International Journal of Engineering Trends and Technology
ISSN: 2231–5381 / https://doi.org/10.14445/22315381/IJETT-V72I4P124

Volume 72 Issue 4, 226-237, April 2024


*Original Article*

# New Hybrid Critical Plane Approach for Fatigue Life Estimation and Damage Assessment of Multiaxial Cyclic Loading

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Received: 20 November 2023 Revised: 20 February 2024 Accepted: 01 April 2024 Published: 24 April 2024

***Abstract*** *- To ensure products can withstand multiple cycles of loading over a long period of time, researchers are continually motivated to develop new models. Existing critical plane methods and fatigue life prediction models are often not precise enough for complicated multiaxial loading scenarios. The investigation into the creation of a new hybrid stress-strain critical plane model is very crucial. The hybrid stress-strain critical plane model signifies a paradigm shift in fatigue estimate stress and strain parameter combinations, particularly in components exposed to multiaxial cyclic loading. This model hybridizes the stress and strain parameters to determine the likely plane of crack origination. Thus, the method provides a more precise prediction of fatigue life and damage assessment parameters. The new model has been validated using both numerical and MATLAB simulations, which establishes the credibility and practical applicability of this innovative hybrid stress-strain critical plane model. The proposed critical plane model offers a dependable and original approach for estimating fatigue in components undergoing multiaxial cyclic loading. Its integration of stress and strain parameters provides a holistic perspective, ensuring accurate predictions and advancing the understanding of fatigue mechanisms. This hybrid critical plane model is the original approach for fatigue estimation in components with multiaxial cyclic loading.*

***Keywords*** *- Fatigue, Multiaxial fatigue model, Hybrid critical plane approach, Damage assessment parameter.*

## 1. Introduction to Multiaxial Fatigue Models

Engineering components often face multiaxial fatigue failure due to the diverse load conditions they experience, especially in machinery where external factors and geometric considerations contribute to this phenomenon. Unlike uniaxial fatigue, the analysis of multiaxial fatigue is complex, involving complicated considerations of stress and strain states, load histories, and fatigue damage assessment parameters governing the component's fatigue life. Other components often experience complex and time-variable loading conditions, leading to multiaxial stress and strain states that can damage the assessment of the material near crack initiation sites. Traditional modeling approaches focused on uniaxial loading may not be sufficient, encouraging researchers to develop methodologies to assess fatigue lives accurately under complex loading paths. However, evaluating multiaxial fatigue is challenging due to testing complexities, material behavior, and other factors, with no universally accepted model in the literature. Understanding non-proportional loading conditions, where the principal stress and strain axes vary, is essential. Non-proportional loading can lead to additional cyclic hardening, affecting fatigue life. Constitutive equations have been developed to model non-proportional cyclic behavior, with attention given to predicting cyclic hardening. The use of non-proportionality factors is crucial in accounting for additional hardening. However, the key challenge remains in developing multiaxial fatigue damage assessment criteria based on loading history and material properties, with various models proposed but no standardized method established. Despite decades of research, there is a lack of a universally accepted multiaxial fatigue damage assessment parameter, with proposed parameters having limitations tied to specific scenarios [1-5]. Multiaxial fatigue damage assessment parameters generally fall into three categories: stress-based, strain-based, and energy-based. Stress-based parameters are suitable for high-cycle fatigue where plastic deformation is minimal, relying on material coefficients derived from experiments [32]. Strain-based approaches excel in low-cycle fatigue where plastic deformation is significant, and they can be extended to both low and high-cycle fatigue regimes, as stress-based and strain-based methods converge in high-cycle fatigue.

Energy-based damage assessment parameters, considering strain energy as a fatigue damage assessment quantity, express the product of strain and stress components. While existing literature thoroughly reviews these parameters, this paper introduces two original fatigue damage assessment parameters based on the critical plane approach to address observed limitations in current models. Proposed original fatigue damage assessment parameters aim to contribute to ongoing efforts to enhance accuracy and reliability in multiaxial fatigue assessment, highlighting the need for continued research in this complex and crucial domain.

