Original Article

A Study on Shear Frame Structure by Modal Superposition Method

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Abstract *- The current study uses a normal mode to transform the coupled differential equation system into a set of uncoupled differential equations. As a result, the modal superposition method reduces the problem of determining the response of a multi-degree-of-freedom system to that of a single-degree-of-freedom system. The undamped multi-degrees-of-freedom system is analyzed using free modal vibrations. To determine the natural frequencies, mode shapes, time period, modal mass contribution, and mode participation factors of structures under free vibrations. Two cases are considered in the present study, first a single-storey, single-degree-of-freedom system (SDOF) & second case, a three-dimensional three-storey multi-degree of freedom (MDOF) building modeled as a shear building, idealized as a lumped spring-mass model. Modal analysis of shear frames for single and multi degrees of freedom (MDOF) structures are studied. The results are compared first theoretically, secondly by MATLAB program, and finally by ETABS software. The first case is a single-storey reinforced concrete 2D frame of 6m wide and 3m high & the dimension of each column and the beam is 300 mm square section. In the second case, a three-dimensional three-storey is considered. The effective length of the beam in the X-direction is 3.6m and 4.7m along the Y-direction, and the column's height is 3m. The effective slab thickness is 120mm, and the grade of concrete $f_{ck}=25N/mm^2$.*

Keywords *- ETABS, Lumped mass model, MATLAB, Mode period, Natural frequency.*

1. Introduction

Free vibration response for single and multiple degrees of freedom system caused by initial conditions (no external force). It started oscillating by applying initial conditions like displacements and velocities at different levels of the structure. This oscillation is known as free vibration response [1]. The basic idea of modal analysis describes the free vibration response as the sum of simple harmonic motions, and the shape of each mode motion is called mode shape.

The main objective of the modal analysis of frames is to determine the frame's natural frequencies, mode shapes, and structure period during free vibration. The modal analysis requires the effect of the mass and stiffness of the structure. The structure's mass depends on the self-weight of the beam (2D) and slabs in three-dimensional frames. The distributed mass is idealized and lumped at the levels of the floors as concentrated mass. The stiffness matrix will be coming from the structural geometry. This stiffness is formulated from the material, cross-section, member properties, and structural configurations, such as controlling parameters like boundary conditions [2]. The shear frame idealized helps govern the equations of motion for undamped free vibrations of single and multi-degree freedom systems [6].

This study is based on modal analysis of shear frames for single and multi degrees of freedom (MDOF) structures using the analytical method, MATLAB program, and ETABS software to study the natural frequency and deflection pattern for different cases.

2. Parametric Study of SDOF

2.1. Lumped mass model for SDOF

A single-storey reinforced concrete 2D frame 6m wide and 3m high is idealized as a lumped mass model. The response of this structure by modal analysis is shown in Figure 1. Each column and beam has a 300 mm square section. The basic idea of modal analysis describes the free vibration response, such as natural frequency, mode period, and mode shapes. Natural frequency depends on the mass and stiffness parameters. The lateral stiffness of the SDOF can be determined for three cases [1].

Case (1): When the beam stiffness is rigid ($EI_b =\infty$)

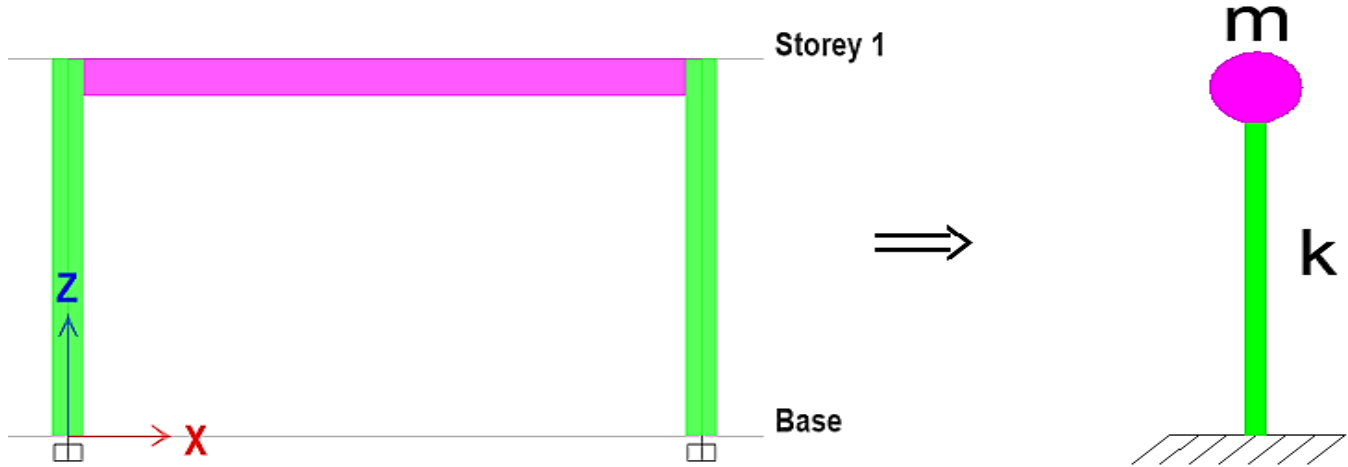The modal analysis is carried out based on the following assumptions:

