International Journal of Engineering Trends and Technology
ISSN: 2231–5381 / https://doi.org/10.14445/22315381/IJETT-V71I6P223

Volume 71 Issue 6, 221-234, June 2023

Original Article

A New Technique of Reduction for the SISO and MIMO System Model for the Compensator Design

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Received: 24 February 2023 Revised: 07 May 2023 Accepted: 23 May 2023 Published: 25 June 2023

Abstract *- This paper aims to introduce a novel approach for minimizing the order of time-invariant SISO & MIMO high-order systems. A novel mixture of two methods is proposed to acquire the preferred approximated stable model, which conserves the basic structures of the high-order actual plant. The suggested approach retains the original plant dominant poles to acquire approximated model denominator using the advantages of the reciprocal transformation. However, the numerator of ROM is acquired by diminishing the inaccuracy between the transitory part of responses of HOS and the reduced model using the particle swarm optimization algorithm (PSO). Particularly the proposed reduction method is appropriate to the systems which are controlled by large magnitude poles as the usual dominant pole retention methods will lead to bad approximant in such cases. Further, the compensation is designed using the approximated reduced model acquired from the advised reduction methodology for controlling the high-order plant using a new algorithm. The efficacy of the suggested methodology is confirmed by matching responses of time & frequency of the larger and the reduced order models. The suggested technique's accuracy is verified by calculating performance error indices. Taking into account three standard examples, the performance, usefulness and accuracy of work suggested to reduce system order were validated and also showed that the suggested reduction methodology is suitable for the design of compensation.*

Keywords *- Reduced order model, Compensator, Particle swarm optimization algorithm, Dominant pole, Reciprocal transformation.*

1. Introduction

In present days, system analysts and control engineers are facing a great challenge while dealing with physical dynamic systems such as aviation systems, chemical plants, power system networks, multi-layer systems, digital communication networks, and control systems etc. To get the simple, illustrative and appreciable interpolation and estimation of these processes, it is essential to convert these systems into mathematical models. This mathematical modelling involves several number of simultaneous equations of higher order with constant coefficients. The analysis, synthesis and designing of the controller of such higher-order systems are both tedious and costly and time-consuming economically and computationally. Also, because of the complexity, it gets hard to gain a good acceptance of the behaviour of large-scale systems. Hence, working on this high-dimensional system in its original form is very difficult. Thus, to save costs and design time and simplify implementation, low dimensional system models are very desirable for engineers to use for the synthesis and control purpose of higher dimensional Plants. The higher order system(HOS) should be approximate as it must retain the basic features of the full order plant. MOR techniques are widely used in engineering and science to simplify higher-dimensional systems [1,2,3,4,5,6,7]. Two approaches have been used for approximating the large-scale linear systems. The reduction of the transfer function that takes place is called the frequency domain method, whereas in other approaches, the higher order state space model is reduced, called the time domain method.

In the time domain approach, several model order reduction (MOR) methods are proposed, such as the Aggregation method, Singular Perturbation method given by, Modal analysis, Minimum realization algorithm, Balanced Realization Approach, Hankel Norm Approximation, etc., for reducing large-scale system [8,9,10,11,12]. Among these, the balance realization technique is usually used to reduce high-scale original systems. This method truncated the smallest state of controllability and observability to find the reduced-order model (ROM). At times there is no matching between the stable state response of the ROM acquired by this approach with the actual model. Thus, various MOR methods were proposed to solve this problem [13].

In the frequency domain approach, initially [12] introduced the reduced technique for the dynamic linear system. Techniques like Pade approximation, Routh method, pole clustering [14,15,16,17,18,19,20] etc., are commonly applied methods for reduction in the frequency domain. It has been experienced that these methods have some boundaries, like the pade method may provide an unstable compact model even if the actual plant of order high is stable. There is a similar problem found in the continued fraction expansion method, which also does not provide the assurance of the steadiness of ROM, although the actual plant is steady and also not able to approximate the transient response very well. The moment matching method had not given a satisfactory performance of transient response in ROM, and there is no guarantee of stability of ROM. The Routh approximation method does not produce a good result for order reduction of the non-strictly transfer function. The retention of the dominant pole for a non-minimum phase system is not possible in the case of the method of Routh stability. The method of stability equation has not given the appropriate result for the reduction of non-minimum phase large models. Therefore, a combined procedure was used to overcome these limitations. In this technique, using the methods of preserving stability, the denominator of the low-order model is achieved, whereas using the classic reduction methods, low-order numerator coefficients are determined. Several methodologies have been proposed using different combinations of two methods for simplification of the high-scale system such as [21] suggested the mixed reduction technique of the mixture of stability equation and Genetic Algorithm methods, [22] has made the grouping of pade approximation for numerator reduction whereas stability equation for denominator reduction, [23] has used the combination of eigen permutation algorithm and improved approximations. [24] proposed the new combination of the clustering method and pade approximation. Model order reduction is also made using the eigen algorithm and factor division methods [25]. Further, [26] has also done the reduction using an improved pole cluster method for denominator and pade for numerator reduction.[27] applied the same modified clustering method in combination with an evolutionary algorithm to obtain better results. Then, [20] made the improvement in the pole clustering method and reduced the large order system. Further [28] has made a reduction by conserving the dominant poles in the ROM. [29] has also applied the order reduction technique on the SISO and discrete large order systems by conserving the dominant poles effect in the ROM. More new combinations have been introduced for the reduction of order. [30].

Among these methodologies, mostly MOR approaches are found based on conserving poles of the dominant nature of the large order plant in the minimized order model; for example, [45,46], the dynamical systems having the dominant poles are accountable to lead oscillation behavior and non-dominant poles also called insignificant poles are accountable to the speedy decomposition of time-response. Hence it is concluded that the stability of plants affects by dominant poles. The mixed combination proposed by [29,33,34] has given a good approximation to the original model but cannot be sight-seen in the controller design field.

This paper has introduced a novel combination of approximation methods by preserving the poles of the dominant nature of the large system in the approximate model along with the particle swam optimization (PSO) algorithm for obtaining the preferred stable reduced model. Further, this approximated system from the proposed reduction methodology is used for the controller design. The novelty of the suggested technique is that it provides a good approximation to the systems controlled by large magnitude poles, as the usual dominant pole retention methods will lead to bad approximant in such cases. The dominant poles of the original plant are preserved in the suggested approach by using the advantages of reciprocal transformation to achieve the denominator coefficient. Whereas the ROM numerator is acquired by diminishing the inaccuracy between the transitory response part of HOS and low order model by applying the algorithm of particle swam optimization (PSO). The author tested the proposed methodology on the SISO and MIMO large-scale plants to achieve the approximated low-order system, which retains the basic features of the real-order plant. The paper is ordered as follows: Segment 2 consists of the statement of SISO and MIMO problems. A description of the reduction approach used in this paper to get the preferred approximated system is given in segment 3. In segment 4, the procedure of designing a compensator is explained. In segment 5, one standard numerical SISO and one numerical MIMO are considered from the literature to authenticate the advised reduction method. Segment 5 also includes one example of the SISO system to validate the suitability of the advised method for the compensator design. Finally, in segment 6 conclusion of the paper is mentioned.

