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Original Article

Review of the Techniques for Life Prediction and Cumulative Damage of Multiaxial Fatigue Loading

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Abstract *- Engineering parts and discontinuities in structures with geometries can support multipart with variable loads, resulting in multiaxial stress–strain states. This assessment provides a summary of the techniques used to evaluate life prediction methods for multiaxial fatigue loading. This systematic review includes current studies on stress life, strain life, energy, and critical plane methods under constant, variable, and random changing amplitudes of multiaxial fatigue loading. This study provides clear core ideas and the latest advances in multiaxial fatigue analysis. Finally, this review discusses the specifics of various approaches that help researchers work on multiaxial fatigue analysis in real-world engineering applications and the challenges and gaps for future research.*

Keywords *- Critical plane approach, Energy-based approach, Multiaxial fatigue loading, Stress/Strain based approach.*

1. Introduction

Multiaxial fatigue is a critical phenomenon observed in several structures that are used daily [1], [3], [5]. To accurately assess the effects of fatigue loading, it is essential to employ modified models that incorporate specific mechanisms [4], [6], [9], [10], [12]. Local stresses and strains within mechanical structures, specifically at their bears, connections, and joints, often exhibit multiaxial loading stress stat1es [38], [40]. This understanding is particularly crucial when considering the integrity of various unsafe structural components in engineering applications such as engines, turbines, and automotive components that are subjected to complex stresses [6], [7]. Accordingly, undertaking multiaxial fatigue load evaluation of important components has become an energetic area of study, allowing for the most efficient use of the load-bearing abilities of materials [18], [24-30]. Multiaxial fatigue involves several loading conditions that cause structural components to experience complicated stress states [2-5]. These complex load states can result in the cracks [33]. Understanding how those strains impact the life of structures is vital for ensuring their reliability and protection. Understanding multiaxial fatigue loading allows researchers to understand the overall performance of structural components and materials [31], [37]. Traditional fatigue evaluation methods that frequently focus on uniaxial loading scenarios do not sufficiently address the actual working situations experienced by these systems. Multiaxial fatigue evaluation allows for additional complete knowledge of the structural reaction, helping to improve the design, material, and preservation tactics. In addition, improvements in computational techniques and simulation approaches have significantly contributed to the understanding of multiaxial fatigue [41], [51-59]. Finite Element Analysis (FEA) and other numerical processes enable researchers to simulate and calculate the performance of structures under complicated loading situations [48], [76]. These simulations help individuals identify vital areas exposed to fatigue damage, allowing targeted assessments, repairs, or adjustments for increasing the fatigue life of the structure [52].

Multiaxial fatigue is a full-size technical observation aimed at understanding the results of complicated loading situations in structural components. Analysis of multiaxial fatigue is essential to assess the integrity of various structures, particularly those subjected to unsafe conditions and critical applications [54], [91]. Designers and researchers have gained deeper knowledge of multiaxial fatigue by incorporating new models and using superior computational techniques. This permits them to enhance designs, enhance protection, and optimize the use of material load-bearing abilities [12], [24-30]. This failure was prompted by the use of different factors, including different loading values, stress levels, strain ratios, and coupling effects in each loading combination. Compared with uniaxial loading, multiaxial fatigue difficulties are more complicated [6], [7], [109]. Researchers have performed multiple types of fatigue studies, which are categorized into the following three categories:

The first class of multiaxial fatigue research involves crack initiation and growth. It is critical to understand how cracks begin and propagate under complex loading conditions. Accurate estimation requires addressing the complexities associated with multiaxial loading conditions [38], [83]. Multiple factors, including stress concentrations, material types, and loading parameters, are critical for analyzing and predicting fatigue life [20], [24], [26].

Researchers have investigated the impact of various load components on crack propagation and the interplay between cracks. This research enables designers to identify crucial areas at risk of fatigue damage and develop approaches to mitigate the growth of the crack. The second category of multiaxial research focuses on the impact loading parameter. In many real-world applications, systems and components are subjected to dynamic or effective loads that can significantly affect fatigue life [42], [52]. Researchers have examined the consequences of impact loading on multiaxial fatigue failure and expanded the methodologies to include the aforementioned results in fatigue life prediction [36-40].

The third category of research focuses on techniques used to estimate the fatigue life of components to ensure the safety and dependable operation of multiaxial loading. Researchers have improved and developed prediction methods and methodologies that do not overlook complicated multiaxial loading situations [15], [33]. These models incorporate elements such as load ratios, mean stress, load series effects, and combination loads to provide more accurate fatigue life estimations [9-11], [109].

By understanding the complexities of multiaxial fatigue, designers can optimize the layout, material selection, and preservation techniques of structures to improve their strength and safety [22], [34-37]. Multiaxial fatigue research involves examining various causes that affect the failure, including cracks, effective loading parameters, and prediction approaches. Understanding the behavior of systems under complex loading conditions has practical implications for ensuring the reliability and safety of engineering components [40], [65-67]. By conducting studies in these regions, designers can increase techniques to mitigate fatigue, optimize designs, and extend the ability lifecycle of systems exposed to complex loading situations.

In addition to factors referred to in advance, the presence of geometric discontinuities, including notches, material flaws, and other structural complexities, can also complicate the multiaxial alternating nation of stress skilled with the aid of components [12]. These complexities contribute to the predominance of fatigue fractures because of the primary failure mode in mechanical and structural parts, such as those located in aerospace engines and vehicle transmissions [13-14]. During service development, fatigue damage accumulates steadily, compromising the fatigue energy of the building and increasing the possibility of fracture. Consequently, performing multiaxial fatigue analysis in the engineering field has become essential for structural systems. Historically, fatigue assessments under torsion/axial loading conditions with a consistent amplitude load (CA) have been assumed to help clarify real-world engineering systems [7], [18]. Conversely, the utilization of classical uniaxial fatigue models in multiaxial fatigue analysis has been shown to result in full-size errors in engineering systems [9], [20]. Uniaxial models fail to capture the complicated load states and interactions in multiaxial loading situations. Consequently, they frequently offer inaccurate predictions of fatigue life, and this can result in unexpected failures. This highlights the necessity for specialized multiaxial fatigue models that do not overlook the interactions between one-of-a-type load components and different applicable factors.

To address these situations, scholars have developed several multiaxial fatigue models that consider the complexities of actual loading conditions. These models aim to provide more accurate predictions of fatigue life by considering the specific traits of multiaxial strain states, including critical aerospace techniques, energy-based total standards, and multiaxial fatigue damage parameters [102], [108]. By incorporating these advanced models and standards into the analysis, designers can perform more accurate checks of fatigue behavior. This enables them to make knowledgeable selections regarding the layout, preservation, and replacement of these components [88]. The presence of geometric discontinuities, material flaws, and other structural complexities contributes to complicated multiaxial alternating strain states, which can be accomplished with the aid of components in real-world scenarios. Classical uniaxial models are inadequate because they predict multiaxial fatigue, mainly because of the development and models needed to address this challenge. By using these advanced approaches, designers can enhance their understanding of multiaxial fatigue behavior and make more reliable predictions to prevent unexpected failures.

In recent decades, there has been an emphasis on measuring multiaxial fatigue, which has resulted in the development of different experimental techniques [23-28]. These measurement methods include stress, strain, critical plane, and energy-based methods. The critical plane method is closely associated with the stress-life approach [28], [31-35].

The stress-life method involves subjecting specimens or components to different multiaxial stress states, and measuring multiaxial fatigue involves determining the fatigue life. In contrast, the strain-based method replicates the fatigue damage induced by uniaxial loading using similar parameters [41], [55]. Although this method does not provide physical justification, it provides an estimated fatigue life for both multiaxial and uniaxial loading scenarios.

The energy-based method, which has gained popularity since its initiation, is based on the hypothesis of irreversible material degradation and the energy associated with crystal displacements [102], [106]. The energy-based approach considers the dissipated strength during cyclic loading to expose damage as a trademark of fatigue damage [21-23], [26]. To account for this nonproportional loading effect, researchers introduced nonproportional components to the multiaxial damage parameters for proportional loading conditions at the beginning of the study.

These modified damage parameters aim to provide accurate evaluations of fatigue life in multiaxial loading scenarios [29-30], [36], [41-44]. Among the various multiaxial fatigue evaluation approaches, the stress-based total technique has a more potent theoretical foundation. This approach considers irreversible material degradation and the energy related to crystal dislocations as essential elements for predicting fatigue life. This technique has undergone substantial adoption and usage in the field of multiaxial fatigue research [102], [110].

Recent progress in multiaxial fatigue measurement approaches has resulted in the incorporation of nonproportional components as multiaxial damage parameters. While the stress-life method predicts the existence of fatigue without bodily justification, the energy-based technique is grounded in the assumption of irreversible material degradation and the energy of crystal dislocations. This energy-primarily based technique has received recognition and is extensively utilized in multiaxial fatigue studies.