Understanding fatigue loading is crucial for researchers due to its potentially serious consequences. The excess of available models presents a challenge in selecting the most suitable one, as they not only differ in the equations they offer but also in the critical criteria they utilize [1], [2-4]. Researchers can enhance their decision-making by gaining knowledge about multiaxial fatigue commonly observed in everyday structures. In this context, various fatigue models have been proposed for engineering applications, each addressing specific aspects of fatigue phenomena. Among the noticeable fatigue models, the Smith-Watson-Topper (SWT) model stands out for its utilization of the innovative concept of equivalent stress and strain energy. This model verifies particularly effective in calculating fatigue life, especially under the demanding conditions posed by complex loading scenarios [3-5], [30]. The Brown-Miller Model takes into account the decrease in strength caused by a nonzero stress mean, incorporating a stress mean correction factor [9], [31]. It offers several advantages, including its simplified approach for easy implementation and understanding, suitability for quick fatigue life estimations in scenarios with limited data, provision of a good initial approximation for certain loading conditions, and versatility across various materials and loading scenarios in engineering analyses.

However, the model also has limitations, such as reliance on simplifying assumptions that may not fully capture real-world fatigue behavior complexity, limited accuracy compared to advanced models in complex scenarios, lack of consideration for local stress and strain conditions in critical areas, potential inaccuracies in high-cycle or low-cycle fatigue predictions, and limited validation with experimental data that may impact the reliability of fatigue life predictions in practical applications. Another significant model is the Fatemi-Socie Model, which is based on the strain-life method and employs a critical plane approach to determine the life under multiaxial fatigue loading conditions [24]. This model based on the strain-life method with a critical plane approach offers significant advantages in fatigue analysis under multiaxial loading conditions, including consideration of the complex nature of multiaxial loading, accurate prediction of fatigue life by analyzing the critical plane, and versatility in application to various materials and loading scenarios. However, the model also faces limitations such as computational complexity, data requirements for accurate predictions, sensitivity to parameters and assumptions, and challenges in validation under real-world multiaxial loading conditions, which may impact its practical implementation and reliability in engineering applications [24], [29]. The Smith-Miller-Neuber Model combines the principles of SWT and Neuber's rule, offering an estimation of fatigue life in situations involving multiaxial fatigue loading while considering the influence of local plasticity on fatigue damage assessment [31]. The Smith-Miller-Neuber Model offers a comprehensive approach by integrating the principles of the Smith-Watson-Topper (SWT) model and Neuber's rule. This combination allows for accurate estimations of fatigue life in multiaxial fatigue loading situations, with a specific focus on considering the impact of local plasticity on fatigue damage assessment [31].

The Smith-Miller-Neuber Model offers several advantages in fatigue analysis, including its comprehensive approach that integrates the Smith-Watson-Topper model and Neuber's rule to consider both high-cycle and low-cycle fatigue aspects, improved accuracy through the combination of established models, versatility in application to various materials and loading scenarios, consideration of local stress and strain conditions for a detailed understanding of fatigue failure mechanisms, and validation through experimental data, enhancing reliability in practical engineering situations. However, the model also has limitations, such as increased complexity due to the integration of multiple models, data requirements for accurate predictions, reliance on assumptions that may not always hold true, sensitivity to input parameters that could impact predictions, and limitations in extreme loading conditions or for materials with behavior deviating significantly from the model assumptions. When selecting a fatigue model for a particular material and loading scenario, the Carpinteri-Spagnoli Model emerges as a noteworthy choice [16], [22]. It is a critical plane model used for predicting fatigue failure under multiaxial loading conditions. It offers a comprehensive analysis by considering both normal and shear stress components on the critical plane.

The model can handle non-proportional loading, is sensitive to material properties, and considers the influence of mean stress on fatigue life. It has been successfully applied to various materials and loading conditions, making it versatile for engineering applications. However, the model has limitations such as computational complexity, reliance on assumptions that may not capture real-world complexities, and the need for validation with experimental data. It may also have a limited scope in extreme loading conditions or for materials with non-linear behavior. While the Carpinteri-Spagnoli model provides a thorough approach to predicting fatigue failure, its accuracy and applicability depend on the specific characteristics of the material and loading conditions being analyzed. This model is designed to provide reliable predictions by tailoring its considerations to the unique characteristics of the material and loading conditions in question. In contrast to the models that incorporate strain and stress mean amplitude, several others exhibit limitations in effectively addressing the particulars of multiaxial stress states and cyclic plasticity [6], [8], [29]. Although these models take into account both strain and stress mean amplitudes, their failure to consider the complex interaction of multiaxial stress states and cyclic plasticity imposes constraints on their accuracy in predicting fatigue life. Conversely, strain-based approaches, exemplified by the Coffin-Manson relation, are characterized by a focus on strain as the primary determinant of fatigue life [12], [14-17]. However, a notable drawback of these approaches is the lack of consideration for stress state effects. By concentrating predominantly on the strain, they overlook the significant influence that variations in stress states can exert on the fatigue behavior of the material.