Fig. 1 Two-dimensional frame with lumped mass model

1. Masses are lumped at the storey level, the mass of the columns is not considered, and the whole mass of the beam is lumped at the beam level.
2. Columns are flexible in the lateral direction.
3. The beam is rigid in a lateral direction.

Case (2): When the beam stiffness is negligible ($EI_b = 0$)

In this case, the beam's flexural rigidity (*EI*) is taken as zero and assumes all columns are flexible in the lateral direction, as mentioned in case (1).

Case (3): When the beam stiffness is very high ($EI_b \neq 0$)

The *EI* of the beam is assumed to be the same as column flexural rigidity. The length of the beam is equal to twice the column height. Based on the above assumptions mentioned in case (1) to case (3), the natural frequency and corresponding mode period are computed and tabulated in Table 3.

2.2. Calculation of Natural Frequency and Mode Period by Analytical Method

Acceleration due to gravity, g= 9.80665 m/s², f_{ck} = 25 N/mm² and density of RCC= 25 kN/m³. Young's modulus of concrete and the column's moment of inertia is calculated as given in (1) and (2), respectively. Equation (3) gives the solution of an undamped SDOF system for free vibration. The natural frequency and mode period are computed as shown in (4) and (5), respectively [6].

The effective length of the beam, i. e. *l = 6-0.3 = 5.7 m,* and column height is 3m, have been taken for load calculation.

$$E_c = 5000\sqrt{f_{ck}} = 5000\sqrt{25} = 2.5 \times 10^{10} \text{ N/m}^2 \quad (1)$$

$$I_c = I_{XX} = \frac{bd^3}{12} = \frac{0.3 \times 0.3^3}{12} = 6.75 \times 10^{-4} \text{ m}^4 \quad (2)$$

$$m\ddot{x} + kx = 0 \quad (3)$$

$$\omega_n = \sqrt{\frac{k}{m}} \quad (4)$$

$$T = \frac{2\pi}{\omega_n} \quad (5)$$

$$\text{Beam,B} = 0.3 \times 0.3 \times 5.7 \times 25 = 12.825 \text{ kN}$$
$$= \frac{12.825 \times 1000}{9.80665} = 1307.79 \text{ kg} \quad (6)$$

$$\text{Column,C} = 0.3 \times 0.3 \times 3 \times 25 = 6.75 \text{ kN}$$
$$= \frac{6.75 \times 1000}{9.80665} = 688.308 \text{ kg} \quad (7)$$

The lateral stiffness of columns for cases (1), (2), and (3) are calculated using (8) to (10). Also, the calculated values of mass and stiffness for different cases are tabulated in Table 1.

$$k = \sum_{\text{columns}}(12E_cI_c/h^3) =$$
$$(2 \times 12 \times 2.5 \times 10^{10} \times 6.75 \times 10^{-4})/3^3$$
$$= 15 \times 10^6 \text{ N/m} \quad (8)$$

$$k = \sum_{\text{columns}}(3E_cI_c/h^3) =$$
$$(2 \times 3 \times 2.5 \times 10^{10} \times 6.75 \times 10^{-4})/3^3$$
$$= 3.75 \times 10^6 \text{ N/m}$$
$$\quad (9)$$

$$k = 96E_cI_c/7h^3$$
$$= (96 \times 2.5 \times 10^{10} \times 6.75 \times 10^{-4})/(7 \times 3^3)$$
$$= 8.5714 \times 10^6 \text{ N/m}$$
$$\quad (10)$$

Table 1. Mass and stiffness values for different cases

Different Cases	Lumped mass, m (kg)	Lateral stiffness, k (N/m)
Case (1)	1307.79	15 x 10^6
Case (2)	1307.79	3.75 x 10^6
Case (3)	[mass of beam + no. of columns x (1/2) x mass of column)] (1307.79 +688.308) = 1996.09	8.57 x 10^6

2.3. Calculation of natural frequency and mode period by MATLAB

The natural frequency and corresponding mode period for SDOF can be calculated using the command [v, d] = eig (K, M) in the MATLAB command window as a calculator. Where v = Eigenvector (mode shapes) and d = Eigenvalue (Circular frequency), M and K are the mass and stiffness matrix of the frame. The Eigenvalue and vector calculations are shown in Figure 2.