2. Statement of SISO and MIMO Problem

2.1. SISO-System

The n^{th} order transfer function be considered as

$$G(s) = \frac{Y(s)}{X(s)} = \frac{b_0 + b_1 s + \cdots + b_{n-1} s^{n-1}}{c_{0+} c_1 s + c_2 s \ldots + c_n s^n} \tag{1}$$

The key purpose of the paper is to decrease the order of the high-scale system G(s) into a simple reduced model $G_r(s)$ through construction that does not cause significant harm to accuracy and, however, preserves the primal structures of the real high-order plant.

ROM transfer function is

$$G_r(s) = \frac{Y_r(s)}{X_r(s)} = \frac{v_0 + v_1 s + \cdots + v_{r-1} s^{r-1}}{u_{0+} u_1 s + u_2 s^2 \ldots + u_{r-1} s^{r-1} + u_r s^r} \tag{2}$$

2.2. Multi-Variable System

The n^{th} order multi-variable system transfer matrix is considered as

$$[G(s)] = 1/D_n(s)\begin{bmatrix} h_{11}(s) & h_{12}(s)\dots & h_{1l}(s) \\ h_{21}(s) & h_{22}(s)\dots & h_{2l}(s) \\ \vdots & \vdots & \vdots \\ h_{m1}(s) & h_{m2}(s)\dots & h_{ml}(s) \end{bmatrix} \quad (3)$$

$$[G(s)] = [g_{ij}(s)]_{m\times l} \quad (4)$$

Where i=1,2,3,…….. m; j=1,2,3,…………., l

and l is the input variable, whereas m is the output variable, respectively. The $[g_{ij}(s)]$ is written as

$$g_{ij}(s) = \frac{Hij\ (s)}{D_n(s)} \quad (5)$$

The goal is to achieve the approximate model of the r^{th} order (r‹ n) as given in equation (5) from the system mentioned in equation(3) can be expressed by the transfer matrix given as

$$[G_r(s)] = 1/D_r(s)\begin{bmatrix} r_{11}(s) & r_{12}(s)\dots & r_{1l}(s) \\ r_{21}(s) & r_{22}(s)\dots & r_{2l}(s) \\ \vdots & \vdots & \vdots \\ r_{m1}(s) & r_{m2}(s)\dots & r_{ml}(s) \end{bmatrix} \quad (6)$$

Where $[G_r(s)] = [g_{r_{ij}}(s)]$ is a $m \times l$ transfer matrix and $g_{r_{ij}}(s)$ is expressed as:

$$g_{r_{ij}}(s) = \frac{N_{ij}(s)}{D_r(s)} \quad (7)$$

$$where\ \ i = 1,2,3,\dots\dots m\ \ and\ j = 1,2,3,\dots..l$$

3. Illustration of the Suggested Reduction Methodology

The proposed method includes a modified dominant pole technique applied to identify the reduced denominator using the benefits of reciprocal transformation. Reciprocal transformation allows high and low-magnitude poles to be kept in reduced-order models. This method is fitted to the systems having the dominance of large magnitude poles because the normal dominant pole approach will not provide a good approximant in such cases. Whereas the reduced numerator is determined by ISE error minimization using the particle swarm optimization (PSO) method.

3.1. Finding of Coefficient of Denominator Polynomial of ROM by Modified Dominant Pole Retention Method

Step 1: Write down a characteristic equation of G(s) of order 'n'and find its roots

$$X(s) = (s+\lambda_{1)}(s+\lambda_2)(s+\lambda_3)\dots.(s+\lambda_n) \quad (8)$$

Step 2: By applying reciprocal transformation in the denominator X(s) to get the reciprocate denominator as

$$\widetilde{X(s)} = (\lambda_n s+1)(\lambda_{n-1}s+1)\dots\dots(\lambda_1 s+1) \quad (9)$$

Step 3: Select the r_1 number of dominant poles from equation(8) represented by $X_{r1}(s)$ and the r_2 number of dominant poles from (9) represented by $X_{r2}(s)$, to find the desired 'r^{th}' order denominator polynomial $X_r(s)$ given as

$$X_r(s) = X_{r_1}(s)X_{r_2}(s) \text{ where } r = r_1 + r_2 \quad (10)$$

$$= u_0 + u_1 s + u_2 s^2 \dots.+u_{r-1}s^{r-1} + u_r s^r$$

where $X_{r2}(s)$is reciprocal of $\widetilde{X_{r2}}(s)$

3.2. Calculation of Coefficient of the Numerator of ROM by Minimizing Error using PSO

Step 1: The step response of unity feedback original system in s domain is given as:

$$Y(s) = \frac{b_{n-1}s^{n-1} + \dots + b_1 s + b_0}{s(s+\lambda_1)(s+\lambda_2)(s+\lambda_3)\dots.(s+\lambda_n)}$$

$$Y(s) = \frac{k_0}{s} + \frac{k_1}{s+\lambda_1} + \frac{k_2}{s+\lambda_2} + \cdots \frac{k_{n-1}}{s+\lambda_{n-1}} + \frac{k_n}{s+\lambda_n} \quad (11)$$

Step II: The reduced order step response having unity feedback in the s domain is given as:

$$Y_r(s) = \frac{v_{r-1}s^{r-1} + \dots + v_1 s + v_0}{s(s+\lambda_1)(s+\lambda_2)(s+\lambda_3)\dots.(s+\lambda_r)}$$

$$Y_r(s) = \frac{k_0{}'}{s} + \frac{k_1{}'}{s+\lambda_1} + \frac{k_2{}'}{s+\lambda_2} + \cdots \frac{k_{r-1}{}'}{s+\lambda_{r-1}} + \frac{k_r{}'}{s+\lambda_r} \quad (12)$$

Step III: Taking the inverse Laplace transform of equations equation(11) and equation(12) gives

$$y(t) = k_0 + k_1 e^{-\lambda_1 t} + \dots + k_{n-1}e^{-\lambda_{n-1}t} + k_n e^{-\lambda_n t} \quad (13)$$

$$y_r(t) = k_0{}' + k_1{}' e^{-\lambda_1 t} + \dots + k_{r-1}{}' e^{-\lambda_{r-1}t} + k_r{}' e^{-\lambda_r t} \quad (14)$$

Where $y(t) =$ unit step responses of HOS
$y_r(t) =$ unit step response of ROM

$k_0\ and\ k_0{}'$ are the steady state part of responses of HOS and ROM, respectively, and the remaining terms represent the transitory part of the system output. The HOS and ROM steady state output matching, condition $k_0 = k'_0$ must be satisfied.

Step IV: Integral Square error (ISE) of transient responses are determined by applying the expression of error index 'E' as

$$E = \int_0^\infty [y(t) - y_r(t)]^2 dt \quad (15)$$

This 'E' is the function of the unknown coefficient $'v_i'$of reduced numerator polynomial.

Step V: Consider the numerator structure of ROM as $v_i s + v_0$.

Also, consider $v_0$equal to $k_0 = k_0{}'$ to match the steady state response of HOS and ROM.

