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Original Article

Reliability Analysis of Production in an Engineering System

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Abstract *- This study aims to compare different reliability indices in order to enhance industry production processes. In this case, we are looking at a system with three parallel units. This study centers on reliability analysis, taking into account the availability, estimated busy period, Mean Time to System Failure (MTSF), and expected number of repairman visits. There is a backup unit for the primary unit. Semi-Markov and regenerative point methods have been used to assess the system's performance. In the beginning, all three units will be functional, and a single repair facility will handle all required repairs. The repair time distribution is thought to be universal, but the unit failure time distribution is found to be exponential with variable parameters. Mean residence time, MTSF, system utilization time, steady state availability, and other critical reliability characteristics are examined. Graphs were used in the investigation process to further increase the study's adaptability.*

Keywords *- Reliability, Markov process, Failure rate, Repair rate, Laplace transforms, Regenerative points.*

1. Introduction

The investigation of reliability and analysis of working systems holds significant importance in enhancing performance and profitability in engineering systems. In today's competitive environment, it is critical to ensure a system's reliable operation and maintenance throughout its expected lifespan. Reliability is essential for the efficient utilization and maintenance of any technical system, particularly in industrial manufacturing, where multiple products and equipment are involved. Despite extensive research in reliability theory, there remains a deficiency in comprehensive analyses of intricate repairable systems under varying failure and repair distributions. This study aims to address this gap by focusing on a system comprising three parallel units, incorporating a backup unit for the primary unit, and employing a singular repair facility. The integration of regenerative point methods and semi-Markov process to evaluate multiple reliability indices in such a system configuration is a novel approach compared to previous research that has focused on various aspects of reliability. Investigators have evaluated their work in the field of reliability in the past by studying the performance of complex systems under various failure and repair distributions. Zuhair et al. [1] conducted a comprehensive examination of the reliability analysis concerning two distinct units, focusing on their performance metrics and operational dependability. In a related study, Sharma [2] explored the availability of a standby system, categorizing itnto three different types of failures to better understand its resilience and efficiency in various scenarios. Meanwhile, Kumar et al. [3] developed a sophisticated model to evaluate the performance of the furnace draft air cycle within a thermal power plant, providing valuable insights into its operational dynamics and potential areas for optimization. These studies collectively enhance our understanding of reliability and availability in complex systems. Bhatia et al. [4] focused on reliability modeling for a three-unit cold standby system (induced draft fan) operating at both full and reduced capacities. Naithani et al. [5] explored the probabilistic assessment of a three-unit induced draft fan system that includes one warm standby unit and a repair prioritization scheme for functional units. Gupta and Gupta [6] investigated the reliability model of a single-unit system, considering factors such as post-inspection, post-repair, preventive maintenance, and replacement. Rizwan et al. [7-8] examined the dependability, availability, and profitability of various systems. Wang, Xie, and Yang [9] analyzed the reliability of a two-unit warm standby repairable system, incorporating usage priority. Lastly, Yusuf, Yusuf, and Suleiman [10] assessed the reliability of a repairable system under both online and offline preventive maintenance strategies. Goyal et al. [11] conducted a study on the dependability, sensitivity and maintainability of the physical processing component of a wastewater treatment facility. Moreover, Gupta, Saini and Kumar [12,13] assessed the operational availability of generators in steam turbine power plants and a single-unit system considering degradation and inspection. Dahiya et al. [14] modelled and analyzed a concrete mixing plant following the coverage factor and robust reliability strategy. Fairly. Our previous study is about the dependability of various anode-rodding strategies in the aluminum industry.

Building on these approaches, one can explore the integration of Semi-Markov and regenerative point methods with artificial neural networks [15] and deep neural networks [16, 17]. This combination has the potential to significantly enhance the model's ability to account for temporal dependencies and state transitions, particularly in dynamic environments where events occur irregularly. By adopting this innovative methodology, researchers could gain deeper insights into system behaviors and improve predictive accuracy.

Systematic testing and refinement of this integration will be crucial for assessing its effectiveness and uncovering new applications in various fields. The researchers discussed above focused on modelling and analyzing a concrete mixing plant by employing a coverage factor and robust reliability strategy to ensure efficient and reliable operation under various conditions. They also examined the dependability of different anode-rodding techniques in the aluminum industry, aiming to identify methods that enhance operational reliability and performance.

Together, these studies provide valuable insights into improving processes in their respective fields. The following reliability measures are obtained using semi-Markov processes and the technique based on regenerative points. Mean Time of System Failure (MTSF), Availability of the system (A_0), An expected busy period for the repairman to repair (B_0), the estimated frequency of visits required for repairs (V_0), and the profit analysis for the system (Profit).