### 2.4 Calculation of natural frequency and mode period by ETABS software

Based on the assumption mentioned in cases (1) to (3), the property modification factors are used for modeling and analyzing the frame with the help of ETABS

software. Table 2 gives the property modification factors for beams and columns for different cases. A rigid diaphragm is assigned for all joints to maintain lumped mass at floor level.

2.5 Results and Discussion

Table 3 presents the comparison of natural frequencies and mode periods of an SDOF for different cases. Figure 3 shows the mode shape for case (1), and Figure 4 shows the mode shapes for cases (2) and (3), respectively.

Natural frequencies and mode period values are the same in analytical and MATLAB. The difference is around 2% in frequency and mode period for analytical and MATLAB, compared to ETABS software.
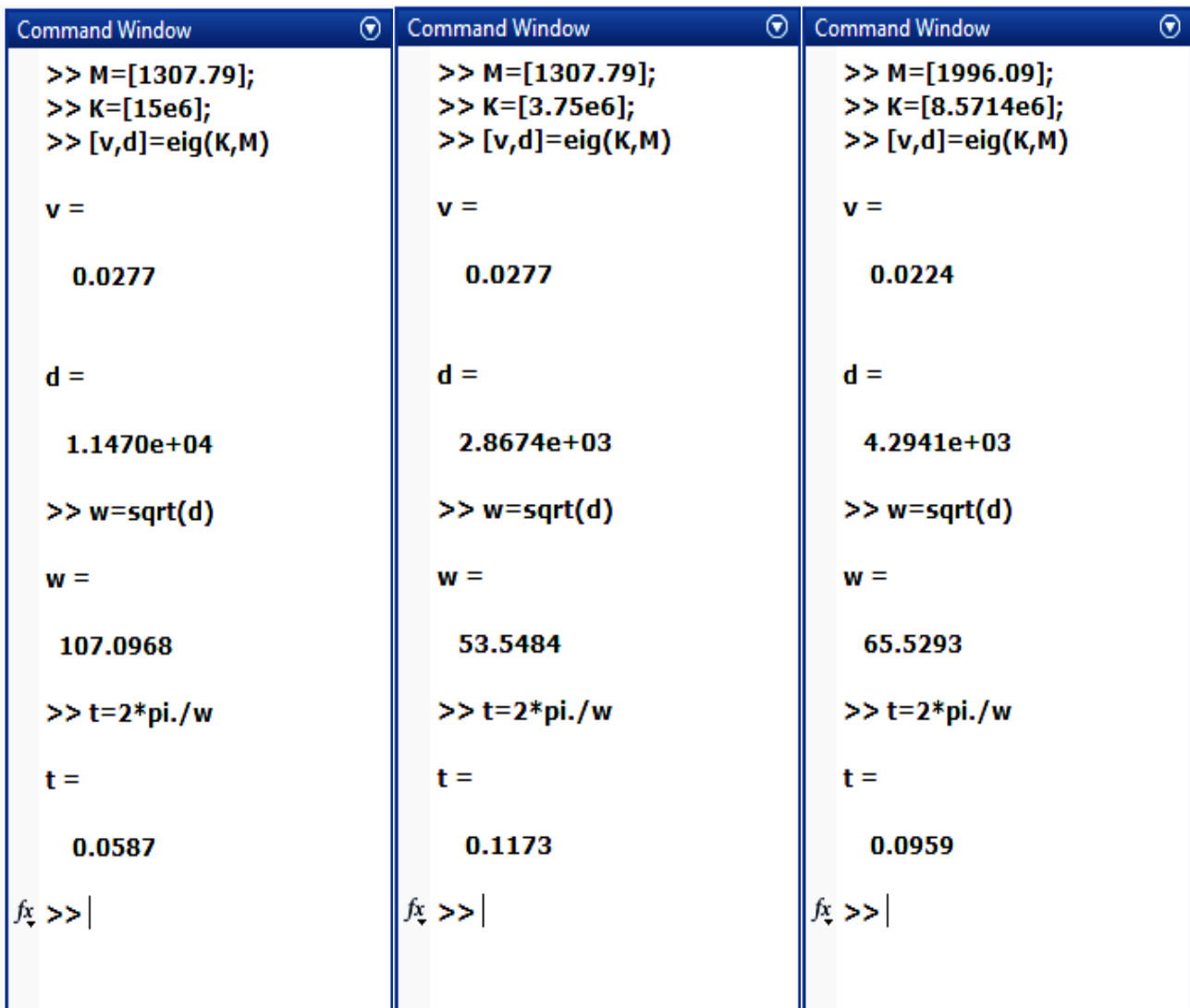
Fig. 2 Eigenvalue and vector calculation in MATLAB command window