Then determine the numerator coefficient $'v_i'$ of ROM in order to get the minimum value of error index E, which would be done by applying an artificial intelligence technique called particle swarm optimization.

Consider the PSO fitness function given as

$$Fitness = min(ISE) \quad (16)$$

The following steps of the PSO algorithm, as shown in Figure 1, are the unknown numerator coefficient 'v_i'of ROM is obtained.

Step VI: Hence the desired ROM numerator polynomial is found as $G_r(s) = v_i s + v_0$.

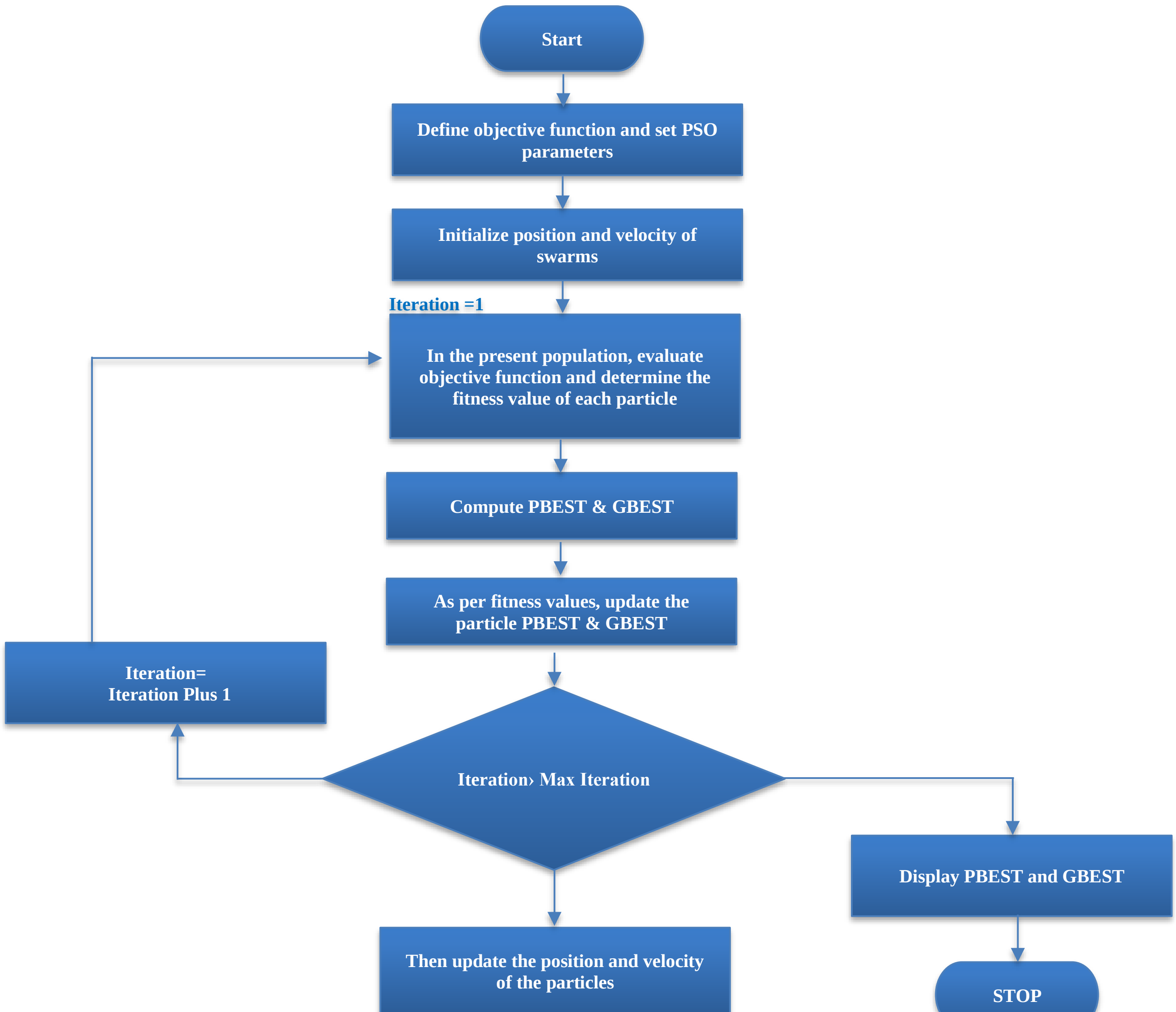


Fig. 1 Algorithm of PSO

Fig. 2 Overall closed loop controlled unity feedback system with reduced order compensator

4. Compensator Design

The large-scale dynamic system involves many differential equations, leading to a rise in system order. In such cases, the task of analyzing, designing a controller and modeling a large-scale system is a very tedious job. This problem can be solved by approximating it into lower order because it reduces computational efforts, cost savings and design time. A lower-order controller has been designed to efficiently control the large dimensional system using the order reduction technique. The complete system obtained is of low order and is simple to recognise [35,36,37]. Also, in HOS, for designing the feedback compensator, a huge number of sensors are desirable to detect state variables of the real high-order plant. Therefore, series controllers are recommended for feedback controllers. A reference model R(s) must be developed based on certain specifications to acquire the desired performance of the real-time dynamical arrangement. It is necessary that the unit feedback closed loop-controlled system response must match with the considered reference model response. As per the desired specification, the computing procedures of the reference system in more detail are given in [38,39]. The approach followed for designing the compensator is, firstly, to design the reduced order compensator $C_r(s)$ using the ROM $G_r(s)$ acquired by the suggested reduction method as it reduces the mathematical computation and simulation time. Then, according to the block diagram shown in Figure 2, the overall controlled unit feedback system was obtained.

4.1. Steps to design a compensation Model

Step I: The compensator structure is considered as

$$C_r(s) = \frac{K(1 + K_A s)}{s(1 + K_B s)} \tag{17}$$

Step II: Open-loop reference model $\overline{R(S)}$ is determined from $R(s)$ for designing the compensator.

$$\overline{R(s)} = \frac{R(s)}{1 - R(s)} \tag{18}$$

Step III: To obtain the controlled parameters K, K_A and K_B, matching between the response of the open loop controlled system shown in Figure 2 with the open loop reference model obtained by equation(18) needs to be done. Therefore

$$\overline{R(s)} = G_r(s)\, C_r(s) \tag{19}$$

$$C_r(s) = \frac{\overline{R(S)}}{G_r(S)} = \sum_{i=0}^{\infty} e_i s^i \tag{20}$$

The power series expansion coefficients are e_i where, $i = 0, 1, 2$ are about s = 0. Using the pade sense of approximated model matching given by [40], the unknown power series coefficients are obtained.

Step IV: Compute the compensator parameters K, K_A and K_B after finding the power series coefficient and substituting them in equation(20) and comparing them with equation(17). The desired reduced order compensator structure is determined.

