rate for Pump 1 after fixing IB:
$\alpha_3 = 0.172$

Pump 2:

Estimated rate of failure of Pump2 due to MSL failure:
$\lambda_{21} = 0.00156$
Estimated rate of failure of Pump 2 due to SB:
$\lambda_{22} = 0.0009$
Estimated restoration rate for Pump 2 after fixing MSL repair:
$\alpha_4 = 0.121$
Estimated restoration rate for Pump 2 after fixing SB repair:
$\alpha_5 = 0.77$

Pump 3:

Estimated rate of failure of Pump 3 due to WRD failure:
$\lambda_{31} = 0.00192$
Estimated rate of failure of Pump 3 due to MSL:
$\lambda_{32} = 0.00162$
Estimated rate of failure of Pump 3 due to IB:
$\lambda_{33} = 0.0009$
Estimated restoration rate for Pump 3 after fixing WRD repair:
$\alpha_6 = 0.093$
Estimated restoration rate for Pump 3 after fixing MSL repair:
$\alpha_7 = 0.1$
Estimated restoration rate for Pump 3 after fixing IB:
$\alpha_8 = 0.102$.

4. Transition State Diagram

The following is a description of the states for each pump:

Pump 1:

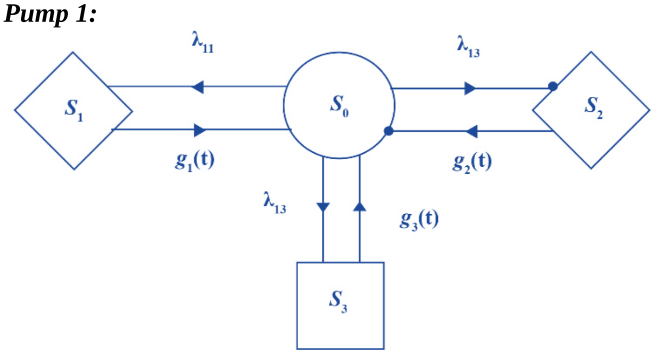

Fig. 1

Pump 2:

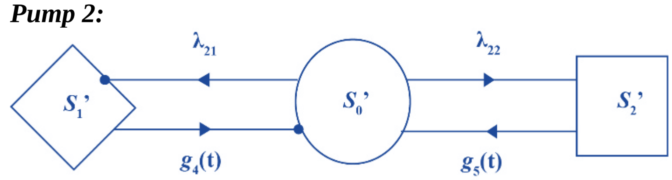

Fig. 2

Pump 3:

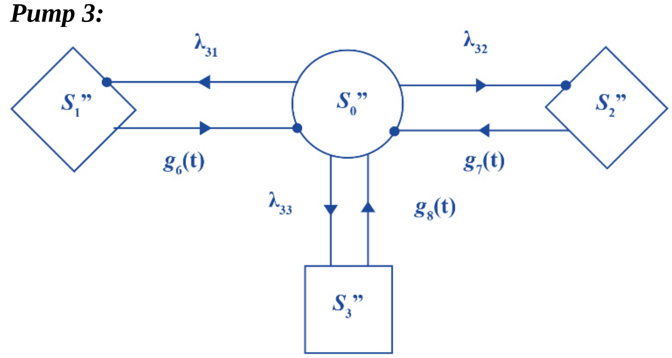

Fig. 3

5. Transition Probabilities and Mean Sojourn Times of Pumps 1, 2 & 3

5.1. The Transition Probabilities and the Mean Sojourn Times for Pump 1

The transition probabilities of possible states of Pump 1 are given by,

$$dQ_{01}^1(t) = \lambda_{11}e^{-(\lambda_{11}+\lambda_{12}+\lambda_{13})\,t}dt,$$
$$dQ_{02}^1(t) = \lambda_{12}e^{-(\lambda_{11}+\lambda_{12}+\lambda_{13})\,t}dt,$$
$$dQ_{03}^1(t) = \lambda_{13}e^{-(\lambda_{11}+\lambda_{12}+\lambda_{13})\,t}dt,$$
$$dQ_{10}^1(t) = g_1(t)dt,$$
$$dQ_{20}^1(t) = g_2(t)dt,$$
$$dQ_{30}^1(t) = g_3(t)dt,$$

(1-6)