1.1. Fatigue Classification

Fatigue can be categorized as either uniaxial or multiaxial [32], [37]. The detailed classification is shown in figure 1.

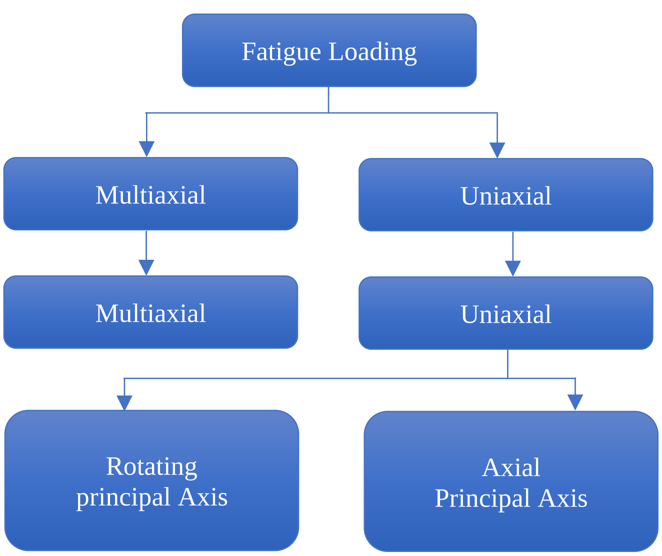

Fig. 1 Fatigue loading classification [37]

2. Loading Condition of Multiaxial Fatigue

Figure 2 illustrates the complicated nature of multiaxial stress, which significantly affects the design and overall performance of structures and components across several industries, including aerospace, motors, and automotive components. It is critical to understand how multiaxial loads behave and interact with various stress components to optimize the overall performance and reliability of these structures [55], [102].

Multiaxial loading involves the simultaneous application of multiple load components from unexpected instructions on a material or structure. Unlike uniaxial stress, it has a single dominant force. The multiaxial load introduces additional complexities and challenges for layout and analysis. The interactions between one of the two types of stress components can lead to stress concentrations, local deformations, and capacity failure mechanisms that are absent in uniaxial stress situations.

To ensure the ultimate performance and strength of systems and components under multiaxial loads, it is crucial to recognize these stress interactions and develop techniques to manage or mitigate their critical consequences [31]. Designers can acquire this knowledge by comprehending how one of several stress components impacts the other and designing materials and structures capable of effectively withstanding multiaxial loading situations.

This may additionally include the selection of suitable materials with suitable mechanical properties, optimization of geometries, and implementation of advanced computational modelling and simulation equipment [58], [76]. As technological advancements continue to increase the demand for complicated and efficient designs, attention to and mitigation of multiaxial stress have become increasingly vital [101].

Multiaxial loading conditions are becoming more common throughout industries, including aerospace, automobiles, and electronics, as producers' goals include enhancing performance, reducing weight, and enhancing typical product performance. Neglecting the results of multiaxial loads can result in unexpected failures, decreased product lifetimes, and compromised safety [82], [102]. Multiaxial loads are complicated phenomena that strongly affect the design and function of systems and components across various industries. Understanding the interactions between different stress components and effectively managing their effects is crucial for maximizing performance and ensuring structural integrity. With the growing demand for complex designs and efficient products, the consideration and mitigation of multiaxial stress will become increasingly critical in engineering and design processes [50], [79-84]. Equation (1) shows the stress-strain relationship.

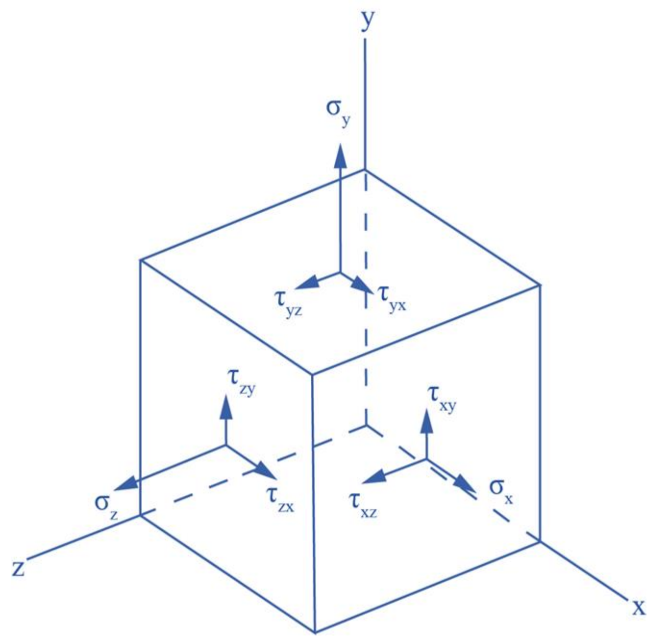

Fig. 2 State of the multiaxial stresses [31]

$$[\sigma] = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix} [\varepsilon] = \begin{bmatrix} \varepsilon_x & \frac{\gamma_{XY}}{2} & \frac{\gamma_{XY}}{2} \\ \frac{1}{2}\gamma_{XY} & \varepsilon_y & \frac{\gamma_{XY}}{2} \\ \frac{1}{2}\gamma_{XZ} & \frac{\gamma_{XY}}{2} & \varepsilon_z \end{bmatrix} \quad (1)$$

$\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{yz}$ *and* $\varepsilon_{xz}, \varepsilon_x, \varepsilon_y, \varepsilon_z, \gamma_{xy}, \gamma_{yz}$*and*$\gamma_{xz}$, as shown in figure 2's X-Y-Z coordinate system.

Metal production commonly occurs under complex loading, which poses challenges in determining the component life cycles, and conducting experimental tests under uniaxial fatigue is comparatively easier [15], [20]. Consequently, researchers have developed several multiaxial fatigue criteria in recent decades to explain the causes of multiaxial fatigue failure, which were initially not well understood [25], [33].

To incorporate static strength theory into the analysis of complex loads, an equivalent stress/ strain concept of the criteria was developed. This method employs empirical or semi-empirical formulations to equate the multiaxial stress variable with an equivalent uniaxial stress variable [38]. An increase in the strain energy of plastic was identified as the root cause of fatigue failure and permanent material damage [47]. The difficulty and cost involved in conducting experiments under multiaxial fatigue have motivated researchers to rely on uniaxial fatigue tests, which are simple to perform. However, this approach does not fully capture the complexity and interactions of multiaxial stress states experienced by metal structures in real-world applications [35]. To address this knowledge gap, researchers have developed multiaxial fatigue criteria that enhance failure in the presence of complex loading conditions.

The formulations used in these criteria are often empirical or semi-empirical and are derived from experimental observations and fitting data. This accumulated energy contributes to fatigue damage and eventual failure. Understanding and quantifying fatigue life is crucial for predicting fatigue life and designing structures with appropriate fatigue resistance [27].

Various approaches have been established in order to address the challenges and costs of multiaxial fatigue testing. These criteria aim to explain the causes and estimate fatigue life using uniaxial data. The accumulation of plastic strain energy is a significant factor in fatigue failure and material damage [36]. These developments have enhanced our understanding of multiaxial fatigue behavior and improved the design of durable and reliable metal structures.

3. Brief Reviews of Multiaxial Fatigue Criteria

3.1. Stress Life Approach and Stress-Based Critical Plane Criteria

The stress-life approach and the stress-based critical plane criterion are two commonly used methodologies in multiaxial fatigue analysis. These approaches offer insights into components subjected to complex multiaxial loading conditions [31], [38]. The stress-life approach, also known as the S-N method, is broadly used for fatigue analysis. In this approach, a single equivalent stress value is derived from the multiaxial stress components using various theories or criteria to represent the stress amplitude [40].

In the multiaxial fatigue analysis, the equivalent stress was compared with the S-N curve obtained from the uniaxial fatigue tests. These curves illustrate the relationship between stress amplitude and fatigue life at different stress ratios. Using the equivalent stress value, the stress-life approach allows the estimation of fatigue life, even in multiaxial stress states [20]. The stress-based critical plane criterion aims to isolate the plane or direction in which fatigue damage is most likely to occur. Unlike the stress–life approach, these criteria consider both the magnitude of stress and its orientation relative to the crystallographic structure of the material [39].