Table 2. Property/Stiffness modification factors

Property/Stiffness Modifiers for Analysis	Case (1)		Case (2)		Case (3)	
	Beam	**Column**	**Beam**	**Column**	**Beam**	**Column**
Cross-section (axial) Area	100	1	1	1	1	1
Shear Area in 2-direction	100	1	1	1	1	1
Shear Area in 3-direction	100	1	1	1	1	1
Torsional Constant	100	1	1	1	1	1
Moment of Inertia about 2-axis	100	1	0	1	1	1
Moment of Inertia about 3-axis	100	1	0	1	1	1
Mass	1	0	1	0	1	1
Weight	1	0	1	0	1	1

Table 3. Comparison of natural frequencies and mode period

Different Cases	Natural Frequencies, ω_n (rad/s)			Mode Period, T (s)		
	Analytical Method	**MATLAB**	**ETABS Software**	**Analytical Method**	**MATLAB**	**ETABS Software**
Case (1)	107.0968	107.0968	104.9507	0.0587	0.0587	0.0600
Case (2)	53.5484	53.5484	53.3568	0.1173	0.1173	0.1180
Case (3)	65.5293	65.5293	64.8900	0.0959	0.0959	0.0970

3. Parametric study of MDOF

Figure 6 depicts an idealized lumped spring-mass model of a typical three-storey 3D shear building. Slab and beam weights are lumped at floor level only, which is assumed to be much stiffer than columns. Columns are assumed to be massless and considered flexible along the lateral direction. Figure 5 shows the plan view of a three-storey building with a rigid diaphragm.
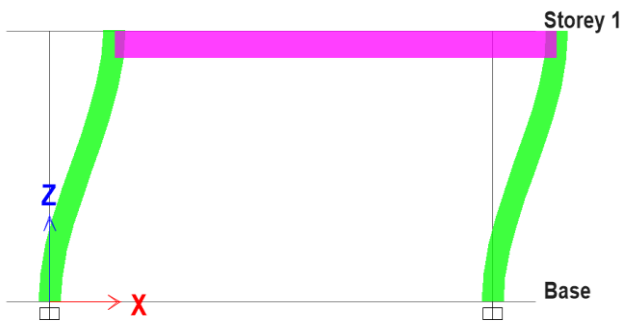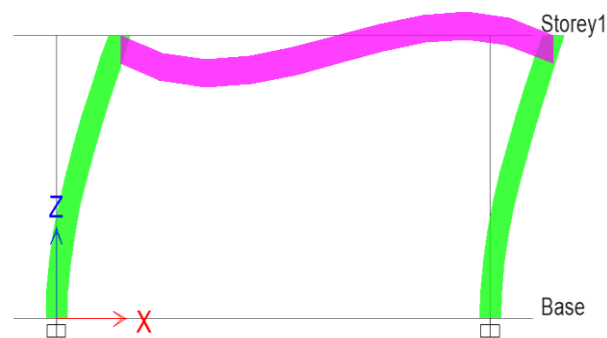

Fig. 3 Mode shapes for case (1)

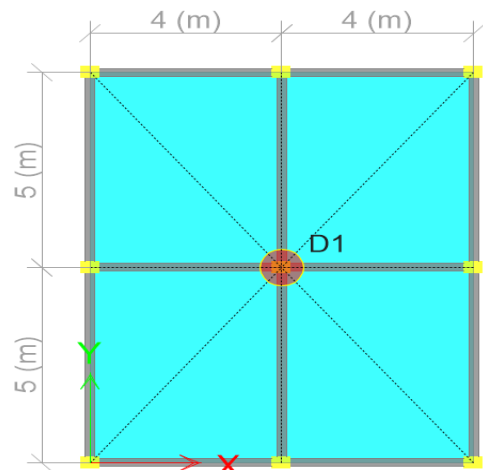**Fig. 4 Mode shapes for case (2) and case (3)**

Fig. 5 Plan view of three storey building

Modal analysis is carried out for three-dimensional undamped free vibrations of three DOF systems. With the help of lumped mass assumption, the frame can be represented as shown in Figure 7.