The transition probabilities $p_{ij}^1$are given below:

$$p_{01}^1 = \lim_{s\to 0} {q^1}^*_{01}(s)$$
$$= \frac{\lambda_{11}}{\lambda_{11}+\lambda_{12}+\lambda_{13}}$$

Similarly,

$$p_{02}^1 = \lim_{s\to 0} {q^1}^*_{02}(s) = \lim_{s\to 0} L[q^1{}_{02}(t)\,] = \frac{\lambda_{12}}{\lambda_{11}+\lambda_{12}+\lambda_{13}}$$

$$p_{03}^1 = \lim_{s\to 0} {q^1}^*_{03}(s) = \lim_{s\to 0} L[q^1{}_{03}(t)\,] = \frac{\lambda_{13}}{\lambda_{11}+\lambda_{12}+\lambda_{13}}$$

(7-9)

By the above probabilities, it may be verified that:

$$p_{01}^1 + p_{02}^1 + p_{03}^1 = 1$$

and

$$p_{10}^1 = \lim_{s\to 0}\frac{\alpha_1}{s+\alpha_1} = 1$$
$$p_{20}^1 = \lim_{s\to 0}\frac{\alpha_2}{s+\alpha_2} = 1$$
$$p_{30}^1 = \lim_{s\to 0}\frac{\alpha_3}{s+\alpha_3} = 1$$

(10-12)

The mean sojourn time, μ_i^1 in the state, 'i' is defined as the stay time before moving to any other state of pump 1

$$\mu_0^1 = \int_0^\infty e^{-(\lambda_{11}+\lambda_{12}+\lambda_{13})\,t}dt = \frac{1}{\lambda_{11}+\lambda_{12}+\lambda_{13}}$$
$$\mu_1^1 = \int_0^\infty \overline{G_1}(t)dt = \int_0^\infty e^{-\alpha_1 t}dt = \frac{1}{\alpha_1},$$

Similarly,

$$\mu_2^1 = \int_0^\infty \overline{G_2}(t)dt = \int_0^\infty e^{-\alpha_2 t}dt = \frac{1}{\alpha_2}$$
$$\mu_3^1 = \int_0^\infty \overline{G_3}(t)dt = \int_0^\infty e^{-\alpha_3 t}dt = \frac{1}{\alpha_3}$$

(13-16)

When time is evaluated from the time point of entry in state 'i', the average time taken by the pump to move any regenerative state 'j' is expressed as follows:

$$m_{ij}^1 = \int_0^\infty t\,dQ_{ij}^1(t) = -{q_{ij}^1}^{*'}(0),$$

and $\sum_j m_{ij}^1 = \mu_i^1$

$$m_{01}^1 = \frac{\lambda_{11}}{(\lambda_{11}+\lambda_{12}+\lambda_{13})^2};$$
$$m_{02}^1 = \frac{\lambda_{12}}{(\lambda_{11}+\lambda_{12}+\lambda_{13})^2};$$
$$m_{03}^1 = \frac{\lambda_{13}}{(\lambda_{11}+\lambda_{12}+\lambda_{13})^2};$$
$$m_{01}^1 + m_{02}^1 + m_{03}^1 = \frac{1}{\lambda_{11}+\lambda_{12}+\lambda_{13}} = \mu_0^1$$

(17-20)

Similarly, m_{10}^1 , m_{20}^1 and m_{30}^1 can be evaluated as

$$m_{10}^1 = \mu_1^1;\quad m_{20}^1 = \mu_2^1;\quad m_{30}^1 = \mu_3^1. \qquad (21\text{-}23)$$

5.2. The Transition Probabilities and the Mean Sojourn Times for Pump 2

The transition probabilities of possible states of Pump 2 are given by,

$$dQ_{01}^2(t) = \lambda_{21}e^{-(\lambda_{21}+\lambda_{22})\,t}dt,$$
$$dQ_{02}^2(t) = \lambda_{22}e^{-(\lambda_{21}+\lambda_{22})\,t}dt,$$
$$dQ_{10}^2(t) = g_4(t)dt,$$
$$dQ_{20}^2(t) = g_5(t)dt,$$

(24-27)