After getting the desired reduced order compensator structure, as shown in fig. 2, it has been directly applied to the original plant G(s) to get the closed system $G_{cl}(s)$ transfer function obtained as

$$G_{cl}(s) = \frac{G(S)C_r(S)}{1 + G(S)C_r(S)} \tag{21}$$

5. Illustrative Examples

Performance indices are measured for the ROMs obtained from the suggested scheme and other known MOR schemes from the literature to confirm the performance of the advised scheme by using the formulas given as

$$IAE = \int_0^{\infty} |y(t_i) - y_r(t_i)| dt \tag{22}$$

$$ISE = \int_0^{\infty} [y(t_i) - y_r(t_i)]^2 dt \tag{23}$$

where IAE= Integral square Error
ISE= Integral Square Error
$y(t_i)$ = HOS unit step response at time t_i.
$y_r(t_i)$ = ROM unit step response at time t_i.

Example 1: Consider 6th order SISO system [35], the proposed MOR method is illustrated as

$$G(s) = \frac{2s^5 + 3s^4 + 16s^3 + 20s^2 + 8s + 1}{2s^6 + 33.6s^5 + 155.94s^4 + 209.46s^3 + 102.42s^2 + 18.3s + 1} \tag{24}$$

5.1. Determination of Denominator Polynomial of ROM

By following section 3.1, the denominator of ROM is obtained as follows:

Step 1: **C**haracteristic equation is obtained from equation(24) is, $2s^6 + 33.6s^5 + 155.94s^4 + 209.46s^3 + 102.42s^2 + 18.3s + 1 = 0$

Roots of the characteristic equation are:

$$s_1 = -0.1, s_2 = -0.2, s_3 = -0.5, s_4 - 1, s_5 = -5, s_6 - 10 \quad (25)$$

Step 2: The reciprocal transformation of the denominator of G(s) is given as:

$$\overline{G(s)} = s^6 + 18.3s^5 + 102.42s^4 + 209.46s^3 + 155.94s^2 + 33.6s + 2$$

$$= (s_1+10)\,(s_2+5)\,(s_3+1)\,(s_4+0.5)\,(s_5+0.2)\,(s_6+0.1) \quad (26)$$

Step 3. By following step 3 of section 3.1 and using equation(10), the denominator of 2nd order (r = 2) reduced model is obtained for various values of r_1 and r_2 as

$$\text{for } r_1 = 1, r_2 = 1, \quad X_2(s)_1 = s^2 + 0.2s + 0.01 \quad (27)$$

$$\text{for } r_1 = 2, r_2 = 0, \quad X_2(s)_2 = s^2 + 0.3s + 0.02 \quad (28)$$

5.2. Numerator ROM is determined as:

In the following section 3.2, the numerator is obtained as follows:

Step 1: Using equation(11), the step response of unity feedback original system in the time domain is obtained as:

$$y(t) = 1 - 1.1002e^{-0.1t} + 0.1687e^{-0.2t} - 0.0487e^{-0.5t} - 0.0772e^{-t} + 0.2794e^{-5t} - 0.2219e^{-10t} \quad (29)$$

and using equation(12), the unit step response of ROM in the time domain is obtained as

for $r_1 = 1$ and $r_2 = 1$,

$$y_r(t)_1 = 1 + (0.101v_1 - 1.01)e^{-0.1t} + (0.01 - 0.101v_1)e^{-10t} \quad (30)$$

for $r_1 = 2$ and $r_2 = 0$,

$$y_r(t)_2 = 1 + (10v_1 - 2)e^{-0.1t} + (1 - 10v_1)e^{-0.2t} \quad (31)$$

where $y(t)$ = step response of HOS with unity feedback ,
$y_r(t)$ = step response of ROM with unity feedback

Step 2: Using equation (15), the corresponding error index E is evaluated as:

for $r_1 = 1$ and $r_2 = 1$,

$$E_1 = 0.0495{v_1}^2 - 1.916v_1 - 5.0104 \quad (32)$$

for $r_1 = 2$ and $r_2 = 0$,

$$E_2 = 73.34{v_1}^2 - 15.806v_1 + 2.80654 \quad (33)$$

Step 3: Numerator polynomial structure is chosen as $v_1 s + v_0$, where v_1 is the unknown coefficient and v_0 is taken as $ko = ko'$ for matching the steady output of the HOS and ROM.

Step 4: The unknown numerator coefficient v_1 of ROM has been evaluated, by applying the PSO algorithm shown in Figure 1, considering the design parameters as

No. of variables = m = 1(swarms)

Population size = n = 500

w_{max}= 0.9 where w_{max} is Inertia weight maximum

w_{min} =0.4 where w_{min} is Inertia weight minimum

Correction factor c_1 and c_2 = 2.05

Max iteration=100

Lower Bound & Upper Bound variables are

LB= [0] and UB= [100], respectively.

Hence, $v_1 = 0.1$ is obtained with the fitness function as shown in Figure 3a for $r_1 = 1$ and $r_2 = 1$and $v_1 = 0.1078$ is obtained with the fitness function, as shown in Figure 3b for r=2 and $r_2 = 0$.

Step 5: The ROM numerator is obtained as

$$Y_2\,(s)_1 = 0.1_1 s + 1, \text{for } r_1 = 1 \text{and } r_2 = 1, \quad (34)$$

$$Y_2\,(s)_2 = 0.1078_1 s + 0.02 \text{ for } r_1 = 2 \text{ and } r_2 = 0 \quad (35)$$

Therefore, the wanted 2nd order reduced system is found using (27), (28), (34) and (35) as

$$G_2\,(s)_1 = \frac{0.1s + 1}{s^2 + 10.1s + 1} \quad (36)$$

$$G_r\,(s)_2 = \frac{0.1078_1 s + 0.02}{s^2 + 0.3s + 0.02} \quad (37)$$

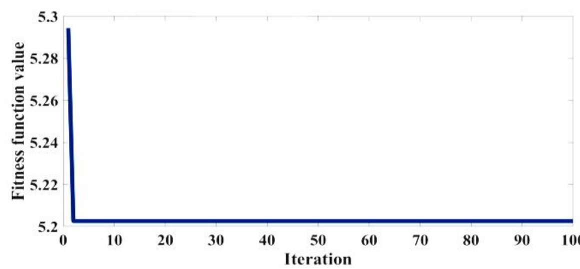


Fig. 3a. PSO Convergence Graph for example 1 (r1 =1, r2= 1)

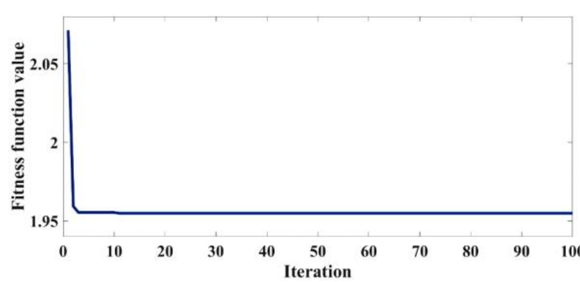


Fig. 3b. PSO Convergence Graph for example 1 (r1 =2, r2= 0)