3.1. Lumped mass model for three degrees of freedom system

Using a free body diagram (FBD), the mass and stiffness matrix can be expressed as follows,

$$m_1\ddot{x}_1 + k_1 x_1 - k_2(x_2 - x_1) = 0 \tag{11}$$

$$m_2\ddot{x}_2 + k_2(x_2 - x_1) - k_3(x_3 - x_2) = 0 \tag{12}$$

$$m_3\ddot{x}_3 + k_3(x_3 - x_2) = 0 \tag{13}$$

It is expressed in matrix form; we get

$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}\begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{Bmatrix} + \begin{bmatrix} (k_1 + k_2) & \text{-}k_2 & 0 \\ \text{-}k_2 & (k_2 + k_3) & \text{-}k_3 \\ 0 & \text{-}k_3 & k_3 \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \{0\} \tag{14}$$

The equation of motion for free vibration of the undamped three degrees of freedom system is given in the form,

$$M\ddot{x} + Kx = 0 \tag{15}$$

The characteristic equation for three degrees of freedom is given by,

$$|K - \omega_n^2 M| = 0 \tag{16}$$

3.2 Calculation of natural frequency and mode period by Analytical method

The effective length of the beam, i. e., *l = 4-0.4 = 3.6 m* along the X-direction and *l =5-0.3 = 4.7 m* along the Y-direction, and the typical column height is 3m, have been taken for load calculation. Here f_{ck} = 25 N/mm^2, the density of RCC = 25 kN/m^3, slab thickness is 120 mm, and Young's modulus of concrete is 2.5 x 10^{10} N/m^2 is used for calculation purposes. Moment of inertia of columns along the X and Y-direction are calculated in (17) and (18), respectively.

$$I_{cx} = I_{XX} = \frac{bd^3}{12} = \frac{0.3 \times 0.4^3}{12} = 1.6 \times 10^{-3} \text{ m}^4 \tag{17}$$

$$I_{cy} = I_{YY} = \frac{db^3}{12} = \frac{0.4 \times 0.3^3}{12} = 9 \times 10^{-4} \text{ m}^4 \tag{18}$$

$$B = 0.2 \times 0.3 \times (6 \times 3.6 + 6 \times 4.7) \times 25 = 74.7 \text{ kN}$$
$$= \frac{74.7 \times 1000}{9.80665} = 7617.28 \text{ kg} \tag{19}$$

$$\text{Slab,S} = 0.12 \times 8 \times 10 \times 25 = 240 \text{ kN}$$
$$= \frac{240 \times 1000}{9.80665} = 24473.19 \text{ kg} \tag{20}$$

The total load at the first-floor level is 7617.28 + 24473.19 = 32090.47 kg

Hence $m_1 = m_2 = m_3$ = 314.7 kN or 32090.47 kg

Now, the lateral stiffness of the column is given by,

$$k_x = \sum_{\text{columns}} \left(12E_c I_{cx}/h^3\right)$$
$$= (9 \times 12 \times 2.5 \times 10^{10} \times 1.6 \times 10^{-3})/3^3$$
$$= 160 \times 10^6 \text{ N/m} \tag{21}$$

$$k_y = \sum_{\text{columns}} \left(12E_c I_{cy}/h^3\right)$$
$$= (9 \times 12 \times 2.5 \times 10^{10} \times 9 \times 10^{-4})/3^3$$
$$= 90 \times 10^6 \text{ N/m} \tag{22}$$