The transition probabilities $p_{ij}^2$are given below:

$$p_{01}^2 = \lim_{s\to 0} {q^2}^*_{01}(s)$$
$$= \lim_{s\to 0} L[q^2{}_{01}(t)\,] = \lim_{s\to 0} L\left[\lambda_{21}e^{-(\lambda_{21}+\lambda_{22})\,t}\right]$$

$$= \lim_{s\to 0}\frac{\lambda_{21}}{s+\lambda_{21}+\lambda_{22}} = \frac{\lambda_{21}}{\lambda_{21}+\lambda_{22}}$$

Similarly,

$$p_{02}^2 = \lim_{s\to 0} {q^2}^*_{02}(s) = \lim_{s\to 0} L[q^2{}_{02}(t)\,] = \frac{\lambda_{22}}{\lambda_{21}+\lambda_{22}}$$

(28-29)

By the above probabilities, it may be verified that:

$$p_{01}^2 + p_{02}^2 = 1$$

and

$$p_{10}^2 = \lim_{s\to 0}\frac{\alpha_4}{s+\alpha_4} = 1$$
$$p_{20}^2 = \lim_{s\to 0}\frac{\alpha_5}{s+\alpha_5} = 1$$

(30-31)

The mean sojourn time, μ_i^2 in the state, 'i' is defined as the stay time before moving to any other state of pump 2

$$\mu_i^2 = E(T) = P(T > t);$$

$$\mu_0^2 = \frac{1}{\lambda_{21}+\lambda_{22}}$$
$$\mu_1^2 = \int_0^\infty \overline{G_4}(t)dt = \int_0^\infty e^{-\alpha_4 t}dt = \frac{1}{\alpha_4}$$
$$\mu_2^2 = \int_0^\infty \overline{G_5}(t)dt = \int_0^\infty e^{-\alpha_5 t}dt = \frac{1}{\alpha_5}$$
(32-34)

When time is evaluated from the time point of entry in state ′i′, the average time taken by the pump to move any regenerative state ′j′ is expressed as follows:

$$m_{ij}^2 = \int_0^\infty t dQ_{ij}^2(t) = -q_{ij}^{2*'}(0),$$
(unconditional time taken to transit),

and $\sum_j m_{ij}^2 = \mu_i^2$

$$m_{01}^2 = \frac{\lambda_{21}}{(\lambda_{21}+\lambda_{22})^2};$$
$$m_{02}^2 = \frac{\lambda_{22}}{(\lambda_{21}+\lambda_{22})^2};$$
(35-36)

and therefore,

$$m_{01}^2 + m_{02}^2 = \frac{1}{\lambda_{21}+\lambda_{22}} = \mu_0^2$$

Similarly, m_{10}^2 and m_{20}^2 can be evaluated as

$m_{10}^2 = \mu_1^2$; $m_{20}^2 = \mu_2^2$.

(37-38)

5.3. The Transition Probabilities and the Mean Sojourn Times for Pump 3

The transition probabilities of possible states of Pump 3 are given by,

$$dQ_{01}^3(t) = \lambda_{31} e^{-(\lambda_{31}+\lambda_{32}+\lambda_{33})\,t}dt,$$
$$dQ_{02}^3(t) = \lambda_{32} e^{-(\lambda_{31}+\lambda_{32}+\lambda_{33})\,t}dt,$$
$$dQ_{03}^3(t) = \lambda_{33} e^{-(\lambda_{31}+\lambda_{32}+\lambda_{33})\,t}dt,$$
$$dQ_{10}^3(t) = g_6(t)dt,$$
$$dQ_{20}^3(t) = g_7(t)dt,$$
$$dQ_{30}^3(t) = g_8(t)dt.$$
(39-44)

The transition probabilities p_{ij}^3 are given below:

$$p_{01}^3 = \lim_{s\to 0} q^{3*}{}_{01}(s) = \lim_{s\to 0} L[q^3{}_{01}(t)] = \lim_{s\to 0} L\left[\lambda_{31} e^{-(\lambda_{31}+\lambda_{32}+\lambda_{33})\,t}\right] = \lim_{s\to 0}\frac{\lambda_{31}}{s+\lambda_{31}+\lambda_{32}+\lambda_{33}} = \frac{\lambda_{31}}{\lambda_{31}+\lambda_{32}+\lambda_{33}}$$