To capture the complex interactions between the stress components leading to fatigue failure, these criteria account for the influence of shear stresses and their interaction with normal stresses on the critical plane. By determining the critical plane, designers can assess the most damaging stress state and make a more accurate estimation of the life of components. Several critical plane criteria have been developed, such as Smith–Watson–Topper (SWT), Findley, Brown–Miller, and Fatemi–Socie criteria [21], [30]. These criteria use mathematical formulations and material-specific parameters to calculate potential critical areas for fatigue damage. Factors such as stress amplitude, mean stress, stress gradient, and material properties are considered to estimate fatigue life more reliably. The stress-based critical plane criterion is valuable when dealing with complex loading conditions in which stress states vary throughout the loading cycle [37]. These criteria help to identify the areas where fatigue damage is likely to happen, providing insights into potential fatigue crack initiation sites. They are useful for designing and assessing structures subjected to multiaxial loading. In contrast, the strain-life approach relies on strain amplitude and fatigue life relation.

It uses equal load values derived from the multiaxial strain components. [49], [67]. In doing so, they offer better expertise in multiaxial fatigue conduct and assist in predicting the existence of fatigue and designing reliable structures. Traditionally, it was believed that brittle materials should comply with the idea of maximum tensile strain, whereas ductile materials must adhere to the von Mises criterion for multiaxial excessive low cycle fatigue (LCF) [41], [53]. However, an opportunity criterion, called the elliptic equation criterion, challenges this differentiation. The elliptic equation criterion offers an opportunity approach for determining the multiaxial excessive cycle fatigue in both brittle and ductile materials.

The behavior of both material types under multiaxial loading situations should be considered, as opposed to conventional assumptions. For brittle materials at critical failure under tensile stress, the most common tensile strain theory is used [71]. This concept states that tragedy happens if the maximum tensile load is greater than the final tensile energy of the material. Moreover, different strain components, such as compressive or shear stresses, cannot be neglected [76]. In comparison, ductile materials that undergo tremendous plastic deformation before failure are regularly analysed using the von Mises criterion. It depends entirely on the concept of equal stress, which combines various stress components. Failure is predicted when the equal strain exceeds an important fee, which is determined by the yield of the material energy [59].

The elliptic equation criterion challenges the differentiation between brittle and ductile materials in a multiaxial excessive cycle fatigue analysis. A unified technique that considers the behavior of each material type is proposed. The criterion uses an elliptic equation that consists of the results of all load components, including tensile, compressive, and shear stresses. By considering the combined effect of all stress components, the elliptic equation criterion provides a complete assessment of the multiaxial stress states [77]. The model aims to capture the crucial strain conditions that lead to fatigue failure regardless of material brittleness or ductility.

The elliptic equation criterion denotes the complicated interactions between stress components and their effects on the accumulation of fatigue damage [83]. This approach offers a more accurate representation of fatigue behavior in brittle and ductile materials under multiaxial loading, allowing advanced predictions of fatigue life and failure analysis. Specifically, traditional approaches, such as the elliptic equation criterion and tensile stress theories, are limited to evaluating the multiaxial fatigue life [88]. This criterion provides a unified framework that considers the combined effects of all stress components to accurately predict fatigue failure regardless of material behavior [91]. Equations (2) and (3) show bending and shear stresses.

$$\left(\frac{\sigma}{\sigma-1}\right)^2 + \left(\frac{\tau}{\tau-1}\right)^2 = 1 \qquad (2)$$

Where, σ-1 and τ-1 represent bending and shear stress, respectively. Following experimental data relating to several materials were used to validate that equation (2) and (3) depend on the biaxial loading phase difference [51]:

$$\left(\frac{\sigma}{\sigma-1}\right)^{\frac{\sigma-1}{\tau-1}A} + \left(\frac{\tau}{\tau-1}\right)^{2A} = 1 \qquad (3)$$

The high cycle fatigue criterion was proposed by equation (4) [52]:

$$\sqrt{J_2} + k\sigma_{H,m} \leq \lambda \qquad (4)$$

k and λ are material constants, and J2 is the second invariant of the stress deviation [58].

The critical plane approach is widely known to accurately estimate fatigue life by considering both crack and fracture processes. This method considers several factors, including the normal tensile strain, shear strain amplitude, and loading conditions [49], [98], [102], and [106], which contribute to its enhanced reliability. This involves evaluating the stress amplitudes, as depicted in Figure 3. These stress components are essential for determining fracture mode failure [50], [109]. By identifying the vital plane, designers can gain insight into the exact location where fatigue damage and crack initiation are most likely to occur. This information is essential for accurately estimating the material life under consideration [102]. The critical plane approach recognizes that the complex interactions between stress and strain structures support failure. Focusing on the critical plane that experiences the most detrimental stress state, this method provides a comprehensive understanding of fatigue behavior [68].

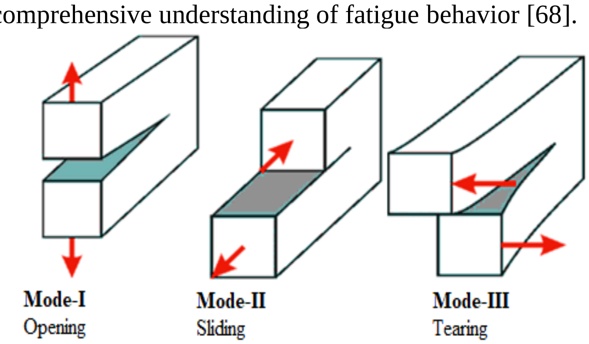

Fig. 3 Material fracture initiation, propagation, and mode of failure of surface cracks [50]

To determine the critical plane involves considering specific loading conditions and material properties. Various criteria, for instance, Smith–Watson–Topper (SWT) Findley, Brown–Miller, and Fatemi–Socie criteria, have been developed to guide this process. In conclusion, the critical plane accurately predicts fatigue life because it considers crack initiation and growth processes [9]. By incorporating tensile and shear normal strain amplitudes and accounting for loading effects, this approach offers a comprehensive understanding of fatigue behavior. Identifying the critical plane, guided by different criteria, allows for a precise estimation of fatigue life and aids in designing structures with improved fatigue resistance [105]. Equation (5) shows maximum normal and shear stresses.

$$max\{\tau_{s,a} + k\sigma_{n,max}\} = f \quad (5)$$

Where the material coefficients are f and k

A fatigue criterion was established depending on the critical plane, which considers the shear and maximum normal stresses. These stresses are crucial for HCF analysis, and equation (6) is used [53], [111]:

$$\left(\tau_{s,a}\right)_{\max} + \frac{\tau_{af}}{2\sigma_{af}}\sigma_{n,\max} = f \qquad (6)$$

The hydrostatic value and amplitude of the shear stresses were determined by a different researcher using the following linear combination-based criterion, equation (7) [54]:

$$\max\left\{\tau_{s,a}\right\} + k\sigma_{H,\max} \le f \qquad (7)$$

Where the material parameters are k, f and $\sigma\, max\left\{\frac{1}{3}\sum_{i=1}^{3}\sigma_{ii}\right\}_{H,max}$, and the hydrostatic stress

The assumptions were established with respect to the angle below and equation (8) [55], [100]:

$$\alpha = 45^{o}\,\frac{3}{2}\left[1-\left(\frac{\tau_{af}}{\sigma_{af}}\right)^{2}\right] \qquad (8)$$

Brown–Miller criterion is a comprehensive approach proposed for predicting fatigue failure in metallic components subjected to cyclic shear and normal strain combinations. This criterion was developed based on theoretical considerations and experimental observations, and several studies have validated it [30], [53], [58], [85], [88-89], [102]. According to the Brown–Miller criterion, the fatigue life of a component decreases exponentially [99]. The Brown and Miller criterion has proven to be an important tool for designers when designing components that are subjected to cyclic shear and normal strain conditions [103], [45]. This approach allows for a comprehensive evaluation of fatigue behavior, considering the interactions between different stress components. This criterion provides a valuable framework for analysing and designing components in various industries where cyclic loading and multiaxial stress conditions are common. By incorporating both shear stress and normal strain effects, designers can optimize their designs and select materials that can withstand the expected cyclic loading conditions [108]. In summary, the Brown and Miller criterion is a comprehensive approach for predicting fatigue failure in metallic components subjected to cyclic shear and normal strain combinations. It considers both shear stress and normal strain, which provides a more accurate estimation of fatigue life. Designers have extensively validated and widely used this criterion to enhance the fatigue life estimation of components in various industries by using equation (9) [17], [30]

$$\frac{\varepsilon_1-\varepsilon_3}{2} = f\left[\frac{\varepsilon_1+\varepsilon_3}{2}\right] \qquad (9)$$

The hypothesis is that the nonlinear relationship between the strain state and fatigue life is generally true. Wang and Brown suggested the following change criterion as shown in equation (10) [45], [46]:

$$\frac{\Delta\gamma_{ns}}{2} + S\Delta\varepsilon_n^{*} = (1+v_e+(1-v_e)S)\frac{\sigma_f'}{E}(2N_f)^b + (1+v_p+(1-v_p)S)\varepsilon_f'(2N_f)^c \qquad (10)$$