In essence, while models incorporating strain and stress mean amplitudes may overlook multiaxial stress states and cyclic plasticity, strain-based approaches like the Coffin-Manson relation may fail to adequately capture the impact of stress state variations on the material's fatigue response. The choice between these models necessitates careful consideration of the specific loading conditions and material characteristics to ensure accurate fatigue life predictions. Generally, navigating the diverse array of fatigue models requires a comprehensive understanding of multiaxial fatigue and careful consideration of each model's unique features. The SWT, Brown-Miller, Fatemi-Socie, and Smith-Miller-Neuber models each contribute distinct perspectives, enabling Researchers to make informed decisions tailored to the specific challenges posed by complex loading conditions.

Critical plane-based models, especially those incorporating the consideration of Strain Energy Density (SED), introduce a distinct perspective in fatigue analysis. These models pinpoint critical planes where fatigue damage assessment initiation occurs, and the strain energy density is quantified for these planes. The acquired information is then employed to formulate estimations for fatigue life [21-26]. By directing attention to critical planes and integrating the concept of strain energy density, these models provide valuable insights into the specific regions within the material that are most disposed to fatigue damage assessment.

In general, the choice of a fatigue model is contingent upon the specific characteristics of the material and the loading scenario under consideration. While comprehensive solutions are offered by models such as the Smith-Miller-Neuber Model and Carpinteri-Spagnoli Model, others may exhibit limitations in addressing factors like multiaxial stress states, cyclic plasticity, or stress state effects. Researchers must conduct a thorough evaluation of the distinctive requirements of their applications to discern and select the most appropriate fatigue model for accurate predictions and reliable fatigue life assessments [7], [13].

The approaches that consider multiaxial stress states demonstrate capability but often rely on empirical formulations to ascertain the material's 'critical plane.' This reliance introduces heightened complexity and computational costs to simulations [1-5], [8-10], [20-23]. While these models provide a means to account for multiaxial stress states, the determination of the critical plane through empirical formulations adds an additional layer of intricacy to the analysis. Conventional fatigue models were initially developed based on uniaxial fatigue data, limiting their applicability when confronted with non-proportional multiaxial loading conditions. The rotation of principal stress/strain axes in non-proportional loading introduces extra damage assessment mechanisms not adequately addressed by these conventional models [17].

Consequently, the effectiveness of these models diminishes in scenarios involving complex loading conditions. For a fatigue model to be robust, it must not only consider both stress and strain effects but also account for non-proportional loading conditions. Additionally, the model should be calibrated based on a minimum number of physically meaningful parameters to ensure broad applicability across diverse materials and loading conditions [18], [27-28]. This holistic approach aims to enhance the model's versatility, making it adept at capturing the complexities of fatigue under various circumstances while maintaining computational efficiency.

## 2. Limitations of Existing Critical Plane Model

Current critical plane models exhibit several shortcomings that hinder their ability to accurately predict fatigue life in complex loading conditions [15]. These limitations arise from their reliance on stress or strain parameters independently, neglecting their combined effect. Furthermore, the identification of the critical plane solely based on maximum shear stress or strain amplitude disregards other critical parameters. Additionally, these models often overlook material properties and mean stresses, restricting their practical utility [21-24].