Similarly,

$$p_{02}^3 = \lim_{s\to 0} q^{3*}{}_{02}(s) = \lim_{s\to 0} L[q^3{}_{02}(t)] = \frac{\lambda_{32}}{\lambda_{31}+\lambda_{32}+\lambda_{33}}$$
$$p_{03}^3 = \lim_{s\to 0} q^{3*}{}_{03}(s) = \lim_{s\to 0} L[q^3{}_{03}(t)] = \frac{\lambda_{33}}{\lambda_{31}+\lambda_{32}+\lambda_{33}}$$
(45-47)

By the above probabilities, it may be verified that:

$p_{01}^3 + p_{02}^3 + p_{03}^3 = 1$

and

$$p_{10}^3 = \lim_{s\to 0}\frac{\alpha_6}{s+\alpha_6} = 1$$
$$p_{20}^3 = \lim_{s\to 0}\frac{\alpha_7}{s+\alpha_7} = 1$$
$$p_{30}^3 = \lim_{s\to 0}\frac{\alpha_8}{s+\alpha_8} = 1$$
(48-50)

The mean sojourn time, μ_i^3 in the state, 'i' is defined as the stay time before moving to any other state of pump 3

$$\mu_0^3 = \int_0^\infty e^{-(\lambda_{31}+\lambda_{32}+\lambda_{33})\,t}dt = \frac{1}{\lambda_{31}+\lambda_{32}+\lambda_{33}}$$
$$\mu_1^3 = \int_0^\infty \overline{G_6}(t)dt = \int_0^\infty e^{-\alpha_6 t}dt = \frac{1}{\alpha_6},$$

Similarly,

$$\mu_2^3 = \int_0^\infty \overline{G_7}(t)dt = \int_0^\infty e^{-\alpha_7 t}dt = \frac{1}{\alpha_7}$$
$$\mu_3^3 = \int_0^\infty \overline{G_8}(t)dt = \int_0^\infty e^{-\alpha_8 t}dt = \frac{1}{\alpha_8}$$
(51-54)

When time is evaluated from the time point of entry in state ′i′, the average time taken by the pump to moving any regenerative state ′j′ is expressed as follows:

$$m_{ij}^3 = \int_0^\infty t dQ_{ij}^3(t) = -q_{ij}^{3*'}(0),$$
(unconditional time taken to transit),

and $\sum_j m_{ij}^3 = \mu_i^3$

$$m_{02}^3 = \frac{\lambda_{32}}{(\lambda_{31}+\lambda_{32}+\lambda_{33})^2};$$
$$m_{03}^3 = \frac{\lambda_{33}}{(\lambda_{31}+\lambda_{32}+\lambda_{33})^2}$$
(55-57)

and therefore,

$$m_{01}^3 + m_{02}^3 + m_{03}^3 = \frac{1}{\lambda_{31}+\lambda_{32}+\lambda_{33}} = \mu_0^3$$
(58)

Similarly, m_{10}^3, m_{20}^3 and m_{30}^3 can be evaluated as

$m_{10}^3 = \mu_1^3$; $m_{20}^3 = \mu_2^3$; $m_{30}^3 = \mu_3^3$.

(59-61)

6. System Measures

6.1. Mean Time Between Failure for Pump 1 & 3

Applying the justifications for regenerative processes and considering the failed states as absorbing states when a failure results from the broken impeller, the following recursive relation for the mean time to system failures of pump 1, $\zeta_i^1(t)$ is obtained:

$$\zeta_0^1(t) = Q_{01}^1(t)(s)\,\zeta_1^1(t) + Q_{02}^1(t)(s)\,\zeta_2^1(t) + Q_{03}^1(t)$$
$$\zeta_1^1(t) = Q_{10}^1(t)(s)\,\zeta_0^1(t)$$
$$\zeta_2^1(t) = Q_{20}^1(t)(s)\,\zeta_0^1(t)$$

(62-64)

The mean time to system failure (MTSF) is now calculated from the time the unit started in the initial state $\mathcal{S}_0$ as

$$MTSF = \lim_{s\to 0}\frac{1-\zeta_0^{**1}(s)}{s}$$

where

$$\zeta_0^{**1}(s) = \frac{Q_{03}^{**1}(s)}{1 - Q_{01}^{**1}(s)Q_{10}^{**1}(s) - Q_{02}^{**1}(s)Q_{20}^{**1}(s)} = \frac{\mathcal{N}(s)}{\mathcal{D}(s)} \quad (65)$$