2. Symbols and Notations

S_i	i^{th} state mode.
α_i	i^{th} units failure rate.
$p_{i,j}$	The probability of transitioning from regenerative state i to regenerative state j within the time interval $(0, \mathrm{t}]$.
μ_i	Mean duration in the regenerative state before moving to another state.
$\phi_i(t)$	Distribution Function Cumulative (c. d. f) of the initial transition period from a state of regeneration (i) to a state of failure (j).
$A_i(t)$	System availability in the initial state i.
$B_i(t)$	Represent the probability that the repairman is actively working during the inspection time t, given that the system entered regenerative state i at $t = 0$.
$V_i(t)$	The expected number of repairman visits, given that the system entered regenerative state i at $t = 0$.
$M_i(t)$	The likelihood that a system, which was initially in a regenerative state i, will become active once more at time t without changing into a regenerative state.
$W_i(t)$	The chance that the mechanic is involved in regenerative state i at time t, without passing through another regenerative state.
C_0	Revenue obtained for each unit of system uptime.
C_1	Cost per unit operating time that the mechanic is engaged in the repair.
C_2	The price per repairman visit.
A_0	The duration of the system's availability.
B_0	The total time the mechanic spends on the repair.
V_0	The total expected number of visits or mechanics.
$g_i(t)$	The Probability Density Function (PDF) of the repair rate for the i^{th} unit experiencing repairable failures.
$q_{i,j}(t)$	The Probability Density Function (PDF) describes the transition from state i to state j.
$Q_{i,j}(t)$	The Cumulative Distribution Function (CDF) of the first passage time from regenerative state i to failed state j.
©	The symbol of Laplace Convolution.
ⓢ	The Stieltje's Convolution symbol.
$*$	The Laplace Transforms symbol.
$**$	The Laplace Steiltje's transforms symbol.

3. System Assumption and Description

This research explores a multifaceted repairable system composed of three interconnected units arranged in series. Initially, the system functions flawlessly, operating at peak efficiency.

However, if either the second or third unit experiences a failure, it will result in a complete breakdown of the entire system, highlighting the critical interdependence of its components. In the meantime, if the first unit fails, it will not affect the system and will be operational due to the standby unit for the first unit.

If all 2nd and 3rd units fail, the system is in a complete failure state. The subsequent expectations are taken throughout the discussion of the model (Table 1):

1. All of the units are initially in good operating order.
2. Three units are working parallelly, and unit one is the only standby unit of the entire system.
3. The system is inoperable if both units one and two fail.
4. All failure rates are constant and are assumed to follow an exponential distribution.
5. Repairs of minor/significant errors are carried out by the general sales department.
6. A repaired system is assumed to work like a new one.

Table 1. Transition table of the states

States	S_j							
S_i		S_0	S_1	S_2	S_3	S_4	S_5	S_6
	S_0	–	α_1	α_2	α_3	–	–	–
	S_1	$f_1(t)$	–	–	–	α_1	α_2	α_3
	S_2	$f_2(t)$	–	–	–	–	–	–
	S_3	$f_3(t)$	–	–	–	–	–	–
	S_4	–	$f_1(t)$	–	–	–	–	–
	S_5	–	$f_2(t)$	–	–	–	–	–
	S_6	–	$f_3(t)$	–	–	–	–	–

4. Transition Characteristics and Average Sojourn Times

A state transition diagram illustrating the potential states and transitions of the units, with the following transition probabilities and intensities:

$$\sum_{i=1}^{3} dQ_{0i}(t) = \sum_{i=1}^{3} \alpha_i e^{-t(\alpha_1+\alpha_2+\alpha_3)} \quad (1)$$

$$dQ_{10}(t) = \delta_1 e^{-t(\delta_1+\alpha_1+\alpha_2+\alpha_3)} \quad (2)$$

$$\sum_{j=4}^{6} dQ_{1j}(t) = \sum_{i=1}^{3} \alpha_i e^{-t(\delta_1+\alpha_1+\alpha_2+\alpha_3)} \quad (3)$$

$$\sum_{i=2}^{3} dQ_{i0}(t) = \sum_{i=2}^{3} \delta_i e^{-\delta_i t} \quad (4)$$

$$\sum_{i=4}^{6} dQ_{i1}(t) = \sum_{i=1}^{3} \delta_i e^{-\delta_i t} \quad (5)$$

Then, the transition probabilities p_{ij} are given below:

$p_{01} = \lim_{s\to 0} q^*{}_{01}(s) = \lim_{s\to 0} L[q_{01}(t)] = \lim_{s\to 0} L\left[\alpha_1 e^{-t(\alpha_1+\alpha_2+\alpha_3)}\right] = \lim_{s\to 0}\left(\frac{\alpha_1}{s+\alpha_1+\alpha_2+\alpha_3}\right) = \frac{\alpha_1}{\alpha_1+\alpha_2+\alpha_3}$ and

similarly,

$$p_{02} = \frac{\alpha_2}{\alpha_1+\alpha_2+\alpha_3}$$
$$p_{03} = \frac{\alpha_3}{\alpha_1+\alpha_2+\alpha_3}$$
$$p_{10} = \frac{\delta_1}{\delta_1+\alpha_1+\alpha_2+\alpha_3}$$
$$p_{14} = \frac{\alpha_1}{\delta_1+\alpha_1+\alpha_2+\alpha_3}$$
$$p_{15} = \frac{\alpha_2}{\delta_1+\alpha_1+\alpha_2+\alpha_3}$$
$$p_{16} = \frac{\alpha_3}{\delta_1+\alpha_1+\alpha_2+\alpha_3}$$

From the probabilities above, the following equations can be varied:

$p_{0,1} + p_{0,2} + p_{0,3} = 1$
$p_{1,0} + p_{1,4} + p_{1,5} + p_{1,6} = 1$
and $p_{2,0} = p_{3,0} = p_{4,1} = p_{5,1} = p_{6,1} = 1$

5. Mean Sojourn Times

The mean sojourn time in a regenerative state refers to the expected duration spent in that state before transitioning to another. This concept can be understood through two main approaches. The first involves integrating the time spent in the state, which helps calculate the average duration by considering all possible times until leaving. The second approach utilizes the Laplace transform, providing an alternative method to analyze this duration by examining how the transform behaves as it approaches a specific limit. Together, these perspectives offer a thorough understanding of mean sojourn time, which is essential for modelling and analyzing regenerative processes in fields such as queueing theory, reliability engineering, and Markov processes.

The mean sojourn time (u_i)

$$\mu_i = \int_0^\infty t dQ_{ij}(t) = -\lim_{s\to 0}\frac{d}{ds}\left(q_{ij}{}^*(s)\right)$$

In a regenerative state, the mean sojourn time refers to the duration spent in that state before transitioning to another. If we consider the sojourn time in this state, it represents the average amount of time one typically remains in state i before moving on to a different state. This concept is important for understanding how systems operate and can help in modelling transitions between various states. Consider T denotes the sojourn time in regenerative state i, then:

$$\mu_0 = \frac{\alpha_1}{(\alpha_1+\alpha_2+\alpha_3)^2}$$
$$\mu_1 = \frac{1}{\delta_1+\alpha_1+\alpha_2+\alpha_3}$$
$$\mu_2 = \mu_5 = \frac{1}{\delta_2}$$
$$\mu_3 = \mu_6 = \frac{1}{\delta_3}$$
$$\mu_4 = \frac{1}{\delta_1}.$$

The unconditional mean time (m_{ij}),

$$m_{i,j} = \int_0^\infty t dQ_{i,j}(t) = \lim_{s\to 0} -\frac{d}{ds}\left(q_{i,j}{}^*(s)\right) = -q_{i,j}^{\prime *}(0)$$

taken by the system to transition to any for any regenerative state j, starting from the epoch entry into state i , is mathematically stated as:

$$m_{0,1} = \frac{\alpha_1}{(\alpha_1+\alpha_2+\alpha_3)^2}$$

$$m_{0,2} = \frac{\alpha_2}{(\alpha_1+\alpha_2+\alpha_3)^2}$$

$$m_{0,3} = \frac{\alpha_3}{(\alpha_1+\alpha_2+\alpha_3)^2}$$

$$m_{1,0} = \frac{\delta_1}{(\delta_1+\alpha_1+\alpha_2+\alpha_3)^2}$$

$$m_{1,4} = \frac{\alpha_1}{(\delta_1+\alpha_1+\alpha_2+\alpha_3)^2}$$

$$m_{1,5} = \frac{\alpha_2}{(\delta_1 + \alpha_1 + \alpha_2 + \alpha_3)^2}$$

$$m_{1,6} = \frac{\alpha_3}{(\delta_1+\alpha_1+\alpha_2+\alpha_3)^2},$$

$$m_{2,0} = m_{51} = \frac{1}{\alpha_2}$$

$$m_{3,0} = m_{61} = \frac{1}{\alpha_3}$$

$$m_{4,1} = \frac{1}{\delta_1}.$$

6. The Mathematical Analysis of the System

6.1. Mean Time to Failure of the System (MTSF)

The Mean Time to Failure (MTSF) measures how long a system is expected to operate before failing. To calculate the MTSF, we treat the failed state as an absorbing state, meaning once the system fails, it does not recover. We use two key equations that describe the probabilities of transitioning between operational states and the failed state. By applying Laplace-Stieltjes transforms to these equations, we derive a new expression that helps determine the probability of the system being operational. The MTSF is calculated by taking the limit of this probability as a parameter approaches zero. This approach allows us to capture various transition probabilities and provides valuable insight into the system's reliability and performance. Equations 6 and 7 are obtained using probabilistic reasoning when we consider the system's failed state to be an absorbing state in order to calculate the MTSF of the system.

$$\emptyset_0(t) = Q_{01}(t) Ⓢ \emptyset_1(t) + \sum_{j=2}^{3} Q_{0j}(t) \quad (6)$$

$$\emptyset_1(t) = Q_{10}(t) Ⓢ \emptyset_0(t) + \sum_{j=4}^{6} Q_{1j}(t) \quad (7)$$

Taking Laplace Stieltje's transforms of the number of Equations 6 and 7 and solving for $\emptyset_0^{**}(s)$, MTSF is obtained.

$$MTSF = \lim_{s\to 0} \frac{1-\emptyset_0^{**}(s)}{s} = \frac{N}{D}$$

$N = m_{02} + m_{03} + m_{10}p_{01} + m_{14}p_{01} + m_{15}p_{01} + m_{16}p_{10} + m_{01}p_{14} + m_{01}p_{15} + m_{01}p_{16}$

$D = 1 - p_{01} - p_{10}$

6.2. Availability Analysis of the System

Availability analysis evaluates the likelihood that a system is operational at any given time. It employs recursive relationships to calculate the probability of the system entering various states, starting from an initial operational state. These equations capture the transitions between operational and failed states. To simplify the analysis, we use the Laplace transform, which helps convert time-domain relationships into a more manageable form. By evaluating the transformed equation, we can derive the availability of the initial state.