The lateral stiffness of columns along the X and Y-direction are calculated using (21) and (22). Also, the calculated values of mass and stiffness at different floor levels are tabulated in Table 4.
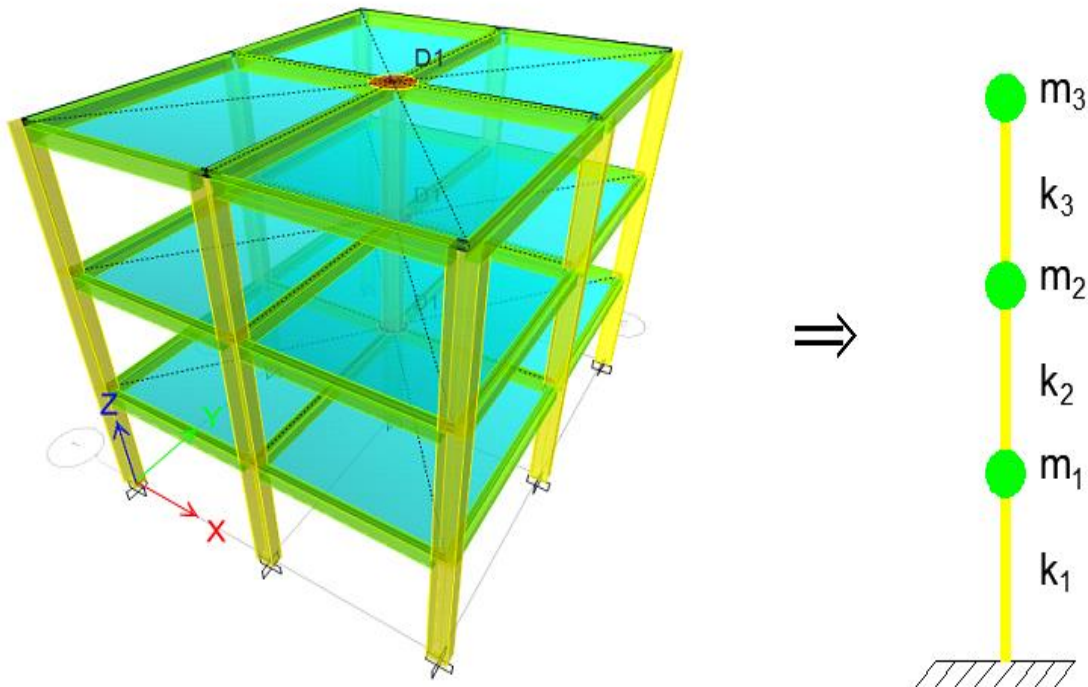

Fig. 6 Three-dimensional building with lumped mass model

Table 4. Mass and stiffness values on each floor

Storey	Lumped mass, m (kg)	Lateral stiffness, k (N/m)	
		k_x	k_y
1	m_1=32090.47	k_1=160 x 10^6	k_1=90 x 10^6
2	m_2=32090.47	k_2=160 x 10^6	k_2=90 x 10^6
3	m_3=32090.47	k_3=160 x 10^6	k_3=90 x 10^6
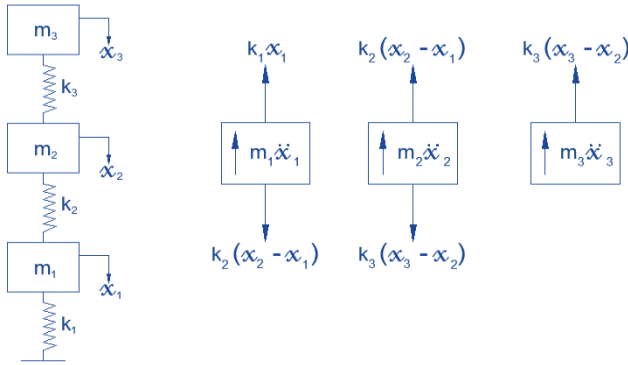
Fig. 7 Lumped spring-mass system with a free-body diagram

Fig. 8 MATLAB program for three degrees of freedom

Modal mass contribution for structure is given by [5],

$$M_k = \frac{[\sum_{i=1}^{n}(W_i \varphi_{ik})]^2}{g(\sum_{i=1}^{n}(W_i (\varphi_{ik})^2))} \tag{22}$$

Where ϕ_{ik} = coefficient of Eigenvalue at floor i and n= number of floors.

Eigenvalues are extracted from MATLAB to calculate the mass contribution for three degrees of freedom. These values are tabulated in Table 5.

3.3 Calculation of natural frequency, mode period, and modal mass contribution by MATLAB

The natural frequency and corresponding mode period for three degrees of freedom can be calculated using the MATLAB program. This program calculates the Eigenvalues and Eigenvectors for the three degrees of freedom system, as shown in Figure 8.

3.4 Calculation of natural frequency, mode period, and modal mass contribution by ETABS software

Based on the assumption mentioned in case (1) from SDOF, the property modification factors are used for modeling and analyzing the 3D frame with the help of ETABS software. A rigid diaphragm is assigned for all joints and slabs to maintain lumped mass at floor level [9].

Modal mass contribution results are extracted from ETABS and compared with MATLAB and tabulated in Table 8. This clears that the effective mass decreases with an increase in the mode number.

3.5 Results and Discussion

Table 6 gives the first three modes for lateral stiffness along the Y-direction and the following three modes along the X-direction.