Where the normal shear strain (Δγns), fatigue strength exponent (b), fatigue ductility exponent (c), Young’s modulus (E), fatigue strength coefficient (σ’f), number of cycles (Nf), fatigue ductility coefficient (ε’f), elastic Poisson’s ratio (νe) and plastic Poisson’s ratio (νp) are obtained, the Fatemi-Socie criteria are given by equation (11) or (12) [107]:

$$\gamma_{ns,a}\left(1+\frac{k\sigma_{n,\max}}{\sigma_y}\right) = (1+v_e)\frac{\sigma'_f}{E}(2N_f)^b + \frac{k}{2}(1+v_e)\frac{\sigma'^2_f}{E\sigma_y}(2N_f)^{2b} + (1+v_p)\varepsilon'_f(2N_f)^c + \frac{k}{2}(1+v_p)\frac{\varepsilon'_f\sigma'_f}{\sigma_y}(2N_f)^{b+c} \qquad (11)$$

Alternatively, Δγmax of equation (11) can be rewritten as equation (12)

$$\frac{\Delta\gamma_{\max}}{2}\left(1+k\frac{\sigma_{n,\max}}{\sigma_y}\right) = \left[(1+v_e)\frac{\sigma'_f}{E}(2N_f)^b + (1+v_p)\varepsilon'_f(2N_f)^c\right]\cdot\left[1+k\frac{\sigma'_f}{2\sigma_y}(2N_f)^b\right] \qquad (12)$$

When the structure’s stress level is primarily in the elastic region, the stress-life approach performs well in estimating fatigue life, as shown in figure 4.

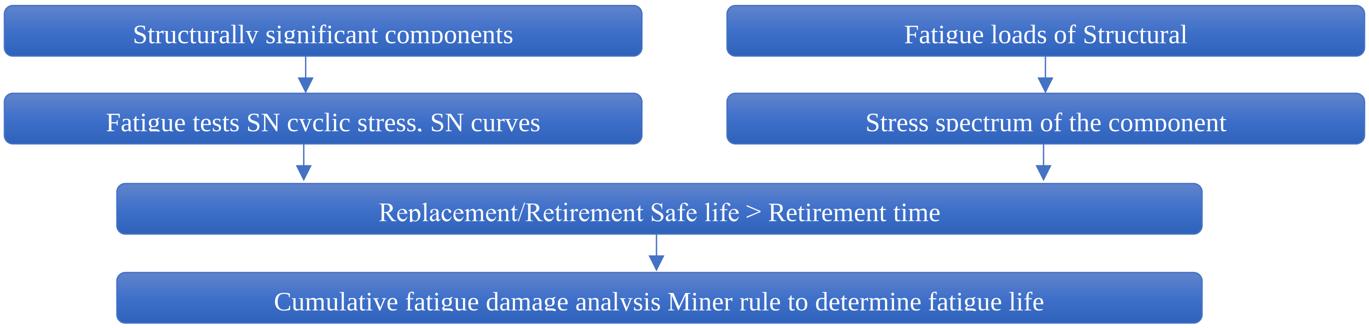

Fig. 4 The general technique for calculating fatigue, the safe life test, involves the use of stress-life methods [56].

The critical plane method focuses on stress measurements to characterize material damage (HCF). It plays a central role throughout the entire process of the critical plane method, and figure 5 likely illustrates a general evaluation structure related to stress-based analyses. In fatigue analysis, this method provides insights into the local stress conditions that contribute to fatigue failure. Researchers have made significant progress in their studies related to the critical plane method and have obtained experimental confirmation of its effectiveness in stress-based analyses [56–57]. By refining their understanding of stress distribution and its influence on fatigue behavior, researchers have been able to apply the critical plane method and models that consider the complex interactions between stress components. An important load course based on strain analysis provides a valuable method for fatigue prediction. This enables the identification of essential regions and mechanisms that are likely responsible for fatigue degradation, resulting in effective mitigation policies and approaches to enhance structural reliability [58]. Researchers working on this challenge have made first-rate progress, obtaining experimental proof of the effectiveness of load-primarily based analysis, critical load paths specializing in stress components and their distribution, critical load paths providing fatigue existence estimate, and advanced knowledge of fatigue behavior at some stage in factors [71]. The stresses are estimated by calculating their equivalent using the formula equation (13) below σeq (t), and Figure 5 shows the critical plane technique based on stress.

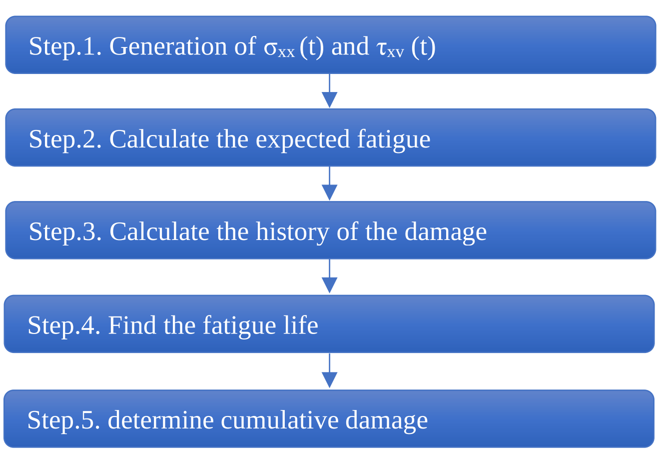

Fig. 5 The critical plane technique based on stress [56]

The damage is indicated by equation (14).

$$\sigma_{eq}(t) = \sigma_{xx}(t)\, sin(2\alpha) + 2\tau_{xy}(t).\, cos(2\alpha) \quad (13)$$

α is the angle that establishes where the critical plane.

$$D_i = \begin{cases} \dfrac{1}{2N_0\left(\dfrac{\sigma_{af}}{\sigma_{eq,i}}\right)^{-\frac{1}{m}}} & ,\sigma_{eq,i} \geq 0.5\sigma_{af} \\ 0 & ,\sigma_{eq,i} \leq 0.5\sigma_{af} \end{cases} \quad (14)$$

The nonlinear cumulative damage method is given by equation (15) [63]:

$$D = \sum_{i=1}^{n} D_i^r \quad (15)$$

Where r is the material constant.

3.2. Strains-life Approach and Strain-Based Critical Plane

Strain-based methods and strain-based critical plane evaluations are outstanding techniques for measuring fatigue failures in materials and structures. In addition, strain-based critical plane approaches are defined in advance; consequently, these techniques specifically focus on stress and its consequences for fatigue behavior. This involves determining the significance of strain skill for the properties of a material or system and relating it to the examined material or system. In the stress-existence approach, fatigue life is usually calculated using SN curves, in which the logarithm (N) is used instead of the logarithm of the amplitude (ε). These curves offer an empirical representation of fatigue conduct and permit designers to calculate the fatigue existence of a thing or tool based entirely on the performed strain amplitudes [90]. This approach considers elastic and plastic suggestions obtained toward cyclic loading. Plastic accumulation and its impact on fatigue damage accumulation are considered.

However, stress components and their effects on fatigue behaviours are considered as opposed to strain components. Significant stress-based research includes the evaluation of enabling the enhancement of fatigue resistance in the format of materials and systems shown by equations (16) and (17) [61], [62], [112]. Strain components and their relationships with fatigue [75]. The essential aerospace is determined using the load that results in the most damage.

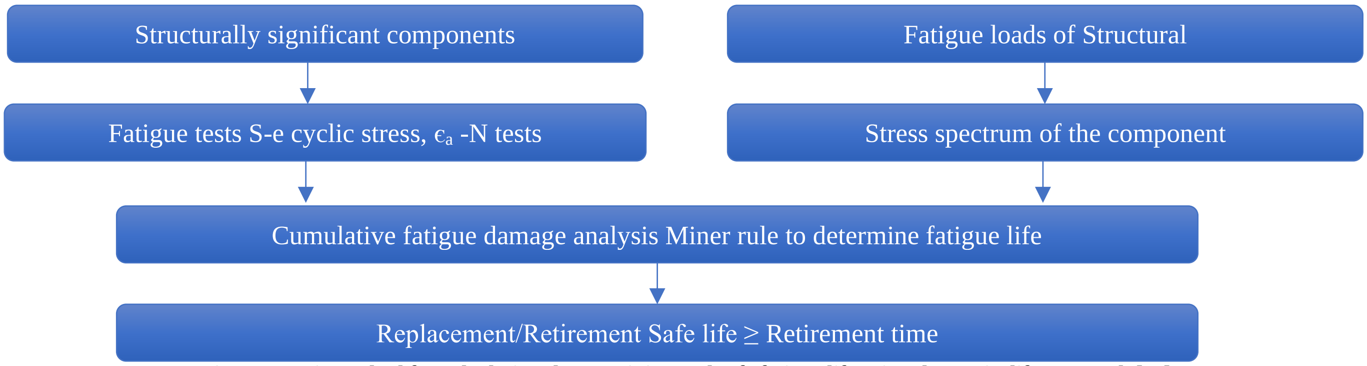

Fig. 6 Generic method for calculating the remaining and safe fatigue life using the strain-life approach [20]