This analysis is essential for understanding system reliability and identifying potential downtime. Equations 8-11 provide the following recursive relations, which yield $A_i(t)$. These expressions are derived by organizing the probabilities and defining $A_i(t)$ as the probability that the unit enters state i at time t, starting from the regenerative state at time zero.

$$A_0(t) = M_0(t) + \sum_{j=1}^{j=3} q_{0,j}(t) © A_j(t) \quad (8)$$

$$A_1(t) = M_1(t) + q_{1,0}(t) © A_0(t) + \sum_{j=4}^{j=6} q_{1,j}(t) © A_j(t) \quad (9)$$

$$\sum_{i=2}^{3} A_i(t) = \sum_{i=2}^{3} q_{i,0}(t) © A_0(t) \quad (10)$$

$$\sum_{i=4}^{6} A_i(t) = \sum_{i=4}^{6} q_{i,1}(t) © A_1(t) \quad (11)$$

Where,
$M_0(t) = e^{-t(\delta_1+\alpha_2+\alpha_3)}$

$M_1(t) = e^{-t(\delta_1+\alpha_1+\alpha_2+\alpha_3)}$

then taking the Laplace transforms of equations and solving them for $A^*(s)$, then the equation (12) is obtained:

$$A_0 = \lim_{s\to 0} sA_0{}^*(t) = \lim_{s\to 0} \frac{sN(s)}{D(s)} = \frac{N_1}{D_1} \quad (12)$$

Where,
$N_1 = u_0 + u_1p_{0,1} - u_0p_{1,4}p_{4,1} - u_0p_{1,5}p_{5,1} + u_0p_{1,6}p_{6,1}$

$D_1 = m_{0,1} + m_{1,0}p_{0,1} + m_{2,0}p_{0,2} + m_{3,0}p_{0,3} + m_{4,1}p_{1,4} + m_{5,1}p_{1,5} - m_{6,1}p_{1,6} + m_{0,2}p_{2,0} - m_{4,1}p_{0,2}p_{1,4}p_{2,0} - m_{5,1}p_{0,2}p_{1,5}p_{2,0} + m_{6,1}p_{0,2}p_{1,6}p_{2,0} + m_{0,3}p_{3,0} - m_{4,1}p_{0,3}p_{1,4}p_{3,0} - m_{5,1}p_{0,3}p_{1,5}p_{3,0} + m_{6,1}p_{0,3}p_{1,6}p_{3,0} + m_{1,4}p_{4,1} - m_{2,0}p_{0,2}p_{1,4}p_{4,1} - m_{3,0}p_{0,3}p_{1,4}p_{4,1} - m_{1,4}p_{0,2}p_{2,0}p_{4,1} - m_{0,2}p_{1,4}p_{2,0}p_{4,1} - m_{1,4}p_{0,3}p_{3,0}p_{4,1} - m_{0,3}p_{1,4}p_{3,0}p_{4,1} + m_{1,5}p_{5,1} - m_{2,0}p_{0,2}p_{1,5}p_{5,1} - m_{3,0}p_{0,3}p_{1,5}p_{5,1} - m_{1,5}p_{0,2}p_{2,0}p_{5,1} - m_{0,2}p_{1,5}p_{2,0}p_{5,1} - m_{1,5}p_{0,3}p_{3,0}p_{5,1} - m_{0,3}p_{1,5}p_{3,0}p_{5,1} - m_{1,6}p_{6,1} + m_{2,0}p_{0,2}p_{1,6}p_{6,1} + m_{3,0}p_{0,3}p_{1,6}p_{6,1} + m_{1,6}p_{0,2}p_{2,0}p_{6,1} + m_{0,2}p_{1,6}p_{2,0}p_{6,1} + m_{1,6}p_{0,3}p_{3,0}p_{6,1} + m_{0,3}p_{1,6}p_{3,0}p_{6,1}$

6.3. Expected Busy Period Analysis of the System

Equations 13 to 16 show the recursive relations that can be obtained by applying probabilistic arguments. Using the unit entering regenerative state i at $t = 0$, define $B_0{}^*(s)$ as the probability that the repairman is busy with repair at instant t.

$$B_0(t) = \sum_{j=1}^{j=3} q_{0j}(t) © A_j(t) \quad (13)$$

$$B_1(t) = W_1(t) + q_{10}(t) © B_0(t) + \sum_{j=4}^{j=6} q_{1j}(t) © B_j(t) \quad (14)$$

$$\sum_{i=2}^{3} \mathrm{B_i(t)} = \sum_{i=2}^{3} \mathrm{W}_i(\mathrm{t}) + \mathrm{q_{i0}(t)©B_0(t)} \quad (15)$$

$$\sum_{i=4}^{6} \mathrm{B_i(t)} = \sum_{i=4}^{6} \mathrm{W}_i(\mathrm{t}) + \mathrm{q_{i1}(t)©B_1(t)} \quad (16)$$