To enhance the precision of fatigue life predictions across various materials, a comprehensive approach is essential. This involves considering material-dependent fatigue strength, mean stress correction factors and critical plane rotation. In response to these considerations, a hybrid critical plane approach is proposed, integrating shear stress and shear strain criteria across multiple planes [6], [11]. This approach takes into account material properties, mean stresses, and critical plane rotation, leading to improved accuracy and a broader range of applicability. The hybrid critical plane model aims to overcome the limitations of existing models by incorporating a more holistic set of factors, offering a more comprehensive and accurate prediction of fatigue life under diverse loading conditions. Ensuring the reliability of the proposed hybrid critical plane model in industrial applications necessitates a robust validation process. In this regard, simulations and two numerical examples have been incorporated, and their results are compared with experimental data. This comprehensive validation approach is crucial to instil confidence in the model's accuracy and effectiveness. Therefore, the proposed hybrid critical plane model, backed by thorough validation, holds the potential to significantly enhance the precision in determining the critical plane most likely to experience damage assessment under multiaxial loading conditions. By aligning model predictions with empirical data through rigorous validation, researchers and practitioners can trust the model's reliability and make more informed decisions in designing components subjected to complex loading scenarios in industrial settings.

Glinka et al. proposed a damage assessment parameter that can be written in the form of equation (1):

$$GlinkaDamageparameter(G_{DP}) = \frac{\Delta\gamma_{12}}{2}\cdot\frac{\Delta\sigma_{12}}{2}\left(\frac{\tau'_f}{\tau'_f-\sigma_{12}^{max}} + \frac{\sigma'_f}{\sigma'_f-\sigma_{12}^{max}}\right)_{max} \quad (1)$$

Fatemi–Society proposes fatigue damage assessment parameter of multiaxial fatigue criterion, which uses shear based can be represented mathematically by equation (2) and it involves multiaxial loading conditions [27]. Maximum fatigue damage assessment parameter (FS) is observed that involves parameters such as shear strain amplitude, Δγ/2, σy-yield stress, and K-constant material.

$$FP_{FS} = \frac{\Delta\gamma_{max}}{2}\left(1 + k\frac{\sigma_{max}^{n}}{\sigma_y}\right) \quad (2)$$

## 3. New Hybrid Critical Plane Model

The developed hybrid critical plane model presents an innovative and advanced methodology for precisely predicting the critical plane most likely to experience damage assessment, thereby enhancing the durability and reliability assessment of components subjected to multiaxial cyclic loading conditions.

Equation (3) mathematically defines three crucial parameters related to normal and shear strains/stresses [4]. This hybrid critical plane model stands out for its ability to offer a comprehensive understanding of the intricate interactions occurring in multiaxial loading scenarios. By accurately identifying the critical plane most susceptible to damage assessment, the model provides valuable insights into potential failure mechanisms andenables researchers to make informed decisions regarding the design, durability, and reliability of components under diverse multiaxial cyclic loading conditions. The incorporation of equation (3) [4] further enhances the model's precision by quantifying critical strain and stress parameters essential for thorough fatigue analysis.

$$[\sigma] = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix}, [\varepsilon] = \begin{bmatrix} \varepsilon_x & \frac{\gamma_{xy}}{2} & \frac{\gamma_{xy}}{2} \\ \frac{1}{2}\gamma_{xy} & \varepsilon_y & \frac{\gamma_{xy}}{2} \\ \frac{1}{2}\gamma_{xy} & \frac{\gamma_{xy}}{2} & \varepsilon_z \end{bmatrix} \quad (3)$$

### *3.1. Three-Dimensional Principal Stresses*

The principal stresses can be denoted by (σ') to distinguish them from other forms of stress (σ). Mathematically expressed in equation (4) [8]:

$$\sigma' = q.\sigma.q^T \quad (4)$$

Where q is the transformation matrix

### *3.2. Transformation stress coordinates in three dimensional geometry*

It is essential to have an understanding of coordinate transforms when dealing three dimensional geometries. These transformations are necessary to manipulate in three-dimensional space (5) properly [15]

$$\sigma' = \begin{bmatrix} \sigma'_{aa} & \sigma'_{ab} & \sigma'_{ac} \\ \sigma'_{ab} & \sigma'_{bb} & \sigma'_{bc} \\ \sigma'_{ac} & \sigma'_{bc} & \sigma'_{cc} \end{bmatrix} = \begin{bmatrix} q_{aa} & q_{ab} & q_{ac} \\ q_{ab} & q_{bb} & q_{bc} \\ q_{ac} & q_{bc} & q_{cc} \end{bmatrix} \begin{bmatrix} \sigma_{aa} & \sigma_{ab} & \sigma_{ac} \\ \sigma_{ab} & \sigma_{bb} & \sigma_{bc} \\ \sigma_{ac} & \sigma_{bc} & \sigma_{cc} \end{bmatrix} \begin{bmatrix} q_{aa} & q_{ba} & q_{ca} \\ q_{ab} & q_{bb} & q_{cb} \\ q_{ac} & q_{bc} & q_{cc} \end{bmatrix} \quad (5)$$