Natural frequency and mode period values are the same in the analytical and MATLAB program. The difference is around 5% in the mode period for analytical and MATLAB programs compared to ETABS software.

The first fundamental period is acting along the Y-direction because less stiffness acts in this direction, followed by X-direction and torsion, as tabulated in Table 7.

Modal mass contributions are compared with MATLAB and ETABS, considering the first three fundamental modes. The percentage of modal mass contribution values is almost the same in MATLAB and ETABS software for lumped mass three degrees of freedom system.

Figure 9 and 10 shows the deflection pattern or mode shapes of three degrees of freedom.

Table 5. Percentage of mass contribution

Storey	Floor Weight, W_i (kN)	Mode-1 (0.278 s)			Mode-2 (0.214 s)			Mode-3 (0.202 s)		
		(ϕ_i)	$(W_i\phi_{ik})$	$(W_i\phi^2_{ik})$	(ϕ_i)	$(W_i\phi_{ik})$	$(W_i\phi^2_{ik})$	(ϕ_i)	$(W_i\phi_{ik})$	$(W_i\phi^2_{ik})$
1	314.70	0.4450	140.04	62.31	-1.2470	-392.43	489.36	1.8017	567.00	1021.55
2	314.70	0.8019	252.35	202.36	-0.5550	-174.65	96.93	-2.2470	-707.13	1588.92
3	314.70	1.0000	314.70	314.70	1.0000	314.70	314.70	1.0000	314.70	314.70
Σ	**944.10**		**707.09**	**579.37**		**-252.38**	**901.00**		**174.57**	**2925.17**
Percentage of mass contribution		$(707.09)^2/(579.37)$=862.96 kg $(862.96/944.10)$ x 100=91.40%			$(-252.38)^2/(901)$=70.69 kg $(70.69/944.10)$ x 100=7.50%			$(174.57)^2/(2925.17)$=10.41 kg $(10.41/944.10)$ x 100=1.10 %		

Table 6. Comparison of natural frequencies and mode period

Modes	Natural Frequencies, ω_n (rad/s)			Mode Period, T (s)			Lateral stiffness, k (N/m)
	Analytical method	*MATLAB*	*ETABS software*	*Analytical method*	*MATLAB*	*ETABS software*	
1	23.5686	23.5686	22.6337	0.2666	0.2666	0.2780	Y-direction
2	66.0378	66.0378	63.9856	0.0951	0.0951	0.0980	
3	95.4274	95.4274	93.6548	0.0658	0.0658	0.0670	
1	31.4248	31.4248	29.3680	0.2000	0.2000	0.2140	X-direction
2	88.0504	88.0504	83.4678	0.0714	0.0714	0.0750	
3	127.2365	127.2365	123.2717	0.0494	0.0494	0.0510	

Table 7. Mode period for different direction

Modes	Mode Period, T (s)	Direction
1	0.278	Y
2	0.214	X
3	0.202	Torsion
4	0.098	Y
5	0.075	X
6	0.071	Torsion
7	0.067	Y
8	0.051	X
9	0.049	Torsion