Where,
$\mathrm{W_1(t)} = e^{-t(\delta_1+\alpha_1+\alpha_2+\alpha_3)}$,

$\mathrm{W_2(t)} = \mathrm{W_5(t)} = e^{-\delta_2 t}$,

$\mathrm{W_3(t)} = \mathrm{W_6(t)} = e^{-\delta_3 t}$

$\mathrm{W_4(t)} = e^{-\delta_1 t}$

Then, taking the Laplace transform of the Equations (13-16) and solving them, the busy period of the repairman is given by Equation 17 as the following:

$$\mathrm{B_0} = \lim_{s\to 0} s\mathrm{B_0}^*(\mathrm{t}) = \lim_{s\to 0} \frac{\mathrm{sN(s)}}{\mathrm{D(s)}} = \frac{\mathrm{N_2}}{\mathrm{D_1}} \quad (17)$$

Where,
$N_2 = u_1p_{0,1} + u_2p_{0,2} + u_3p_{0,3} + u_4p_{0,1}p_{1,4} + u_2p_{0,1}p_{1,5} - u_3p_{0,1}p_{1,6} - u_2p_{0,2}p_{1,4}p_{4,1} - u_3p_{0,3}p_{1,4}p_{4,1} - u_2p_{0,2}p_{1,5}p_{5,1} - u_3p_{0,3}p_{1,5}p_{5,1} + u_2p_{0,2}p_{1,6}p_{6,1} + u_3p_{0,3}p_{1,6}p_{6,1}$ and $\mathrm{D_1} = m_{0,1} + m_{1,0}p_{0,1} + m_{2,0}p_{0,2} + m_{3,0}p_{0,3} + m_{4,1}p_{1,4} + m_{5,1}p_{1,5} - m_{6,1}p_{1,6} + m_{0,2}p_{2,0} - m_{4,1}p_{0,2}p_{1,4}p_{2,0} - m_{5,1}p_{0,2}p_{1,5}p_{2,0} + m_{6,1}p_{0,2}p_{1,6}p_{2,0} + m_{0,3}p_{3,0} - m_{4,1}p_{0,3}p_{1,4}p_{3,0} - m_{5,1}p_{0,3}p_{1,5}p_{3,0} + m_{6,1}p_{0,3}p_{1,6}p_{3,0} + m_{1,4}p_{4,1} - m_{2,0}p_{0,2}p_{1,4}p_{4,1} - m_{3,0}p_{0,3}p_{1,4}p_{4,1} - m_{1,4}p_{0,2}p_{2,0}p_{4,1} - m_{0,2}p_{1,4}p_{2,0}p_{4,1} - m_{1,4}p_{0,3}p_{3,0}p_{4,1} - m_{0,3}p_{1,4}p_{3,0}p_{4,1} + m_{1,5}p_{5,1} - m_{2,0}p_{0,2}p_{1,5}p_{5,1} - m_{3,0}p_{0,3}p_{1,5}p_{5,1} - m_{1,5}p_{0,2}p_{2,0}p_{5,1} - m_{0,2}p_{1,5}p_{2,0}p_{5,1} - m_{1,5}p_{0,3}p_{3,0}p_{5,1} - m_{0,3}p_{1,5}p_{3,0}p_{5,1} - m_{1,6}p_{6,1} + m_{2,0}p_{0,2}p_{1,6}p_{6,1} + m_{3,0}p_{0,3}p_{1,6}p_{6,1} + m_{1,6}p_{0,2}p_{2,0}p_{6,1} + m_{0,2}p_{1,6}p_{2,0}p_{6,1} + m_{1,6}p_{0,3}p_{3,0}p_{6,1} + m_{0,3}p_{1,6}p_{3,0}p_{6,1}$

6.4. Expected Repair Visits in the System

Let $\mathrm{V}_i(\mathrm{t})$ be defined as the expected number of repair visits in the interval $(0, t]$, assuming the system initially starts in a regenerative state i. The following recursive relationships are derived using probabilistic arguments for $\mathrm{V}_i(\mathrm{t})$:

$$\mathrm{V_0(t)} = \mathrm{Q_{01}(t)}Ⓢ(1 + \mathrm{V_0(t)}) + \sum_{j=2}^{3} \mathrm{Q_{0j}(t)}Ⓢ\, \mathrm{V}_j(\mathrm{t}) \quad (18)$$

$$\mathrm{V_1(t)} = \mathrm{Q_{10}(t)©V_0(t)} + \sum_{j=4}^{j=6} \mathrm{Q_{1j}(t)} ©\left(1 + \mathrm{V_j(t)}\right) \quad (19)$$

$$\sum_{i=2}^{3} \mathrm{V_2(t)} = \sum_{i=2}^{3} \mathrm{Q_{i0}(t)©V_0(t)} \quad (20)$$

$$\sum_{4}^{6} \mathrm{V}_i(\mathrm{t}) = \sum_{i=4}^{6} \mathrm{Q_{i1}(t)©}\left(1 + \mathrm{V_1(t)}\right) \quad (21)$$