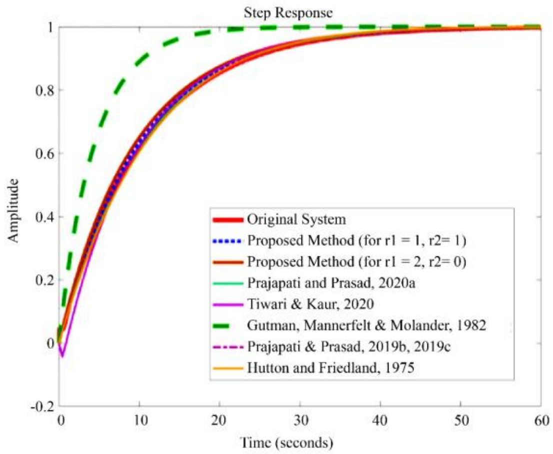


Fig. 4 Example 1- Response of SISO system for a step input

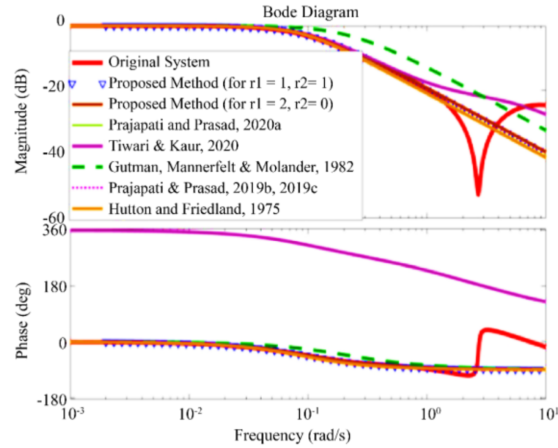


Fig. 5 Example 1 -Bode Plot of SISO system

Table 1. Example 1- Comparison of transient response specifications, gain and phase margin

Reduction Method	Transient Response Specification				Gain Margin	Phase Margin
	Tr	Ts	Mp	TP		
Original System	22.71	40.48	0	75.42	Inf	180°
Proposed Method (for $r_1 = 1, r_2 = 1$)	21.97	39.12	0	105.46	Inf	180°
Proposed Method (for $r_1 = 2, r_2 = 0$)	21.23	38.33	0	68.62	Inf	180°
Prajapati and Prasad, 2020a [41]	21.87	39.23	0	71.58	Inf	180°
Tiwari & Kaur, 2020 [42]	20.621	37.218	0	68.7262	22.1	178°
Gutman, Mannerfelt & Molander, 1982 [43]	10.01	17.9	0	63.66	Inf	180°
Prajapati & Prasad,2019b [44], Prajapati & Prasad, 2019c [2]	22.1	37.07	0.0144	75.99	Inf	180°
Hutton and Friedland, 1975 [16]	22.12	37.09	0.0143	76.45	Inf	180°

Table 2. ISE and IAE comparison of example 1

Reduction Technique	Reduced Order System	Performance Indices	
		ISE	IAE
Proposed Method (for $r_1 = 1, r_2 = 1$)	$\frac{0.1s + 1}{s^2 + 10.1s + 1}$	0.00092	0.076
Proposed Method (for $r_1 = 2, r_2 = 0$)	$\frac{0.1078s + 0.02}{s^2 + 0.3s + 0.02}$	0.0055	0.22
Prajapaand Prasad, 2020a [41]	$\frac{0.0913s + 0.0209}{s^2 + 0.30663s + 0.02}$	0.00178	0.12
Tiwari & Kaur, 2020 [42]	$\frac{-0.4809s + 0.6396}{s^2 + 0.1070s + 0.639}$	0.023	0.38
Gutman, Mannerfelt & Molander, 1982 [43]	$\frac{576s + 360}{2458s^2 + 2196s + 360}$	0.623	2.38
Prajapati & Prasad,2019b [44], Prajapati & Prasad, 2019c [2]	$\frac{0.0880s + 0.011}{s^2 + 0.2012s + 0.01}$	0.0046	0.20
Hutton and Friedland, 1975 [16]	$\frac{0.0879s + 0.011}{s^2 + 0.2012s + 0.011}$	0.0047	0.20

The step and bode responses of the proposed ROM (r_1 =1 & r_2=1) are compared with the case r_1= 2 & r_2= 0 and also compared with the real system and ROM's acquired from the methods offered by [38, 39, 40, 41, 2, 13] are shown in Figure 4 and Figure 5, respectively and hence found the better response. The transient response specification, gain margin and phase margin are obtained and compared, as shown in Table 1, to check the steadiness of the approximated and the original systems. In Table 2, ISE and IAE are calculated and compared for the authentication of the suggested method. Hence all the analyses proved the effectiveness of the suggested method for the case r_1 = 1 & r_2 = 1.

Example 2: To illustrate the extent of the suggested reduction procedure, the example of a MIMO 6th order system given by the transfer matrix is considered from [25],
As

$$[G(s)] = \begin{bmatrix} \frac{2(s+5)}{(s+1)(s+10)} & \frac{(s+4)}{(s+2)(s+5)} \\ \frac{(s+10)}{(s+1)(s+20)} & \frac{(s+6)}{(s+2)(s+3)} \end{bmatrix} \quad (38)$$

$$= \frac{1}{D(s)} \begin{bmatrix} h_{11}(s) & h_{12}(s) \\ h_{21}(s) & h_{22}(s) \end{bmatrix} \quad (39)$$

where

$$D(s) = s^6 + 41s^5 + 571s^4 + 3491s^3 + 10060s^2 + 13100s + 6000 \quad (40)$$

And $h_{11}(s) = 2s^5 + 70s^4 + 762s^3 + 3610s^2 + 770 + 6000$ (41)

$$h_{12}(s) = s^5 + 38s^4 + 459s^3 + 2182s^2 + 4160s + 2400 \quad (42)$$

$$h_{21}(s) = s^5 + 30s^4 + 331s^3 + 1650s^2 + 3700s + 3000 \quad (43)$$

$$h_{22}(s) = s^5 + 42s^4 + 601s^3 + 3660s^2 + 9100s + 6000 \quad (44)$$

Following the steps described in section 3.1, the reduced denominator is obtained as

for r_1 =1 , r_2 = 1, $D_2(s)_1 = s^2 + 21s + 20$ (45)

for r_1 = 2, r_2 = 0, $D_2(s)_2 = s^2 + 3s + 2$ (46)

As per the steps explained in section 3.2, the lower order numerator unknown coefficients are obtained with the fitness function shown in Figure 6 for the cases r_1 =1 and r_2 =1 and r_1 =2 and r_2 =0. Therefore, the reduced numerator is obtained as

for r_1 = 1, r_2 = 1, $\begin{bmatrix} 0.1s + 20 & 3.8438s + 8 \\ 1.002s + 10 & 4.5928s + 20 \end{bmatrix}$ (47)

for r_1 = 2, r=0, $\begin{bmatrix} 1.5563s + 2 & 1.0530s + 0.8 \\ 3.9904s + 1 & 2.3214s + 2 \end{bmatrix}$ (48)

using equation (45) & equation (47), the desired reduced system for r_1 = 1, r_2 = 1 is obtained as

$$[G_r(s)]_1 = \frac{\begin{bmatrix} 0.1s + 20 & 3.8438s + 8 \\ 1.002s + 10 & 4.5928s + 20 \end{bmatrix}}{s^2 + 21s + 20} \quad (49)$$

using equation (46) & equation (48), the desired reduced system for r_1 = 2, r_2 = 0 is obtained as

$$[G_r(s)]_2 = \frac{\begin{bmatrix} 1.5563s + 2 & 1.0530s + 0.8 \\ 3.9904s + 1 & 2.3214s + 2 \end{bmatrix}}{s^2 + 3s + 2} \quad (50)$$

Responses obtained by the proposed methodology and the reduction schemes suggested by [3,41,18,45] for step input are shown in Figure 7. It has been seen that the output for the case r_1 = 1 and r_2 =1 is nearer to the given higher-order plant output than the case r1 = 2 and r2 = 0 and also shows better results than the other standard methods. The same was also determined in the frequency response characteristics, which are shown in Figure 8. From Tables 3 and 4, it has been shown that the ROM found by the suggested method for the case r1 = 1 and r2 = 1 is very close to the higher dimensional system specification with the minimum value of ISE and IAE.