Table 8. Comparison of modal mass contribution

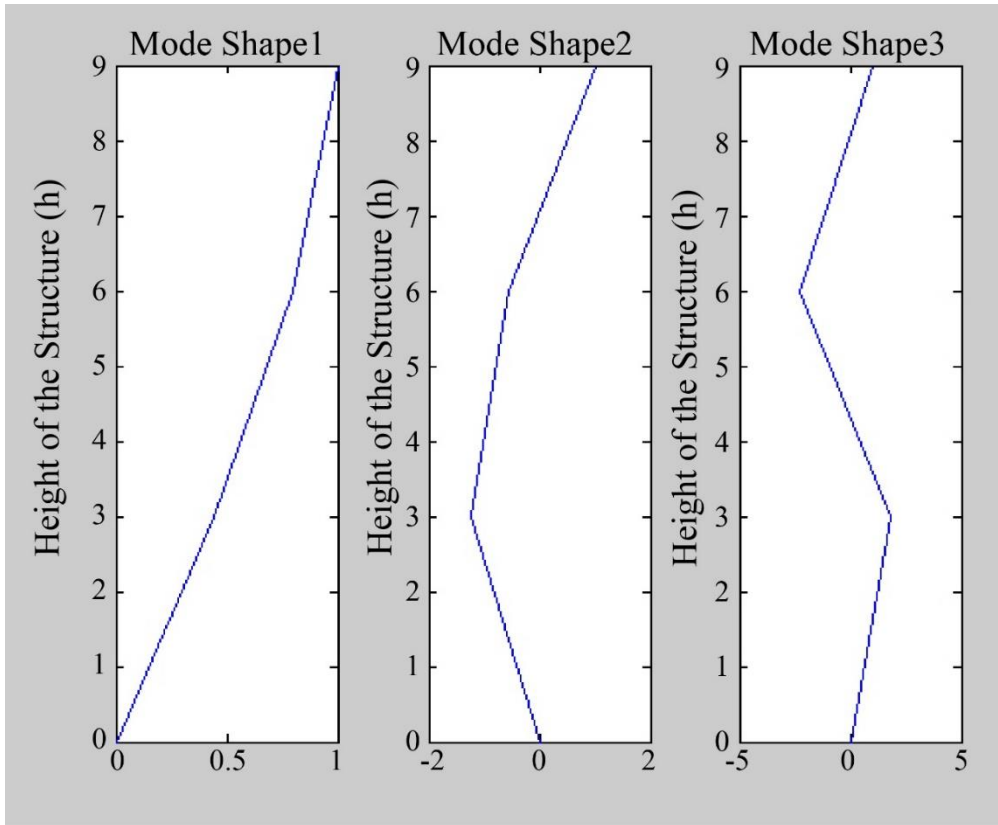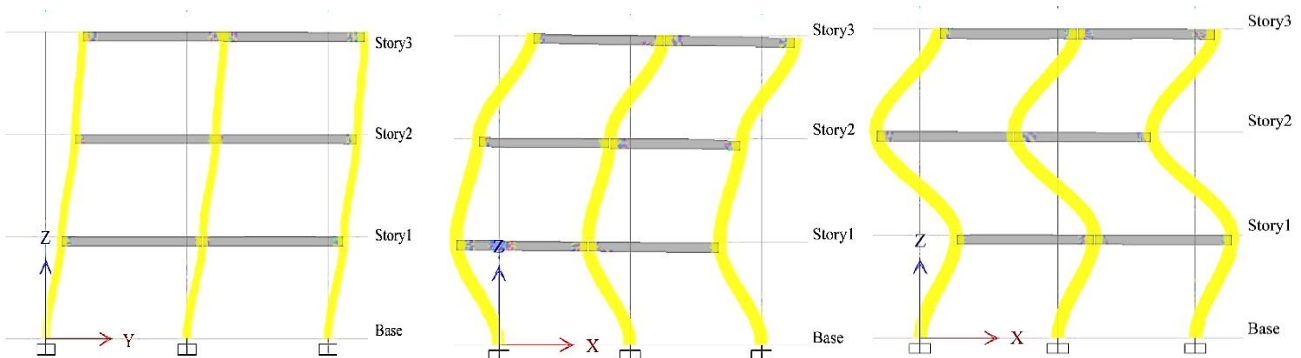| Modes	Percentage of Mass Contribution	
	MATLAB	*ETABS*
1	91.40	90.92
2	7.50	7.88
3	1.10	1.20
The effective mass of the first three modes	100	100

Fig. 9 Mode shapes using MATLAB

Fig. 10 Mode shapes using ETABS software

4. Conclusion

This paper presents the modal analysis for free vibrations of undamped single and three degrees of freedom systems. Based on the results, the following conclusions are drawn.

1) In the SDOF system, dynamic parameters such as natural frequency and mode periods are calculated by three methods, namely, Analytical and MATLAB; the results are the same. Whereas a difference of 2% variation is observed in ETABS software in frequency and mode period compared with analytical and MATLAB.

    A three-dimensional three-storey multi-degree of freedom (MDOF) building modeled as a shear building, idealized as a lumped spring-mass model. The results are compared first theoretically, secondly by MATLAB program, and finally by ETABS software.

2) The time period of the structure is theoretical, and the MATLAB program is identical, whereas the ETABS software has a variation of 4.2% in the X-direction and 3.0% variation in the Y-direction compared with the analytical and MATLAB program.

3) The relative translational displacements are high in the Y-direction compared to the X-direction, as observed in Table 6. The frequency in Y-direction is 23.5686 lower compared to 31.4248 in the X-direction.

4) The mass participation factors in the first mode of vibration contribute to 91.40% in the first mode by MATLAB to 90.92% by ETABS; however, the variation of MATLAB shows a higher percentage of 5.25% variation in comparison to ETABS software. The energy contribution is 100 percent by considering only three modes for the 3D space frame programme.

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