Considering the significance of strain components, stress-primarily based crucial plane evaluation offers insight into commonplace fatigue damage and orientations [101], which offers extra accuracy.

$$\Delta\varepsilon_{eq} = \varepsilon_f'(2N_f)^c \quad (16)$$

$$\Delta\varepsilon_{eq} = \Delta\varepsilon_e + \Delta\varepsilon_p = \frac{\sigma_f'}{E}(2N_f)^b + \varepsilon_f'(2N_f)^c \quad (17)$$

This criterion fails to overlook the effect of load interplay along the load path, making it insignificant [64], [29]. When the impact of solids is calculated, it becomes clear that on common a, everyday loads extensively impact crack initiation and propagation; he proposed the subsequent criterion as follows equation (18) [65]:

$$\gamma_{s,a} = \left(1 + k\frac{\sigma_{n,max}}{\sigma_y}\right) = f \quad (18)$$

Where k and f are the material parameters

A different researcher developed the damage equation (19) or (20) as follows [66], [9]. Strain based multiaxial fatigue model was introduced for both smooth and notched objects [68], [60].

$$\Delta\varepsilon_{eq,a} = \sqrt{\left(\frac{\Delta\varepsilon_n}{2}\right)^2 + \left(\frac{\Delta\gamma_s}{2}\right)^2} = \frac{\sigma_f'}{G}(2N_{fi})^b + \varepsilon_f'(2N_{fi})^c \quad (19)$$

$$D = \sum_{i=1}^{l}\left(\frac{n_i}{N_{fi}}\right)^{\eta_i} \quad (20)$$

It was initially extensively believed that the main planes affecting fatigue life and the maximum damage planes were the planes with the best shear stress, corresponding to the best stresses [68]. These planes have been determined to be crucial in the analysis of fatigue behavior and the foundation of predictive failure. The efficacy of this method was verified through experiments and other experiments.

Researchers observed that with the aid of specializing in these important planes, they were able to identify the areas in a material or structure that had been most seriously cracked in the sequence of fatigue; theory determines this important place. The efficacy of this method has been validated using several experiments. The area of maximum damage in complete fatigue is called the fatigue vital area, calculated using specific techniques or formulas shown in equation (21) [67], [97]. The general procedure used to determine the fatigue life for the strain life approach is shown in figure 6.

$$\frac{\Delta\gamma_i}{2} + k\left(\frac{\sigma n_{n,max}}{E} = \frac{\tau_f'}{G}{}_{fi}{}^{b_0{}'}{}_{f}(2N_{fi})^{c_0}\right) \quad (21)$$

3.3. Energy-Based and Critical Plane Criteria Based on Energy

Linear damage accumulation algorithms are frequently employed in fatigue analysis because of their simplicity. These algorithms are based on three primary assumptions: The material behaves as it would in its original state at the Beginning of each loading cycle. This assumption implies that the material does not undergo any permanent changes or deformation and assumes that the material's response is elastic and reversible [86]. This assumption that the damage accumulated during each cycle remains the same throughout test period and implies that the fatigue behavior of the material is predictable and repeatable. The cycling sequence is independent of the actual sequence, and this assumption assumes that the sequence of loading cycles does not affect the damage accumulation process.

This finding implies that the damage solely depends on the magnitude of the load and not on a specific sequence [78]. These assumptions allow accurate prediction of the failure (HCF) regime, where fatigue failures occur due to microstructural damage and crack initiation. Linear algorithms are widely used in HCF analysis because of their simplicity and satisfactory accuracy in predicting fatigue life [114]. However, these assumptions may not hold true for LCFs because plastic deformation plays a significant role, and the material's behavior deviates from the assumptions of the algorithm. To address the limitations of this approach, a unified theory was proposed to analyze both the HCF and LCF systems. By considering the accumulated strain energy, the unified theory provides a more comprehensive approach to fatigue analysis and prediction [70], [45].

In summary, linear damage accumulation algorithms are commonly used in fatigue analysis because of their simplicity. These methods are based on assumptions that allow accurate predictions in the HCF regime. However, for LCFs, where macroscopic strain is the major cause of failure, a unified theory incorporating the total strain energy density has been proposed to improve fatigue life prediction and damage analysis [70], [45]. The sum of the plastic, ΔWp, and elastic ΔWe strain energies yields the total strain energy per cycle is given by equation 22 as follows:

$$\Delta W_t = \Delta W_p + \Delta W_e \qquad (22)$$

The theory that the plastic component of strain causes damage while the elastic component of tensile stress promotes fracture formation [70] can be applied to both masing and non-masing materials. In this context, masing materials exhibit linear behavior, whereas non-masing materials deviate from this linear behavior. To account for masing and non-masing materials, a master curve can be created [97].

For masing materials, the master curve represents the cyclic response of the material. For non-masing materials, the master curve involves shifting the cyclic response curve along its linear response portion to account for the deviation from linearity [45]. This allows for a unified approach to analyzing and predicting fatigue damage. The hysteresis loops observed in these materials can be attributed to either Masing-type or non-masing-type deformations [71], [45].

Masing refers to materials that exhibit linear results with symmetric hysteresis loops. On the other hand, non-masing-type materials deviate from linearity, leading to asymmetric hysteresis loops. In this context, the plastic component of strain and its accumulation over the loading cycles can be expressed. The specific expression for cyclic plastic strain may vary depending on the material and the analysis method used.

In summary, the plastic component of strain causes damage, and the elastic component of tensile stress promotes fracture formation [99]. By creating a master curve, the cyclic response of the material can be captured, accounting for the deviation from linearity in non-Masing materials. The hysteresis loops observed in these materials can be attributed to either masing or non-masing deformation types given by equations (23) and (24) [38] and [96].

$$\Delta W_P = \frac{1-n^*}{1+n^*}(\Delta\sigma - \delta\sigma_0)\Delta\varepsilon_p + \delta\sigma_0\Delta\varepsilon_p \qquad (23)$$

$$\Delta W_P = \frac{1-n'}{1+n'}\Delta\sigma\Delta\varepsilon_p \qquad (24)$$

Where n* and n' are the cyclic strain hardening exponents of the material curve and the idealized massing material, respectively [45]. Following the performance of fatigue testing under constant strain amplitude, interesting energy-based hypotheses have been proposed [73], [74],[114]. It is important to emphasize the idea that asserts a relationship between the energy density of plastic strain and the rate at which cracks form equation (25) [75-76].

$$r_m = 1 - \sum_{i=1}^{m-1} \frac{W_{fi}}{W_{fm}} . r_i \qquad (25)$$

The Ellyin–Golos approach, which considers progressive damage buildup, was used to develop the nonlinear damage model NLDA model. This model is well suited for practical use and results in significant plastic strain accumulation. However, for metals exhibiting unstable hysteresis behavior, additional consideration must be given to the stress range in addition to the plastic strain range [109]. Unstable hysteresis refers to materials that exhibit asymmetric or non-repeating stress–strain behavior during cyclic loading. In such cases, the stress range, which represents the amplitude of stress fluctuations experienced during cyclic loading, becomes an important parameter for damage evaluation in conjunction with the plastic strain range [78], [79], [45]. For metals with unstable hysteresis, both the plastic strain range and stress ranges must be considered to assess damage accumulation and predict fatigue life accurately.In summary, the Elylin approach accounts for progressive damage buildup. This model experienced low cycles with significant plastic strain. The plastic strain range is a suitable parameter for evaluating damage, especially for metals with stable hysteresis. However, metals are also considered for accurate damage assessment [78], [79], [45]. Cumulative fatigue damage (ψ) is given by equation (26).

$$\varphi = f(\psi, p_m) = \frac{1}{log\ 10\left(\frac{\Delta W_p}{\Delta W_e^+}\right)} \qquad (26)$$

Using the universal slope method, the following equation was derived [80], [81], [111]: where Wp and We are the total plastic strain energy at failure can be calculated by equation (27) [77].

$$\Delta\varepsilon = \Delta\varepsilon_e + \Delta\varepsilon_p = 3.5\frac{\sigma_B}{E}N_f^{-0.12} + \varepsilon_f^{0.6}N_f^{-0.6} \qquad (27)$$