By applying the Laplace-Stieltjes transform to Equations $(18) - (21)$ and solving for $\mathrm{V_0}^{**}(s)$, the expected number of visits is determined as follows:

$$\mathrm{V_0} = \lim_{s\to 0} s\mathrm{V_0}^{**}(s) = \lim_{s\to 0} \frac{\mathrm{sN(s)}}{\mathrm{D(s)}} = \frac{\mathrm{N_3}}{\mathrm{D_1}} \quad (22)$$

Where,
$N_3 = p_{0,1} + p_{0,1}p_{1,4} + p_{0,1}p_{1,5} + p_{0,1}p_{1,6} + p_{0,3}p_{3,0} - p_{0,3}p_{1,4}p_{3,0}p_{4,1} - p_{0,3}p_{1,5}p_{3,0}p_{5,1} + p_{0,3}p_{1,6}p_{3,0}p_{6,1}$

$\mathrm{D_1} = m_{0,1} + m_{1,0}p_{0,1} + m_{2,0}p_{0,2} + m_{3,0}p_{0,3} + m_{4,1}p_{1,4} + m_{5,1}p_{1,5} - m_{6,1}p_{1,6} + m_{0,2}p_{2,0} - m_{4,1}p_{0,2}p_{1,4}p_{2,0} - m_{5,1}p_{0,2}p_{1,5}p_{2,0} + m_{6,1}p_{0,2}p_{1,6}p_{2,0} + m_{0,3}p_{3,0} - m_{4,1}p_{0,3}p_{1,4}p_{3,0} - m_{5,1}p_{0,3}p_{1,5}p_{3,0} + m_{6,1}p_{0,3}p_{1,6}p_{3,0} + m_{1,4}p_{4,1} - m_{2,0}p_{0,2}p_{1,4}p_{4,1} - m_{3,0}p_{0,3}p_{1,4}p_{4,1} - m_{1,4}p_{0,2}p_{2,0}p_{4,1} - m_{0,2}p_{1,4}p_{2,0}p_{4,1} - m_{1,4}p_{0,3}p_{3,0}p_{4,1} - m_{0,3}p_{1,4}p_{3,0}p_{4,1} + m_{1,5}p_{5,1} - m_{2,0}p_{0,2}p_{1,5}p_{5,1} - m_{3,0}p_{0,3}p_{1,5}p_{5,1} - m_{1,5}p_{0,2}p_{2,0}p_{5,1} - m_{0,2}p_{1,5}p_{2,0}p_{5,1} - m_{1,5}p_{0,3}p_{3,0}p_{5,1} - m_{0,3}p_{1,5}p_{3,0}p_{5,1} - m_{1,6}p_{6,1} + m_{2,0}p_{0,2}p_{1,6}p_{6,1} + m_{3,0}p_{0,3}p_{1,6}p_{6,1} + m_{1,6}p_{0,2}p_{2,0}p_{6,1} + m_{0,2}p_{1,6}p_{2,0}p_{6,1} + m_{1,6}p_{0,3}p_{3,0}p_{6,1} + m_{0,3}p_{1,6}p_{3,0}p_{6,1}$

7. Profit Analysis of the System

For graphical interpretation, consider the specific cases $g_1(t) = \delta_1 e^{-\delta_1 t}$, $g_2(t) = \delta_2 e^{-\delta_2 t}$ and $g_3(t) = \delta_3 e^{-\delta_3 t}$ (23)

The profit incurred by the system can be obtained by using Equation 24:

$$P = C_0A_0 - C_1B_0 - C_2\mathrm{V_0} \quad (24)$$

where, C_0 denotes the revenue per unit time, $C1$ denotes the cost per unit time for which the repairman is busy for repair, and C_2 denotes the cost per visit of the repairman.

Profit analysis is a crucial aspect of reliability studies as it directly impacts the economic viability of the system. In this section, we present a detailed profit analysis based on the reliability measures obtained from the study. The key metrics considered include Mean Time to System Failure (MTSF), System Availability$(\mathrm{A_0})$, Busy Period of the Repairman $(\mathrm{B_0})$, and Expected Number of Visits $(\mathrm{V_0})$.

7.1. Reliability Measures and Their Economic Implications

The system's reliability metrics are as follows:

- Mean Time Plant Failure $(MTSF)$: 12.1298
- System Availability $(\mathrm{A_0})$: 0.357576,
- Busy Period of the Repairman $(\mathrm{B_0})$: 0.158757
- Expected Number of Visits $(\mathrm{V_0})$: 0.0288487

7.2. Mean Time to System Failure (MTSF)

The MTSF value of 12.1298 indicates that the system is expected to operate for an average of 12.13 hours before experiencing a failure. This long MTSF is advantageous as it implies lower downtime and less frequent interruptions, leading to higher productivity and reduced costs associated with unscheduled maintenance.

7.3. System Availability (A_0)
The system availability of 0.357576, or 35.76%, reflects the proportion of time the system is operational. While this availability might seem moderate, it is crucial to understand that it can be significantly improved with strategic investments in maintenance and reliability enhancements. Higher availability translates to increased production capacity and better utilization of resources, ultimately boosting profitability.