Example 3: A 4th order system is considered with the reference model is given in [41] for compensator controller design.

$$G(s) = \frac{s^3 + 12s^2 + 54s + 72}{s^4 + 18s^3 + 97s^2 + 180s + 100} \quad (51)$$

By following the steps given in Section 3, the ROM is obtained as

$$G_r(s) = \frac{0.2970s + 7.2}{s^2 + 11s + 10} \quad (52)$$

The design of the compensator is done according to the steps mentioned in section 4 as follows:
Step I: Considered the compensator structure as given in equation (17)

$$C_c(s) = \frac{K(1 + K_A s)}{s(1 + K_B s)}$$

Step II: Reference model considered as

$$R(s) = \frac{4}{s^2 + 4s + 4} \quad (53)$$

The open-loop reference model is

$$\overline{R(s)} = \frac{R(s)}{1 - R(s)} = \frac{4s^2 + 16s + 16}{s^4 + 8s^3 + 20s^2 + 16s} \quad (54)$$

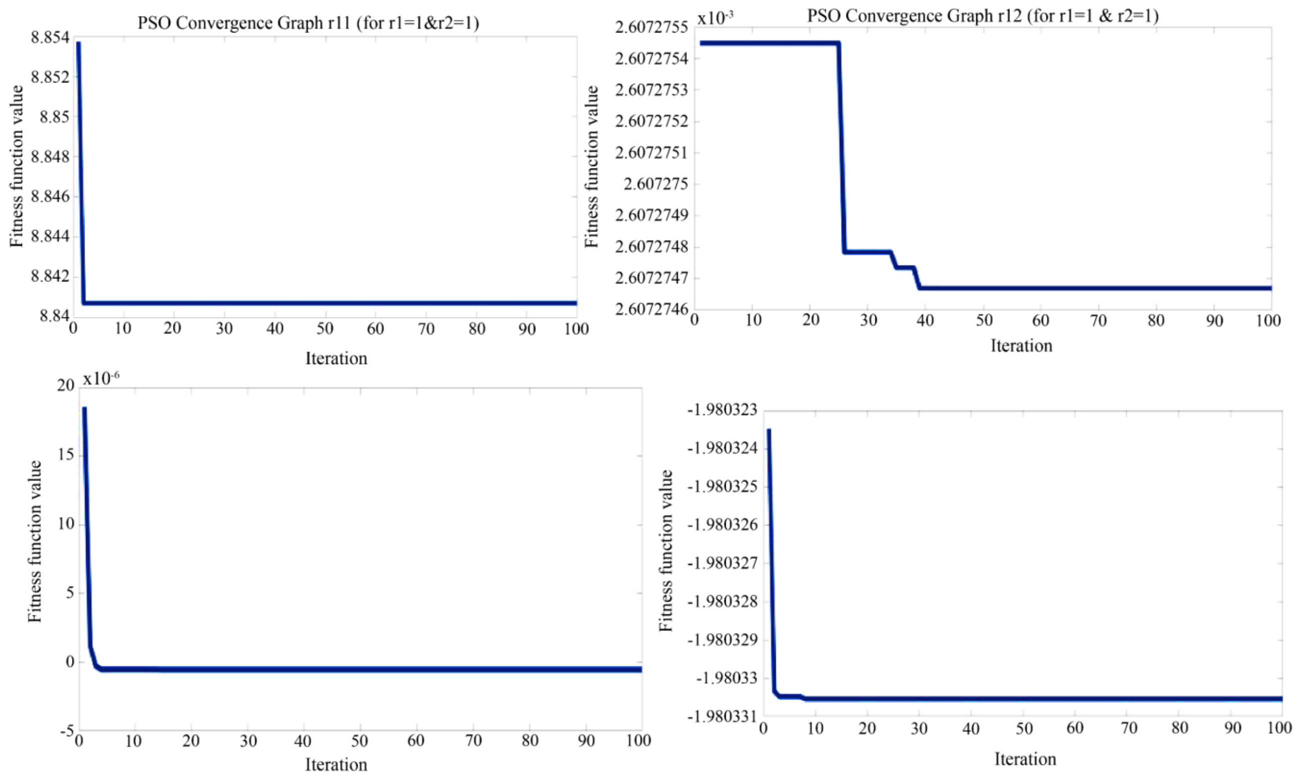


Fig. 6 PSO Convergence graph of example 2

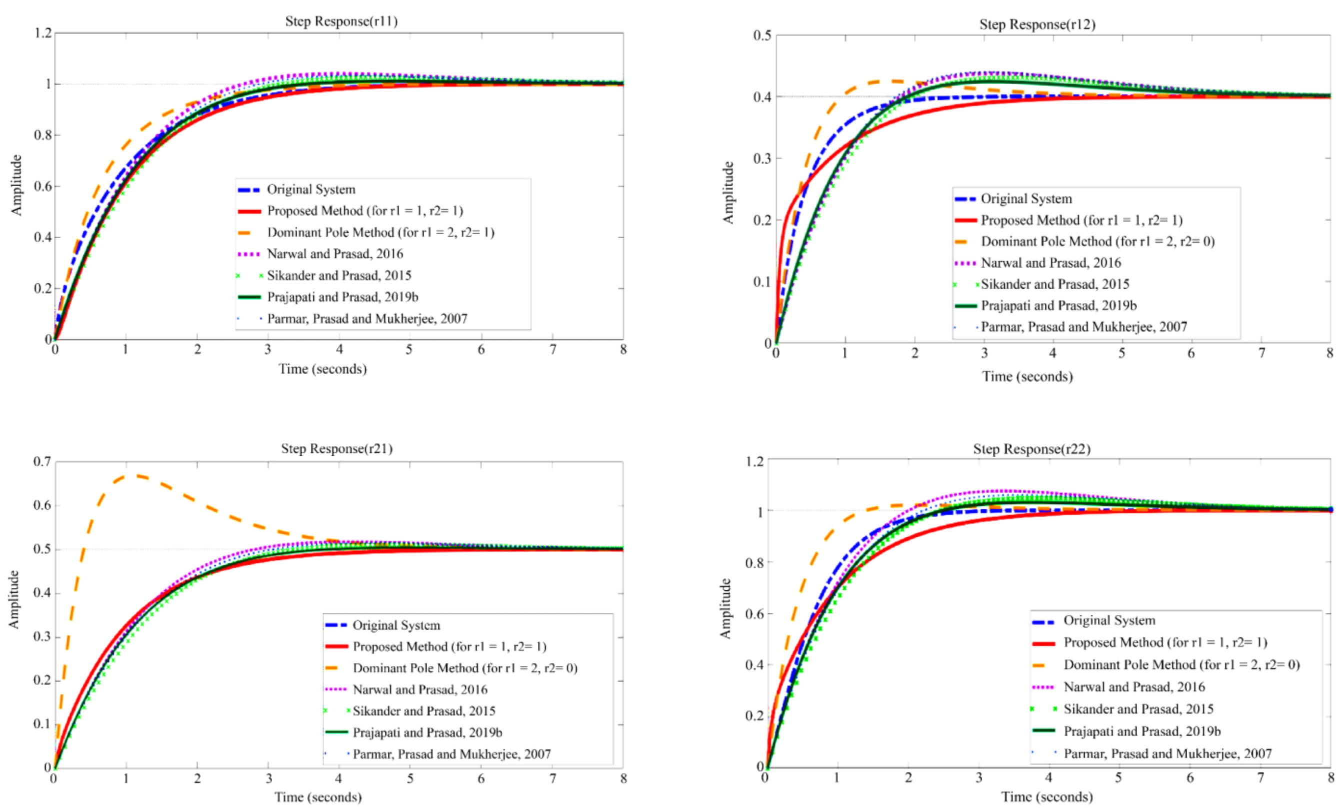


Fig. 7 Example 2- Step Responses

Table 3. Comparison of example (2) original system with different reduction technique based on IAE

Reduction Technique	$r_{11}(s)$	$r_{12}(s)$	$r_{21}(s)$	$r_{22}(s)$
Proposed method (for r_1=1, r_2=1)	0.145	0.093	0.0001	0.24
Proposed method (for r_1=2, r_2=0)	0.181	0.0167	0.728	0.327
Narwal and Prasad, 2016 [34]	0.3101	0.207	0.136	0.317
Sikander and Prasad, 2015[3]	0.283	0.2014	0.1253	0.3143
Prajapati and Prasad, 2019b[44]	0.1793	0.1545	0.0729	0.1971