The other authors developed a modified universal slope approach that considers a variety of different alloys [82], [112]. It can be written as follows for different steels (low-alloyed or unalloyed) by using equation (28) [83], [45]:

$$\frac{\Delta\varepsilon}{2} = \frac{\Delta\varepsilon_e}{2} + \frac{\Delta\varepsilon_p}{2} = 1.5\frac{\sigma_B}{E}\left(2N_f\right)^{-0.087} + 0.59\psi\left(2N_f\right)^{-0.087} \qquad (28)$$

Where, $\psi = 1 when \frac{\sigma_B}{E} \leq 0.003 \psi = 1.375 - 125.0\frac{\sigma_B}{E} when \frac{\sigma_B}{E} > 0.003$

The titanium and aluminium alloys are improved as indicated by equation (29) [45]:

$$\frac{\Delta\varepsilon}{2} = \frac{\Delta\varepsilon_e}{2} + \frac{\Delta\varepsilon_p}{2} = 1.67\frac{\sigma_B}{E}\left(2N_f\right)^{-0.095} + 0.35\left(2N_f\right)^{-0.069} \qquad (29)$$

Given that the energy-based idea was in the first state [47] and correctly represented the material's uniaxial fatigue behavior was developed equations (30-32) [83-84] are used. It reads as follows:

$$\Delta W_p = \int_{cycle} \sigma_d \varepsilon_p + \tau_d \gamma_p \quad (30)$$

$$\Delta W_p = A N_f^B \quad (31)$$

Where A and B are the material constants.

$$\Delta W_t = \Delta W_p + \Delta W_e^+ \quad (32)$$

The location of the plastic strain energy density (PSED) is given by equation (33) [84]

$$\Delta W_p = \int_{cycle} \sigma_{ij} \varepsilon_{p,ij}, (i, j = 1,2,3) \quad (33)$$

The elastic strain energy density (ESED) is positive and given by equation (34) [83-84]

$$\Delta W_e^+ = \int_{cycle} \sigma_{ij} \varepsilon_{p,ij} \left(\sigma_{ij} \varepsilon_{e,ij} \geq 0\right), (i, j = 1,2,3) \quad (34)$$

The Ramberg–Osgood relationship, a widely used stress–strain constitutive model, was employed to calculate the generalized strain energy density (W) in the context of evaluating different energy sources [59]. The strain energy density (W) provides a measure of the accumulated elastic and plastic strain energy within a material. In the context of fatigue life prediction for engineering structures, the strain energy density criterion was applied.

This criterion, which is based on the Ramberg–Osgood relationship, allows for the assessment of fatigue damage accumulation and the prediction of the remaining fatigue life of structures. References [76] and [92] likely provide specific examples or applications that demonstrate the use of this criterion in fatigue life prediction for different engineering structures. The specific details and insights provided in Figure 7 illustrate strain energy density. This figure likely provides a graphical representation or correlation between these parameters, highlighting their interplay and influence on fatigue behavior. In summary, the Ramberg–Osgood relationship was used to calculate the generalized strain energy density (W) as a criterion for evaluating different energy sources. This criterion was subsequently applied in equation (35) the prediction of fatigue life for various engineering structures, as evidenced by the reference [76] and [92]. Figure 8 likely provides insights into the influence of these factors on fatigue behaviour.

$$\Delta W_t = \Delta W_p + \Delta W_e = \frac{\Delta\sigma\Delta\varepsilon_p}{1+n'} + \frac{\Delta\sigma\Delta\varepsilon_e}{2} = \frac{4}{1+n'} k' \left(\frac{\Delta\sigma\Delta\varepsilon_e}{2}\right)^{1+n'} + \frac{\Delta\sigma\Delta\varepsilon_e}{2} \quad (35)$$

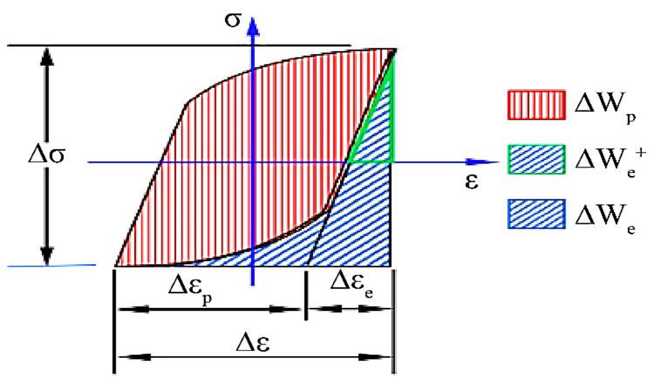

Fig. 7 Diagram illustrating strain energy density [76]

The strain energy density in the critical plane has been proposed as a more effective approach for characterizing fatigue life under uniaxial tension and compression than the energy criterion. While the energy criterion is suitable for evaluating multiaxial fatigue failure mechanisms, it fails to accurately represent the cumulative fatigue failure mechanism due to its scalar nature [87]. This viewpoint has been recognized by other researchers, who have identified two limitations associated with the energy technique. First, the energy technique requires an exact constitutive equation, which may not always be readily available or practical for certain materials or loading conditions. Second, the calculation of plastic stress-energy in the strength approach can be inaccurate when the amount of plastic deformation is small (as indicated through Eq. 36) [88], [89], [82]. To overcome these limitations, strain strength density inside essential aerospace has emerged as a more reliable measure for fatigue prediction below uniaxial tension and compression. By considering the accrued stress-energy density inside the crucial plane, the consequences of both elastic and plastic deformation can be effectively considered, providing a more accurate representation of cumulative fatigue failure mechanisms. Specifically, the load energy density within the critical plane is proposed as a superior method for characterizing fatigue life under uniaxial tension and compression in comparison with the strength criterion. The energy criterion, although suitable for multiaxial fatigue analysis, lacks accuracy in representing cumulative fatigue failure mechanisms because of its scalar nature. Researchers have acknowledged the limitations of this energy technique, including the need for an exact constitutive equation and inaccuracies in plastic strain energy calculations for small plastic deformations. The strain energy density in the critical plane addresses these limitations and offers a more reliable measure for fatigue life prediction, as shown in equation (36) [87], [88], [89], [82].

$$\Delta W_t = \sigma \frac{(\Delta\varepsilon)_{max}}{2}{}_{n,max} \quad (36)$$

A new energy criterion, proposed with the aid of a specific researcher, considers the combined most shear stresses as a sum [90], [94]. A new energy criterion, introduced by a unique researcher, considers the blended elastic and plastic energy densities within aerospace at most shear loads. This criterion considers the sum of those energy densities as a measure for comparing fatigue failure [90], [94]. By considering the maximum shear strain of each of the elastic and plastic energy densities inside the aerospace, mixed results of material deformations inside the direction of cyclic loading can be obtained. The elastic electricity density represents the energy stored inside the material due to elastic deformation, and the plastic strength density reflects or absorbs energy from the material.

The use of combined allows a more comprehensive assessment of fatigue failure mechanisms [89]. It accounts for the relationship between elastic and plastic deformations and is vital in situations where sizable plastic deformation occurs. The precise details and mathematical formulation of this new energy criterion are likely to be provided in equations (37) [90] and [108]. These references may offer further insights into the rationale behind the criterion and its application in fatigue analysis.

In summary, a new energy criterion proposed by different research studies combined plastic and elastic energy. This approach considers the sum of these energy densities as a measure of fatigue failure. By incorporating both elastic and plastic deformations, this criterion provides a more comprehensive assessment of fatigue failure mechanisms.

Further information and details about this criterion can be found in equation (38) [90], [106]. Then, by considering the mean stress, the aforementioned criterion was adjusted as follows Eq. (39) [90]: Another study hypothesized that the maximum shear strain plane behavior contributes differently to failure than normal behavior, and they used a variety of correction techniques to explain material failure as follows [92] accurately, [101], [114].

$$\Delta W_a = \frac{(\Delta\gamma_s)_{max}}{2\frac{\Delta\tau_s}{2}\frac{\Delta\sigma_n}{2}\frac{\Delta\varepsilon_n}{2}} \quad (37)$$

$$\Delta W_a = \frac{(\Delta\gamma_s)_{max}}{2\frac{\Delta\tau_s}{2}\left[\frac{1}{1-\frac{\tau_{s,max}}{\tau_f'}\frac{1}{1-\frac{\sigma_{s,max}}{\sigma_f'}}} = f\right]} \quad (38)$$

$$\Delta W = \frac{(\Delta\gamma_s)_{max}}{2\frac{\tau_{s,a}}{\tau_f'}\frac{2\sigma_{n,max}}{\sigma_y+\sigma_f'\frac{\Delta\varepsilon_n}{2}}} \quad (39)$$