By using equation (6) calculate the maximum shear stress in both two dimensional and three dimensional of any point by considering the maximum and minimum principal stresses [15].

$$\tau_{max} = \frac{\sigma_{max} - \sigma_{min}}{2} \quad (6)$$

### *3.3. Three-Dimensional Principal Strains*

The principal strains (ϵ) can be expressed as ϵ1, ϵ2, and ϵ3 and the normal strain components are typically used ϵaa, ϵbb, and ϵ33, which can be calculated by equation (7) [21].

$$\varepsilon' = q.\varepsilon.q^T \quad (7)$$

Where q is the transformation matrix

### *3.4. Transformation of Strain Coordinates in Three-Dimensional*

The transformation strain coordinates provide a quantitative measure of the deformation undergone by an object under a transformation. They are often used to describe the properties of materials subjected to different types of transformations, such as stretching, shearing, or rotating and expressed by equation (8) as follows [12]:

$$\varepsilon' = \begin{bmatrix} \varepsilon'_{aa} & \varepsilon'_{ab} & \varepsilon'_{ac} \\ \varepsilon'_{ab} & \varepsilon'_{bb} & \varepsilon'_{bc} \\ \varepsilon'_{ac} & \varepsilon'_{bc} & \varepsilon'_{cc} \end{bmatrix} = \begin{bmatrix} q_{aa} & q_{ab} & q_{ac} \\ q_{ab} & q_{bb} & q_{bc} \\ q_{ac} & q_{bc} & q_{cc} \end{bmatrix} \begin{bmatrix} \varepsilon_{aa} & \varepsilon_{ab} & \varepsilon_{ac} \\ \varepsilon_{ab} & \varepsilon_{bb} & \varepsilon_{bc} \\ \varepsilon_{ac} & \varepsilon_{bc} & \varepsilon_{cc} \end{bmatrix} \begin{bmatrix} q_{aa} & q_{ba} & q_{ca} \\ q_{ab} & q_{bb} & q_{cb} \\ q_{ac} & q_{bc} & q_{cc} \end{bmatrix} \quad (8)$$

### *3.5. Maximum Shear Strain*

It is simple to compute the maximum amount of shear strain at any point from the principal strains. Mathematically, it can be calculated using equation (9), and this equation can be applied to both two and three dimensional geometry [21].

$$\gamma_{max} = \varepsilon_{max} - \varepsilon_{min} \quad (9)$$

The application of multiaxial fatigue stresses to critical components in various engineering systems, including engines, turbines, discs, blades, and automotive systems, is a common occurrence. Moreover, the localized stresses and strains at the assumptions, connections, and joints of mechanical structures are often subjected to multiaxial loading [2], [3], [4], [5].

Therefore, conducting multiaxial fatigue load analysis becomes an essential task for engineering components exposed to complex loads, ensuring the full utilization of the load-bearing capabilities of materials. The new approach presented in this context involves the integration of critical planes for both strain and stress, aiming to identify the plane with the maximum likelihood of crack initiation.

This innovative methodology provides a comprehensive solution for analyzing multiaxial fatigue in critical components. To facilitate a deeper understanding of the model, the authors include a mathematical derivation and solved numerical examples. In the context of stress criteria, the critical plane stress criterion is active by examining $\tau_{max}$ (maximum shear stress) and $\sigma_m$ (mean stress). Simultaneously, the strain criterion involves the consideration of γmax and $\varepsilon_m$.

### *3.6. Proposed Damage Assessment Parameter*

The mathematical expression of the proposed damage assessment parameter ($D_{prop}$) captures the combined influence of both linear and nonlinear effects induced by mean stress, further enhancing the model's capacity to comprehensively assess damage assessment in materials subjected to cyclic loading conditions.