$$MTSF = \lim_{s\to 0}\frac{1-\frac{\mathcal{N}(s)}{\mathcal{D}(s)}}{s} = \lim_{s\to 0}\frac{\mathcal{D}(s)-\mathcal{N}(s)}{s\mathcal{D}(s)}$$

$$= \lim_{s\to 0}\frac{\mathcal{D}'(s)-\mathcal{N}'(s)}{s\mathcal{D}'(s)+\mathcal{D}(s)} = \frac{\mathcal{D}'(0)-\mathcal{N}'(0)}{\mathcal{D}(0)}$$

$$= \frac{\mathcal{N}_1}{\mathcal{D}_1}, where \quad (66)$$

$$\mathcal{N}_1 = \mu_0^1 + p_{01}^1\,\mu_1^1 + p_{02}^1\,\mu_2^1 \;\; and\; \mathcal{D}_1 = p_{03}^1.$$

Similarly, the recursive relations for the meantime of system failures of pump 3 be framed as the reasons for failure are the same, except the variations in the failure and repair rates and the meantime of system failures for pump 3, $\zeta_i^3(t)$ is as:

$$\zeta_0^3(t) = Q_{01}^3(t)(s)\,\zeta_1^3(t) + Q_{02}^3(t)(s)\,\zeta_2^3(t) + Q_{03}^3(t)$$
$$\zeta_1^3(t) = Q_{10}^3(t)(s)\,\zeta_0^3(t)$$
$$\zeta_2^3(t) = Q_{20}^3(t)(s)\,\zeta_0^3(t)$$

(67-69)

In the meantime, system failure when the unit started at the commencement of the state $\mathcal{S}_0$", is

$$MTSF = \lim_{s\to 0}\frac{1-\zeta_0^{**3}(s)}{s},$$

where

$$\zeta_0^{**3}(s) = \frac{Q_{03}^{**3}(s)}{1 - Q_{01}^{**3}(s)Q_{10}^{**3}(s) - Q_{02}^{**3}(s)Q_{20}^{**3}(s)} = \frac{\mathcal{N}(s)}{\mathcal{D}(s)} \quad (70)$$

$$MTSF = \lim_{s\to 0}\frac{1-\frac{\mathcal{N}(s)}{\mathcal{D}(s)}}{s} = \lim_{s\to 0}\frac{\mathcal{D}(s)-\mathcal{N}(s)}{s\mathcal{D}(s)}$$
$$= \lim_{s\to 0}\frac{\mathcal{D}'(s)-\mathcal{N}'(s)}{s\mathcal{D}'(s)+\mathcal{D}(s)} = \frac{\mathcal{D}'(0)-\mathcal{N}'(0)}{\mathcal{D}(0)}$$
$$= \frac{\mathcal{N}_3}{\mathcal{D}_3}, where \quad (71)$$

$$\mathcal{N}_3 = \mu_0^3 + \mu_1^3 p_{01}^3 + \mu_2^3 p_{02}^3 \;\; and\; \mathcal{D}_3 = p_{03}^3$$

6.2. Availability Analysis for Pump 1 & 3

The recursive relations for pump 1 are obtained using probabilistic reasoning and let $\Lambda_i^1(t)$ as the probability that the unit is in upstate at the time t, given that the unit entered state i at $t = 0$.

$$\Lambda_0^1(t) = \mathcal{M}_0^1(t) + q_{01}^1(t)©\Lambda_1^1(t) + q_{02}^1(t)©\Lambda_2^1(t) + q_{03}^1(t)©\Lambda_3^1(t)$$
$$\Lambda_1^1(t) = \mathcal{M}_1^1(t) + q_{10}^1(t)©\Lambda_0^1(t)$$
$$\Lambda_2^1(t) = \mathcal{M}_2^1(t) + q_{20}^1(t)©\Lambda_0^1(t)$$
$$\Lambda_3^1(t) = q_{30}^1(t)©\Lambda_0^1(t)$$

(72-75)

where

$$\mathcal{M}_0^1(t) = e^{-(\lambda_{11}+\lambda_{12}+\lambda_{13})t};$$
$$\mathcal{M}_1^1(t) = \overline{G_1}(t);\; \mathcal{M}_2^1(t) = \overline{G_2}(t)$$

(76-78)