Several critical plane energy models have been developed to assess the constant amplitude fatigue life. In particular, a reliable hysteresis loop can be used to calculate the energy parameters directly. The open hysteresis loop is significantly more complicated for changing the amplitude of multiaxial loading, making it challenging to compute the energy parameters [98] and produce an energy parameter that is equivalent to that. The contributions of normal and shear energy are given varying weights by equation (40) [99]:

$$\Delta W_t = \Delta W_p + \Delta W_e = \frac{1}{2E}.\left(\frac{\Delta\sigma_n^*}{2}\right)^2 + w\frac{1}{2G}\left(\frac{\Delta\tau_n^*}{2}\right)^2 + \int_{cycle}\sigma_n d\varepsilon_n^p + w\int_{cycle}\tau_n d\gamma_n^p$$
$$= \frac{\left(\sigma_f' - \sigma_{n,m}\right)^2}{2E}\left(2N_f\right)^{2b} + 4\left(\sigma_f' - \sigma_{n,m}\right)\varepsilon_f'\frac{c-b}{c+b}\left(2N_f\right)^{b+c} \quad (40)$$

$$D = D_{1^{st}} + D_{2^{nd}} + ...D_{N^{th}} = \sum_{i=1}^{i=N}\left\{\frac{1}{\sigma_f'\varepsilon_f'}\Delta\sigma_n\Delta\varepsilon_n + \frac{1}{\tau_f'\gamma_f'}\left(\Delta\tau_{max}.\Delta\frac{\gamma_{max}}{2}\right)_i\right\}$$
$$= \left[\frac{\sigma_f'}{E}.N_f{}^b + \varepsilon_f'\left(N_f\right)^c\right] + \left[\frac{\tau_f'}{G}.N_f{}^{b_0} + \gamma'\left(N_f\right)^{c_0}\right] \quad (41)$$

$$ESE = \left(\tau_{max} + \frac{\sigma_{n,max\,\varepsilon}^*.\tau_f'}{\sqrt{3}\sigma_f'}\right).\sqrt{3\Delta\varepsilon_n^{*2} + \left(\frac{\Delta\gamma_{max}}{2}\right)^2} = \frac{\tau'^2}{G}\left(2N_f\right)^{2b_0} + \tau_f'.\varepsilon_f'\left(2N_f\right)^{b_0+c_0} \quad (42)$$

The effects of loading memory, loading sequence, and micro stress cycles were examined in the context of other researcher's tests, which considered varying amplitude load scenarios in an operational environment. The highest case occurrence for the loading history was proposed to calculate the block loading history [114], [99]: N denotes the number of peak valleys. The efficacy and precision of the method for gauging fatigue life were further verified through the use of several models on various materials and the comparison of different loading histories. In addition, the algorithm for estimating fatigue life under multiaxial random loading is displayed in Figure 8 [85], [113]. By incorporating research on the critical plane method and the fatigue damage parameter, the equivalent strain amplitude (ESA) model was modified to enable multiaxial fatigue life evaluation, resulting in the formation of model equation (42) [87], [92]. This energy criterion is increasingly being employed as the main approach to calculating life.

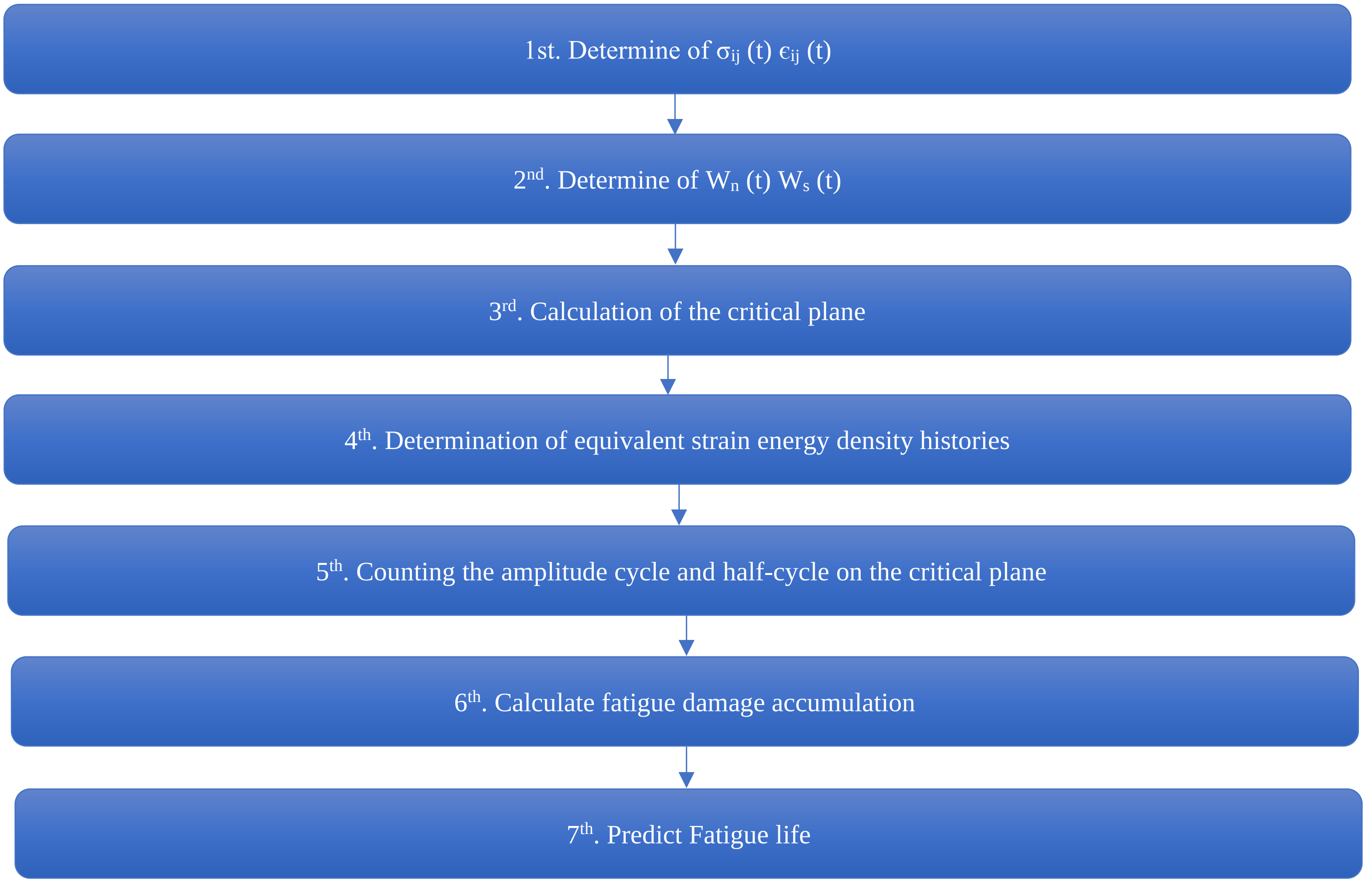

Fig. 8 Algorithm for estimating fatigue life under multiaxial random loading, as described in [85]

4. Method for Cyclic Counting

To accurately predict fatigue life in scenarios involving loading blocks, it is crucial to choose a suitable approach to characterize damage. The cycle counting method is a well-established technique widely studied and implemented for breaking down complex variable amplitude histories into separate loading blocks. Its effectiveness makes it a valuable tool in the field of fatigue life prediction. [93][30], [114], [96], [112]. Linear damage accumulation algorithms are often used because of their simplicity [70], [45].

4.1. The First Approach

To enable cycle counting on various planes, a method has been developed that considers the unique characteristics of each critical plane and incorporates RC cycle counting techniques [96]. Understanding and analyzing the fatigue behavior of materials greatly depends on this relationship [96], [114. Furthermore, in the calculation cycles, auxiliary parameters are collected for each cycle to provide specific information about the loading history. These parameters include key parameters related to the failure mode. Considering shear and ordinary stress records, this approach affords a sophisticated and comprehensive method for cycle calculation and fatigue analysis. Empirical results can be found in [37], [89], [9], [114].

By incorporating these principles and approaches, a more certain understanding of fatigue behavior and cycle values may be obtained.

4.2. Second Approach

The assumption of exceptionally uniform strain has evolved as a solution to overcome the limitations of traditional cyclic estimation techniques. This concept considers relative stresses and uses a vital plane-based total multiaxial cycle calculation approach.

In this technique, load styles representing the beginning and end of each cycle are determined primarily on the basis of the behavior of the uniform stresses, and accurate calculations are centered [35], [48]. The Wang and Brown (WB) cycle counting method is usually used to depend on half cycles from 0 to the most point under uniaxial conditions and to record the converting amplitude in multiaxial cyclic loads [35], [48], [67], [105].

This approach provides a scientific approach for selecting the essential factors within the loading data and deciding on a wide variety of cycles. However, adjustments to the WB method have been proposed in the literature to enhance its applicability and accuracy [97], [111], [113].