7.4. Repairman's Busy Time (B_0)
The busy period of the repairman is 0.158757, indicating that the repair facility is engaged 15.88% of the time. Efficient management of the repairman's time ensures that repairs are conducted promptly without excessive waiting periods. This efficiency reduces the overall downtime and maintenance costs, contributing positively to the system's profitability.

7.5. Expected Number of Visits (V_0)
The repairman is expected to make 0.0288487 visits, suggesting that repair interventions are relatively infrequent. This low frequency of repairs implies a robust system with well-maintained units, which minimizes repair costs and maximizes production uptime.

8. Graphical Interpretation
For the graphical interpretation, we considered specific cases to illustrate the system's behavior under varying failure and repair rates.

8.1. Behavior of MTSF and Availability with Failure Rate
Figures 1 and 2 depict the Mean Time to System Failure (MTSF) and system availability (A_0) as functions of the failure rate (λ_1).

The results show that both MTSF and availability decrease with an increase in the failure rate. As the system encounters more frequent failures, the time until the system fails and its overall availability is reduced. Conversely, system availability increases with a higher repair rate, indicating that effective and timely repairs can mitigate the adverse effects of increased failure rates.

8.2. Growth of Busy Period with Failure Rate
The repairman's busy period (B_0) is depicted in Figure 3, along with the failure rate (λ_1). The graph indicates that when the failure rate rises, the busy period also rises.

This is to be expected since more frequent malfunctions require longer times to fix. Furthermore, the busy period also increases as the repair rate does, indicating that higher failure rates have a significant impact on the repair workload even with enhanced repair capabilities.

8.3. Profit Pattern with Revenue and Cost per Visit
Figure 4 shows the profit pattern for various values of the repairman's cost per visit in relation to revenue (C_0). The financial burden of repair interventions is highlighted by the fact that profit declines as the cost per visit rises. Therefore, keeping repair visits profitable requires effective cost management. It shows that as the cost of repairs increases, profit declines, highlighting the financial strain of repair interventions.

This relationship emphasizes the need for effective cost management to maintain profitability. To keep repair visits financially viable, organizations should consider optimizing repair processes, negotiating better service rates, and implementing preventive maintenance to reduce costly repairs. Additionally, exploring ways to enhance revenue during repair activities can contribute to overall profitability.

8.4. Profit versus Failure Rate and Repair Rate
The relationship between profit (P) and failure rate (α_1) for various repair rates is depicted in Figure 5. Even though the failure rate varies, the profit rises with higher repair rates. This emphasizes how crucial effective repair procedures are to maintaining profitability in spite of the difficulties brought on by system malfunctions.

In summary, the graphical interpretation shows that system availability and MTSF performance decrease with increasing failure rates but that these declines can be avoided with good repair techniques.

Increased failure and repair rates add to the repairman's workload, and increased repair efficiency leads to profitability, which is impacted by repair costs. These revelations emphasize how vital it is to optimize repair procedures and control expenses in order to guarantee the system's dependability and financial sustainability.

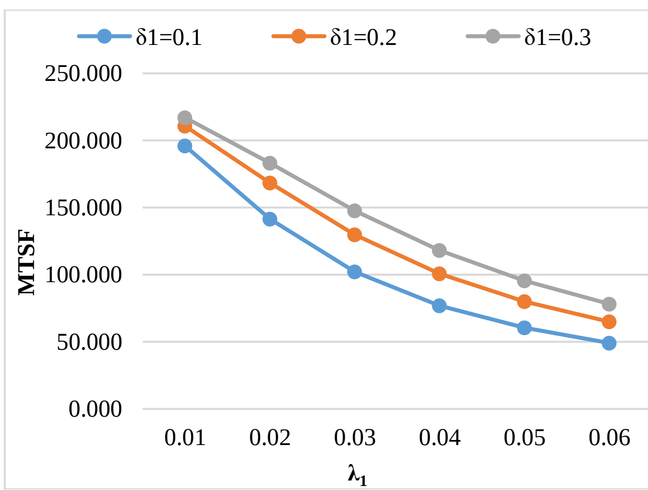

Fig. 1 MTSF versus (λ_1) for different repair rate values(δ_1)

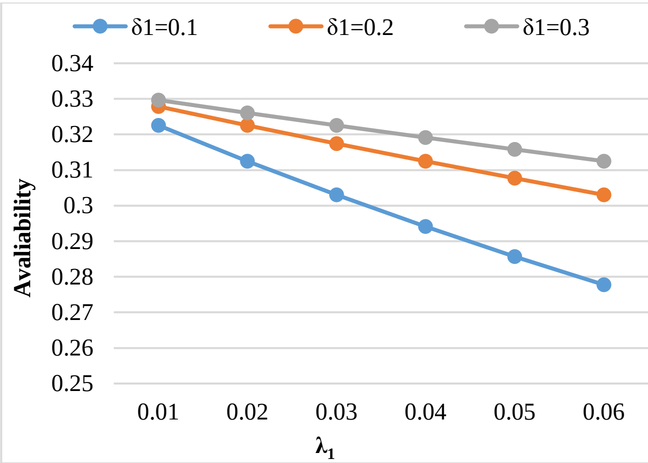

Fig. 2 A_0 versus rate (λ_1) for different values of repair rate(δ_1)