Using the determinants technique, the following is derived by taking the Laplace Transforms of the aforementioned equations and solving them for $\Lambda_0^{*1}(s)$:

$$\Lambda_0^1 = \lim_{s\to 0} s\Lambda_0^{*1}(s) = \frac{\mathcal{N}_1^A}{\mathcal{D}_1^A} \quad (79)$$

where,

$$\Lambda_0^{*1}(s) = \frac{\mathcal{M}_0^{*1}(s) + q_{01}^{*1}(s)\mathcal{M}_1^{*1}(s) + q_{02}^{*1}(s)\mathcal{M}_2^{*1}(s)}{1 - q_{01}^{*1}(s)q_{10}^{*1}(s) - q_{02}^{*1}(s)q_{20}^{*1}(s) - q_{03}^{*1}(s)q_{30}^{*1}(s)} \quad (80)$$

$\mathcal{N}_1^A = \mu_0^1 + p_{01}^1\,\mu_1^1 + p_{02}^1\,\mu_2^1$ and
$\mathcal{D}_1^A = \mu_0^1 + p_{01}^1\,\mu_1^1 + p_{02}^1\,\mu_2^1 + p_{03}^1\,\mu_3^1.$

Similarly, the recursive relations for the Availability of Pump 3 can be framed as the reasons for failure are the same, except for the variations in the failure and repair rates, and the availability for pump 3 is obtained as:

$$\Lambda_0^3(t) = \mathcal{M}_0^3(t) + q_{01}^3(t)©\Lambda_1^3(t) + q_{02}^3(t)©\Lambda_2^3(t) + q_{03}^3(t)©\Lambda_3^3(t)$$
$$\Lambda_1^3(t) = \mathcal{M}_1^3(t) + q_{10}^3(t)©\Lambda_0^3(t)$$
$$\Lambda_2^3(t) = \mathcal{M}_2^3(t) + q_{20}^3(t)©\Lambda_0^3(t)$$
$$\Lambda_3^3(t) = q_{30}^3(t)©\Lambda_0^3(t)$$

(81-84)

where

$$\mathcal{M}_0^3(t) = e^{-(\lambda_{31}+\lambda_{32}+\lambda_{33})t};$$
$$\mathcal{M}_1^3(t) = \overline{G_6}(t);\; \mathcal{M}_2^1(t) = \overline{G_7}(t).$$

(85-87)

The following is derived by taking the Laplace Transforms of the aforementioned equations and solving them for Λ_0^3 (s) using the determinants method:

$$\Lambda_0^3 = \lim_{s\to 0} s\Lambda_0^{*3}(s) = \frac{\mathcal{N}_3^A}{\mathcal{D}_3^A} \quad (88)$$

where,

$$\Lambda_0^{*3}(s) = \frac{\mathcal{M}_0^{*3}(s) + q_{01}^{*3}(s)\mathcal{M}_1^{*3}(s) + q_{02}^{*3}(s)\mathcal{M}_2^{*3}(s)}{1 - q_{01}^{*3}(s)q_{10}^{*3}(s) - q_{02}^{*3}(s)q_{20}^{*3}(s) - q_{03}^{*3}(s)q_{30}^{*3}(s)} \quad (89)$$

$\mathcal{N}_3{}^A = \mu_0^3 + p_{01}^3\,\mu_1^3 + p_{02}^3\,\mu_2^3$ and $\mathcal{D}_3^A = \mu_0^3 + p_{01}^3\,\mu_1^3 + p_{02}^3\,\mu_2^3 + p_{03}^3$.

6.3. Mean Time Between Failure for Pump 2

Applying the justifications for regenerative processes and considering the failed states as absorbing states when a failure results from the broken Shaft, the following recursive relation for the mean time to system failures of pump 2, $\zeta_i^2(t)$ is obtained:

$$\zeta_0^2(\mathrm{t}) = \mathrm{Q}_{01}^2(\mathrm{t})(\mathrm{s})\,\zeta_1^2(\mathrm{t}) + \mathrm{Q}_{02}^2(\mathrm{t}),$$
$$\zeta_1^2(\mathrm{t}) = \mathrm{Q}_{10}^2(\mathrm{t})(\mathrm{s})\,\zeta_0^2(\mathrm{t}). \quad (90\text{-}91)$$