1st. Find the equivalent strain history, using the following equation

$$(\varepsilon_{eq}) = \frac{1}{\sqrt{2(1+v)}}\left[(\varepsilon_x-\varepsilon_y)^2 + (\varepsilon_x-\varepsilon_z)^2 + (\varepsilon_z-\varepsilon_y)^2 + \frac{3}{2}(\gamma_{xy}^2+\gamma_{xz}^2+\gamma_{yx}^2)\right]^{1/2}$$

3rd. Calculate the equivalent relative strain as,

$$\varepsilon_{eq}^{r}(t=T_A) = \max_{0\le t\le T} \varepsilon_{eq}\left(\varepsilon_{ij}(t)-\varepsilon_{ij}(t=0)\right)$$

2nd. Rearranging the original loading block and determining the greatest point of the equivalent strain $\varepsilon_{eq}\,(t=T_M) = \underset{\cap<t<T}{m\,ax}\ \varepsilon_{eq}\,(t)$

5th. If the relative equivalent relative strain is monotonically increasing, the procedure ends.

4th. Obtain the updated equivalent relative strain history with respect to point TA1 and proceed to step 3:

$$\varepsilon_{eq,}^{r}2(t) = \varepsilon_{eq}\left(\varepsilon_{ij}(t) - \varepsilon_{ij}(t=0)\right)\quad t\epsilon[0, T_A - T_{A1}]$$

5th. If the relative equivalent relative strain is monotonically increasing, the procedure ends.

Fig. 9 Wang and Brown's cycle counting process [106]

Figure 9 probably presents a complete assessment of the WB cycle counting technique, explaining the key steps and issues concerning this technique. This approach might also illustrate the method of determining half cycles, converting amplitudes in multiaxial loading, and offering insights into the modifications proposed to enhance the technique's effectiveness [56]. In summary, the concepts of relative equivalent strain and critical plane-based multiaxial cycle counting were developed to overcome the limitations of conventional cycle counting methods. The relative equivalent strain approach focuses on comparable relative stresses. It identifies the starting and finishing points of a loading process on the basis of the behavior of the relative equivalent strain. The Wang and Brown (WB) cycle counting method is commonly used, with modifications proposed to increase its accuracy [30]. Figure 9 likely provides a comprehensive overview of the WB cycle counting approach, offering insights into its steps and modifications.

5. Damage Accumulation Requirement

The calculation of fatigue life in engineering structures depends on how the accumulation of damage is estimated. Generally, two main categories, linear and nonlinear, should be considered.

This hypothesis requires two major components: (1) damage from one cycle and (2) damage from all load levels, as well as a critical damage value. For further information on these hypotheses, refer to [97], [99], [103-114].

5.1. Rule of Linear Cumulative Damage

The linear cumulative damage regulation, also referred to as Palmgren–Miner's legal guidelines, implemented for fatigue evaluation of repeatedly loaded mechanical components, has provided an easy method for estimating damage due to cyclic loading over the years. According to the linear accumulation damage law, damage accumulation is assumed to be linearly proportional to the number of loading cycles experienced through the component [112]. This assumption is based on the observation that the fatigue existence of a material is related to the applied load amplitude on the alternative dating or the wide variety of loading cycles it encounters. To observe the rule of linear accumulative damage, the fatigue life of the material, including the stress-life (SN) curve or the fatigue energy coefficient, needs to be recognized. The rule includes calculating the damage contribution from every cycle of loading and summing them to attain the cumulative damage [106]. The damage contribution of every cycle is usually determined by dividing the number of cycles by the fatigue life factor below steady amplitude loading at that strain degree. It is crucial to consider that the guideline for linear cumulative damage assumes that the damage from each cycle is independent and does not consider the results of load relations, stress, or stress records [107]. Although this simplification can also introduce a few inaccuracies under certain conditions, the rule is used extensively because of its simplicity and practicality. The rule of linear cumulative damage represents a simplified technique for estimating the cumulative damage caused by cyclic loading over the years. It is assumed that damage accumulation is linearly proportional to the number of load cycles experienced by a component. Although certain factors and interactions have been overlooked, this approach has been widely applied for fatigue life prediction and assessment. The concept of linear fatigue damage accumulation is presented by the mathematical equation (43) [104-105].

$$D = \sum_{i=1}^{l} \frac{n_i}{N_{fi}} = \frac{n_1}{N_{f1}} + \frac{n_2}{N_{f2}} + \frac{n_3}{N_{f3}} + \ldots\ldots \frac{n_l}{N_{fl}} = 1 \quad (43)$$

The total number of cycles (N_f) needed until fatigue failure occurs when a material is repeatedly subjected to the same amount of stress or strain. This can be described by the linear cumulative damage rule of Miner [113], which states that failure occurs when D equals one, despite ignoring factors such as the load sequence and loading interaction under random loading [106],[107],[112].

5.2. Damage Accumulation in Nonlinear Systems

Damage accumulation in nonlinear systems involves accounting for the effects of nonlinear behavior and material response during cyclic loading. Unlike linear systems, in which the connection between the implemented strain and strain remains constant, nonlinear structures exhibit one-of-a-type loads and load behaviors under different loading conditions; however, additional factors must be considered in cumulative damage evaluation. In fatigue evaluation, Cumulative damage in nonlinear structures can be assessed through various techniques, some of which might be briefly defined below. Instead of depending entirely on elastic strain, stress-based methods do not overlook accumulated plastic loads as a sign of damage. The cumulative impact of damage in nonlinear systems may be accurately estimated by monitoring plastic accumulation. Energy-based methods, which include critical aerospace load energy density, offer an alternative to different load-based methods. These techniques do not overlook the strength dissipation or absorption of the material during cyclic loading. By calculating both elastic and plastic deformation loads, the methods provide particular damage evaluation in nonlinear structures. Continuous Damage Mechanics (CDM) is a mathematical version of damage development and accumulation mouth in matter. This model considers material damage because of fatigue loading and consists of damage variables in the structural equations. Considering cumulative damage and fatigue in nonlinear structures, researchers should consider increasing material density [108].

Nonlinear Finite Element Analysis (FEA) techniques simulate the change of systems under cyclic loading, consider the nonlinearity of localized stress response measurements of the failure mechanism of damage accumulation in nonlinear structures, and might provide insights [102].Accumulation damage analysis for nonlinear structures is generally more complicated than that for linear structures because it requires consideration of nonlinear material behavior, plastic deformation, and feasible interactions between loading cycles. In summary, cumulative damage in nonlinear systems under cyclic loading considers the results of nonlinear behavior and material response over time [103].

All of the above methods involve nonlinear finite detail evaluation, which is an analytical method for determining the amount of damage accumulation in nonlinear structures. The damage curve concept with capacity for dynamic load evaluation was proposed by individual researchers [25], [29], [14], [107]. Consecutive scholars have built on this idea, proposing a nonlinear damage hypothesis with an exponential method [108], [112], [94]. A variable associated with the ‘i' th loading value, The variable ηi tends to increase slowly as the stress level increases. Several nonlinear damage accumulation models have subsequently been developed [106].

$$\Delta W_p = \frac{1 - n^*}{1 + n^*} (\Delta\sigma - 1) \delta\sigma_0 \Delta\varepsilon_p + \delta\sigma_0 \Delta\varepsilon^p \qquad (44)$$

$$D = \sum_{i=1}^{l} \left(\frac{n_i}{N_{fi}} \right)^{\eta_i} \qquad (45)$$

6. Conclusion

When designing engineering structures, the impact of multiple variables and constant amplitude stress should be considered when calculating fatigue life. An accurate assessment of complex multiaxial loading is essential for developing fatigue design theories and methodologies. The use of energy has grown in popularity for evaluating fatigue because of its effectiveness in describing the fatigue damage process. Nevertheless, a more accurate method for calculating energy must be developed. Fracture initiates and grows from different orientations, as found by research on cracks under multiaxial loading based on the type of material and loading condition. Thus, different types of criteria have been created despite all of these; there is still no globally applicable standard for different materials and loading approaches. The critical plane criterion can be used to measure the status of damage under a direct loading path. Multiaxial loading presents a unique challenge for the rain flow counting method. Many academics have realized significant achievements in the HCF system of multiaxial stress and increasing stress life, and often, the stress equivalence method is applied to evaluate the damage. In contrast, low-cycle fatigue systems often use the strain-based method because of multiaxial constant amplitude fatigue, which results in strain accumulation. The constant amplitude fatigue is physically represented by the critical plane of the energy-based model. Research needs to be conducted to develop a deterministic connection between stress-strain histories under variable and random loading conditions and the corresponding fatigue damage. In addition to that, the latest multiaxial modes should be incorporated into the FEM software.

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