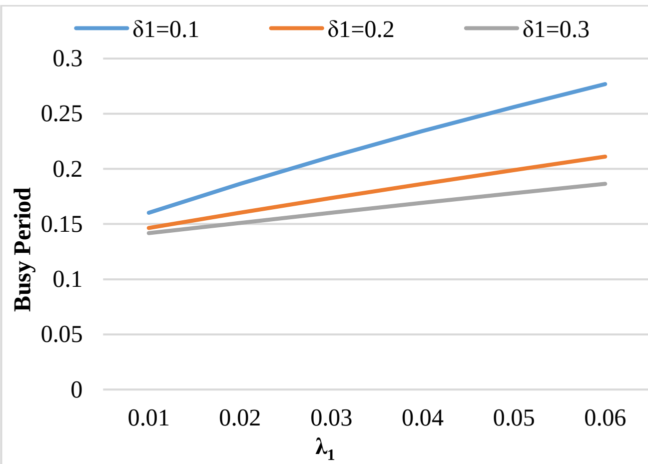

Fig. 3 B_0 versus rate (λ_1) for different values of repair rate(δ_1)

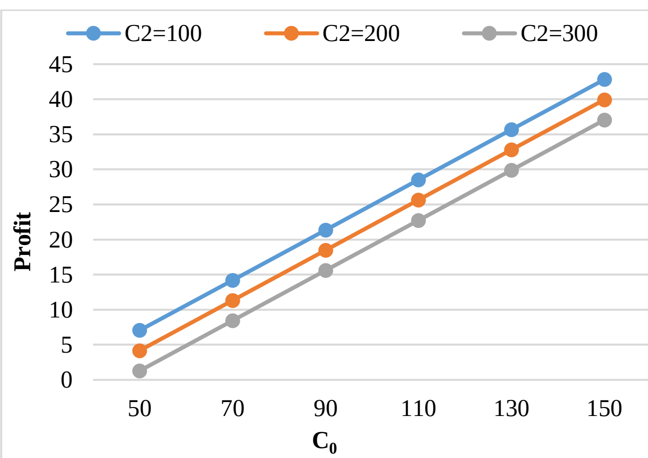

Fig. 4 P versus C_0 for different values of cost per V_0

An understanding of analysis and reliability aids profit development. The building of a robust and stable system necessitates the use of demand forecasting, modes and networks, product design, procurement of components, market research, and organizational capacity to manage operational challenges.

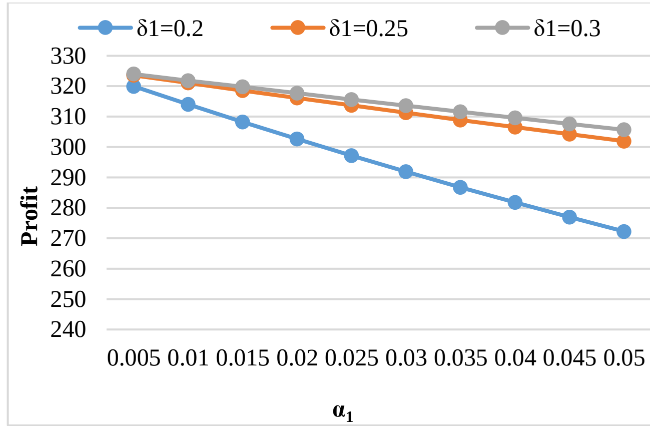

Fig. 5 Profit (P) versus failure rate (α_1) for different repair rates

9. Conclusion

This study conducted a comprehensive reliability analysis of a three-unit parallel system equipped with a backup unit. The focus was on critical performance metrics, including the expected number of visits to the system, the busy period of the repairman, the Mean Time to System Failure (MTSF), and overall system availability. The findings revealed that the system availability is approximately 0.3576, indicating that the system is operational about 35.76% of the time. Additionally, the busy period for the repairman was recorded at 0.1588, suggesting that the repairman is engaged in maintenance activities for roughly 15.88% of the time. The MTSF was determined to be 12.1298, highlighting the average duration the system can function before experiencing a failure. Lastly, the expected number of visits to the system was calculated at 0.0288, reflecting the anticipated maintenance interventions required. The operating probability and performance prediction of engineering systems depend heavily on reliability analysis. As upstream reliability affects downstream operations, reliability metrics are critical for guaranteeing efficacy and efficiency in sectors like crude palm oil supply. A reliable and stable manufacturing process can be established more easily when a variety of reliability models, such as economic, simulation, deterministic, multistage supply, and stochastic models, are used. Manufacturing reliability can be further enhanced by technology and digitization, as they can minimize disruptions and failures. Our graphical interpretations have shown that availability and MTSF improve with higher repair rates but decrease with increasing failure rates. While the rate of failure increases the repairman's workload, an analysis of profit showed that an increase in repair costs results in a decrease in profitability. However, higher repair rates translate into higher profits, demonstrating the financial benefits of raising reliability metrics. The importance of reliability engineering and maintenance optimization in raising system profitability and performance is supported by this study. To further corroborate and improve these results, future studies should look at more complex systems and use multi-objective optimization.

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