In the meantime, system failure when the unit started at the commencement of the state $\mathcal{S}_0{}'$, is

$$MTSF = \lim_{s\to 0} \frac{1 - \zeta_0^{**2}(s)}{s},$$

where

$$\zeta_0^{**2}(s) = \frac{Q_{02}^{**2}(s)}{1 - Q_{01}^{**2}(s)Q_{10}^{**2}(s)} = \frac{\mathcal{N}(s)}{\mathcal{D}(s)} \quad (92)$$

$$MTSF = \lim_{s\to 0} \frac{1 - \frac{\mathcal{N}(s)}{\mathcal{D}(s)}}{s} = \lim_{s\to 0} \frac{\mathcal{D}(s) - \mathcal{N}(s)}{s\mathcal{D}(s)}$$

$$= \frac{\mathcal{N}_2}{\mathcal{D}_2}, where \quad (93)$$

$$\mathcal{N}_2 = \mu_0^2 + p_{01}^2\mu_1^2 \; and \; \mathcal{D}_2 = p_{02}^2.$$

6.4. Availability Analysis for Pump 2

The recursive relations for pump 2 are obtained using probabilistic reasoning and let $\Lambda_i^2(t)$ as the probability that the unit is in upstate at the time t, given that the unit entered state i at $t = 0$.

$$\Lambda_0^2(t) = \mathcal{M}_0^2(t) + q_{01}^2(t)Ⓒ\Lambda_1^2(t) + q_{02}^2(t)Ⓒ\Lambda_2^2(t),$$
$$\Lambda_1^2(t) = \mathcal{M}_1^2(t) + q_{10}^2(t)Ⓒ\Lambda_0^2(t),$$
$$\Lambda_2^2(t) = q_{20}^2(t)Ⓒ\Lambda_0^2(t) \quad (94\text{-}96)$$

where

$$\mathcal{M}_0^2(t) = e^{-(\lambda_{21}+\lambda_{22})t};\ \mathcal{M}_1^2(t) = \overline{G_4}(t). \quad (97\text{-}98)$$

The following is derived by taking the Laplace Transforms of the aforementioned equations and solving them for $\Lambda_0^2(s)$ using the determinants method:

$$\Lambda_0^2(t) = \lim_{s\to 0} s\Lambda_0^{*2}(s) = \frac{\mathcal{N}_2^A}{\mathcal{D}_2^A} \quad (99)$$

where,

$$\Lambda_0^{*2}(s) = \frac{\mathcal{M}_0^{*2}(s) + q_{01}^{*2}(s)\mathcal{M}_1^{*2}(s)}{1 - q_{01}^{*2}(s)q_{10}^{*2}(s) - q_{02}^{*2}(s)q_{20}^{*2}(s)}. \quad (100)$$

$\mathcal{N}_2{}^A = \mu_0^2 + p_{01}^2\mu_1^2$ and $\mathcal{D}_2^A = \mu_0^2 + \mu_1^2 p_{01}^2 + \mu_2^2 p_{02}^2$.

Using the expressions obtained in sections 3 & sections 6.1. and 6.3, the following estimations for pumps MTSF are arrived:

MTSF for Pump 1 = $1432.79\ hours$
MTSF for Pump 2 = $1125.45\ hours$
MTSF for Pump 3 = $1152.07\ hours$

and using the expressions obtained in sections 3 & sections 6.2. and 6.4., the following estimates for pumps availability are arrived:

Availability of Pump 1 = 0.996
Availability of Pump 2 = 0.999
Availability of Pump 3 = 0.992

6.5. Sensitivity Analysis for MTSF of Pumps 1, 2 & 3

Table 1

Parameter (r)	**Sensitivity Analysis** $dM = \frac{\partial(MTSF)}{\partial r}$	**Relative Sensitivity Analysis** $\delta M = \frac{dM \times r}{MTSF}$
Pump 1		
$\boldsymbol{\lambda_{11}}$	1.0206×10^4	0.0086
$\boldsymbol{\lambda_{12}}$	1.1451×10^4	0.0078
$\boldsymbol{\lambda_{13}}$	-2.0166×10^6	−1
$\boldsymbol{\alpha_1}$	−88.7493	−0.0086
$\boldsymbol{\alpha_2}$	−90.3032	−0.0078
$\boldsymbol{\alpha_3}$	0	0
Pump 2		
$\boldsymbol{\lambda_{21}}$	9.1827×10^3	0.0127
$\boldsymbol{\lambda_{22}}$	-1.2505×10^6	−1
$\boldsymbol{\alpha_4}$	−118.3890	−0.0127
$\boldsymbol{\alpha_5}$	0	0
Pump 3		
$\boldsymbol{\lambda_{31}}$	1.1947×10^4	0.0199
$\boldsymbol{\lambda_{32}}$	1.1111×10^4	0.0156
$\boldsymbol{\lambda_{33}}$	-1.2801×10^6	−1
$\boldsymbol{\alpha_6}$	−246.6566	−0.0199
$\boldsymbol{\alpha_7}$	−180	−0.0156
$\boldsymbol{\alpha_8}$	0	0