Parmar, Prasad and Mukherjee, 2007 [21]	0.2794	0.206	0.1214	0.2974

Table 4. Comparison of example (2) original system with different minimization techniques with respect to ISE

Reduction Technique	$r_{11}(s)$	$r_{12}(s)$	$r_{21}(s)$	$r_{22}(s)$
Proposed method (for r_1=1, r_2=1)	0.0096	0.0032	$4.76\text{x}10^{-9}$	0.0173
Proposed method (for r_1=2, r_2=0)	0.0106	9.72*10^-5	0.1764	0.0467
Narwal and Prasad, 2016 [34]	0.1615	0.0897	0.0296	251.36
Sikander and Prasad, 2015[3]	0.1672	0.0958	0.0312	0.2004
Prajapati and Prasad, 2019b[44]	0.0765	0.0595	0.0115	0.0808
Parmar, Prasad and Mukherjee, 2007 [21]	0.1471	0.0884	0.0258	0.1598

By using reduced model $G_r(s)$, the parameters of the compensator are obtained as follows

$$C_r(s) = \frac{\overline{R(s)}}{G_r\,(s)} = \frac{e_0 + e_1 s + e_2 s^2}{s} \tag{55}$$

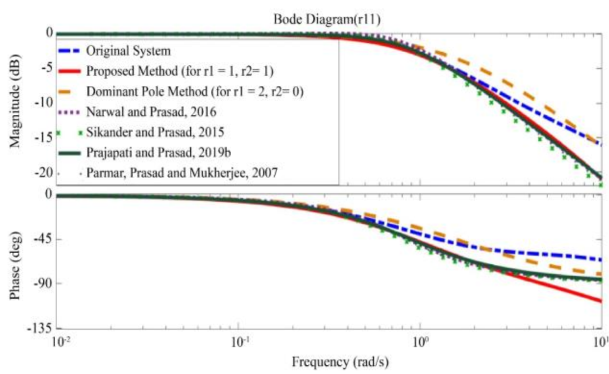
$$= \frac{1.3889 + 1.1227s - 0.2023s^2}{s} \tag{56}$$

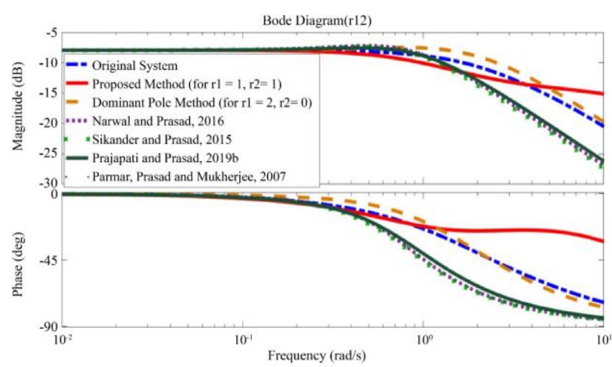
Therefore, the compensator parameters calculated are K= 1.3889, K_A= 0.9885, K_B = 0.1802

Putting the values of K, K_A and K_B in equation(17), the desired reduced order compensator structure is obtained as

$$C_r(s) = \frac{1.3889(1 + 0.9885s)}{s(1 + 0.1802s)} \tag{57}$$

$$G_{cl}\,(s) = \frac{C_r(s)G(s)}{1 + C_r(s)G(s)} = \frac{1.373\,s^4 + 17.86\,s^3 + 90.81\,s^2 + 173.9\,s + 100}{0.1802\,s^6 + 4.244\,s^5 + 36.85\,s^4 + 147.3\,s^3 + 288.8\,s^2 + 273.9\,s + 100} \tag{58}$$