6.6. Sensitivity Analysis for Availability of Pumps 1, 2 & 3

Table 2

Parameter (r)	Sensitivity Analysis $d\Lambda = \frac{\partial(\Lambda_0)}{\partial r}$	Relative Sensitivity Analysis $\delta\Lambda = \frac{d\Lambda \times r}{\Lambda_0}$
Pump 1		
$\boldsymbol{\lambda_{11}}$	0.0287	3.458×10^{-5}
$\boldsymbol{\lambda_{12}}$	0.0322	3.1361×10^{-5}
$\boldsymbol{\lambda_{13}}$	-5.673	-0.004
$\boldsymbol{\alpha_1}$	-2.4966×10^{-4}	-3.4593×10^{-5}
$\boldsymbol{\alpha_2}$	-2.5403×10^{-4}	-3.1373×10^{-5}
$\boldsymbol{\alpha_3}$	0.0234	0.004
Pump 2		
$\boldsymbol{\lambda_{21}}$	0.0094	1.4681×10^{-5}
$\boldsymbol{\lambda_{22}}$	-1.2792	-0.0012
$\boldsymbol{\alpha_4}$	-1.2111×10^{-4}	-1.4671×10^{-5}
$\boldsymbol{\alpha_5}$	0.0015	0.0012
Pump 3		
$\boldsymbol{\lambda_{31}}$	0.0868	1.6807×10^{-4}
$\boldsymbol{\lambda_{32}}$	0.0807	1.3185×10^{-4}
$\boldsymbol{\lambda_{33}}$	-9.2966	-0.0084
$\boldsymbol{\alpha_6}$	-0.0018	-1.6882×10^{-4}
$\boldsymbol{\alpha_7}$	-0.0013	-1.3111×10^{-4}
$\boldsymbol{\alpha_8}$	0.0820	0.0084

Therefore, the decreasing order in which parameter affects the MTSF and Availability of Pumps 1, 2 and 3 as:

Table 3

	Pumps	Decreasing the order of parameters
MTSF	Pump 1	$\lambda_{13}, \lambda_{11}, \alpha_1, \lambda_{12}, \alpha_2, \alpha_3$
	Pump 2	$\lambda_{22}, \lambda_{21}, \alpha_4, \alpha_5$
	Pump 3	$\lambda_{33}, \lambda_{31}, \alpha_6, \lambda_{32}, \alpha_7, \alpha_8$
Availability	Pump 1	$\lambda_{13}, \alpha_3, \alpha_1, \lambda_{11}, \alpha_2, \lambda_{12}$
	Pump 2	$\lambda_{22}, \alpha_5, \lambda_{21}, \alpha_4$
	Pump 3	$\lambda_{33}, \alpha_8, \alpha_6, \lambda_{31}, , \lambda_{32}, \alpha_7$

7. Conclusion

The outcome reveals that the mean time between failures for pump 2 is 1125.45 hours which is the least among the three pumps, whereas pump 1 is the better-performing pump having 1432.79 hours of mean time between failures and hence lasting longer than others. To improve this reliability index, the company needs well-organized preventive maintenance plans. A Root cause analysis of the pump components could further establish the reasons for frequent failing components. It is worth noting that the availability index of pump 2 is the highest, followed by pump 1 and 3, which shows that this pump can be available most of the time but unreliable due to frequent failures. Table 1 and 2 shows the outcomes for the sensitivity and relative sensitivity functions for the meantime of system failures and the availabilities of Pumps 1, 2 and 3. In the case of Pump 1 and 3, the mean time of system failures and availabilities are highly sensitive w.r.t. the Impeller broken failure rate, while for Pump 2, these are more affected by the failure rate of Shaft broken. For drawing inference, the absolute values of both functions are considered and have been shown chronologically in table 3.

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