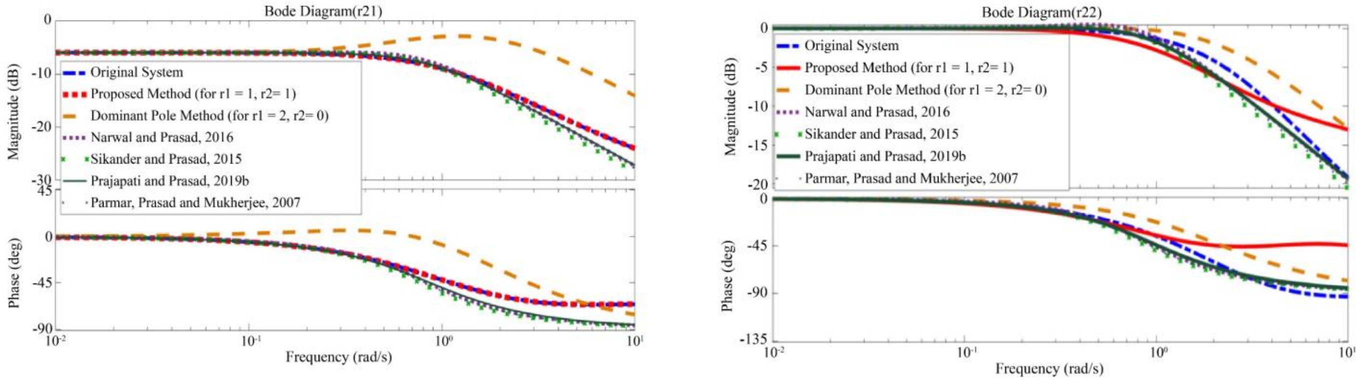


Fig. 8 Bode response of example 2

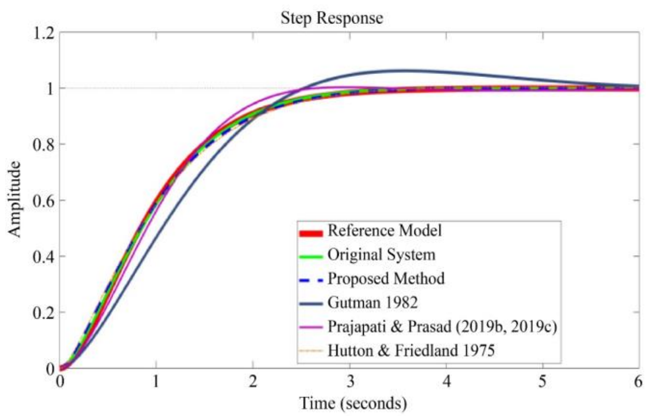


Fig. 9 Example3- Response comparison for a step input

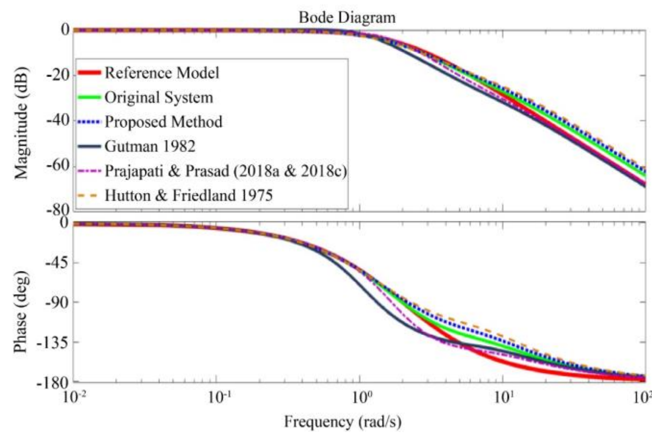


Fig. 10 Example 3-Frequency response comparison

Table 5. Comparison in terms of transient response specifications, Gain Margin & Phase margin of example 3

Reduction Technique	Reduced Order System	Compensator (K, KA, KB)	Transient Response Specification				Performance Indices	
			Tr	**Ts**	**Mp**	**Tp**	**Gain Margin**	**Phase Margin**
–	Reference Model		1.679	2.92	0	4.69	Inf	180°
–	Original System	1.3889,1.0302,0.2302	1.713	2.83	0	4.503	Inf	180°
Proposed method with r_1=1, r_2=1	$\frac{0.297s + 7.2}{s^2 + 11s + 10}$	1.3889,0.9885,0.1802	1.793	2.95	0.039	4.582	Inf	180°
Gutman, Mannerfelt and Molander, 1982[43]	$\frac{216s + 864}{194s^2 + 1080s + 1200}$	1.3889,0.6521,0.2521	1.713	5.30	6.155	3.583	Inf	140°
(Prajapati and Prasad, 2019b[44],Prajapati and Prasad, 2019c[2]	$\frac{1.774s + 8.719}{s^2 + 15.18s + 12.11}$	1.3892,1.2122,0.4131	1.503	2.29	0.311	2.978	Inf	180°
Hutton and Friedland, 1975[16]	$\frac{0.6207s + 0.8276}{s^2 + 2.069s + 1.149}$	1.3882,0.9493,0.1478	1.849	2.98	0.229	4.812	Inf	180°

Table 6. Error comparison of example 3

Reduction Technique	Reduced Order System	Performance Indices	
		ISE	IAE
–	Original System	0.0015	0.020
Proposed method with r_1=1, r_2=1	$\frac{0.297s+7.2}{s^2+11s+10}$	0.0005	0.037
Gutman, Mannerfelt and Molander, 1982[43]	$\frac{216s+864}{194s^2+1080s+1200}$	0.233	0.322
(Prajapati and Prasad, 2019b[44],Prajapati and Prasad, 2019c[2]	$\frac{1.774s+8.719}{s^2+15.18s+12.11}$	0.27	0.103
Hutton and Friedland, 1975[16]	$\frac{0.6207s+0.8276}{s^2+2.069s+1.149}$	0.011	0.058

The compensator is designed using ROM found by the proposed reduction methodology and the other lower-order models obtained from the methods given in the literature [43,44,2,16]. The simulation result confirms that the proposed method's compensator works finely in transient and steady-state conditions compared to the other methods. The transient response specification, gain margin and phase margin are calculated and compared, as shown in Table 5, to check the steadiness of the ROM and the higher-scale system. The step & frequency response comparison of the reference model with different controlled systems obtained by reduced compensators is shown in Figures 9 & 10, respectively.

Table 6 shows the quantitative investigation on the basis of ISE and IAE values. From the analysis, it has been concluded that the offered scheme is more acceptable as compared to the other methods as it gives the minimum values of error indices. It has also been found that compensator design using ROM is easier than designing compensation with a HOS. Therefore, the suggested reduction method is used to design compensation to achieve the basic performance of dynamic systems.

6. Conclusion

This paper offers a low-order approximation methodology for LTI SISO and MIMO systems. A new grouping of two methods is applied to minimize the complexity of a large-order actual system. The numerator of ROM is obtained by using the PSO technique. Whereas high-order plant dominant poles retain in the ROM and take advantage of reciprocal transformation, the denominator of ROM is obtained. The acquired ROM by the suggested technique is compared with the ROMs computed from the other existing latest and standard model reduction techniques available to analyze the suggested technique's usefulness, efficacy and accurateness. The suggested scheme gives the assurance of steadiness of the ROM if the HOS is steady and retains the basic properties of the HOS in the approximated low-dimensional model. From the responses of the time and frequency domain, it has been observed that the suggested methodology gives a nearer approximation to the full-order system.

Furthermore, the ISE and IAE are calculated and compared with the existing reduction methodologies shown in Tables (2, 3, 4 and 5) to confirm the superiority of the suggested method. It has been concluded that the proposed technique is simple to implement, easy to program and requires less execution time as compared to HOS. Furthermore, the compensator is designed using a reduced approximated system. It is directly functional on the original system to provide the minimum value of ISE and IAE compared to other existing compensators obtained by well-known existing reduction techniques. The proposed methodology is applied to three standard examples to illustrate and verify the proposed technique's accuracy. This proposed work may also be extended to systems of high order having complex conjugate poles & discrete systems.

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