The damage assessment parameters, determined through stress-strain relationships, were essentially characterized as normal equivalents, tangential equivalents, or a combination of tangential and normal equivalents. Consequently, these parameters needed to be compared with the corresponding fatigue characteristics. When amplitudes or maximum values are employed in the parameter, the fatigue characteristics are derived from the classic Manson-Coffin-Basquin characteristic. Assume the elastic and plastic strain amplitudes are given by equation (10) and (11), respectively.

$$\varepsilon^{a,e} = \frac{\sigma'_f}{E}\left(2N_f{}^{b}\right) \quad (10)$$

$$\varepsilon^{a,p} = \varepsilon'_f\left(2N_f{}^{c}\right) \quad (11)$$

The strain amplitude (εa) is the sum of equation (10) and (11) as follows:

$$\varepsilon^{a} = \varepsilon^{a,e} + \varepsilon^{a,p} = \frac{\sigma'_f}{E}\left(2N_f{}^{b}\right) + \varepsilon'_f\left(2N_f{}^{c}\right) \quad (12)$$

Where c is an exponent of the plastic fatigue strain, and b is the exponent of the shear fatigue limit.

The Basquin fatigue damage assessment parameter considering stress ($BD_p$) and shear are given by equations (13) and (14), respectively.

$$B_{Dp(Stress)} = \sigma'_f\left(2N_f{}^{b}\right) \quad (13)$$

$$B_{Dp(Shear)} = \tau'_f\left(2N_f{}^{bo}\right) \quad (14)$$

The elastic and plastic shear strain amplitudes are given by equations (15) and (16), respectively.

$$\gamma^{a,e} = \frac{\tau'_f}{G}\left(2N_f{}^{b_0}\right) \quad (15)$$

$$\gamma^{a,p} = \gamma'_f\left(2N_f{}^{c_0}\right) \quad (16)$$

Similar to equation (12), the shear strain amplitude ($\gamma^a$) is the sum of equations (15) and (16) as follows:

$$\gamma^{a} = \gamma^{a,e} + \gamma^{a,p} = \frac{\tau'_f}{G}\left(2N_f{}^{b_0}\right) + \gamma'_f\left(2N_f{}^{c_0}\right) \quad (17)$$

Where co is an exponent of the plastic fatigue shear, and bo is the exponent of the shear fatigue limit. γa is the shear strain amplitude, $\gamma^{a,e}$ is the elastic shear strain amplitude and $\gamma^{a,p}$ is the plastic shear strain amplitude. On the basis of equations (12) and (13), it is possible to determine the damage assessment parameter defined in the normal stress component and can be written as equation (18).

$$DamageParameternormalcomponent(DP^{n}) = \frac{\sigma'^{2}_f}{E}\left(2N_f{}^{2b}\right) + \sigma'_f.\varepsilon'_f\left(2N_f{}^{b+c}\right) \quad (18)$$

Similarly, from equations (14) and (17), the shear damage assessment parameter in the tangential component ($DP^t$) can be obtained and is given by equation (19):

$$Tan\,g\,entialDamageParameter(DP^{t}) = \frac{\tau'^{2}_{f}}{G}\left(2N_{f}^{\,2b_{o}}\right) + \frac{\tau'_{f}.\gamma'_{f}}{2}\left(2N_{f}^{\,b_{o}+c_{o}}\right) \quad (19)$$

From the normal strain amplitude of equation (12) and the shear strain amplitude of equation (17), it is possible to calculate strain damage assessment parameters ($DP^{\varepsilon\gamma}$) as follows:

$$DP^{\varepsilon\gamma} = \frac{\sigma'_{f}}{E}\left(2N_{f}^{\,b}\right) + \varepsilon'_{f}\left(2N_{f}^{\,c}\right) + \frac{\tau'_{f}}{G}\left(2N_{f}^{\,b_{0}}\right) + \gamma'_{f}\left(2N_{f}^{\,c_{0}}\right) \quad (20)$$

The proposed damage assessment parameter ($DP^{prop}$) due to normal and tangential components can be found in equations (18) and (19) as follows:

$$Pr\,o\,posedDamageParameter(DP^{prop}) = \frac{\sigma'^{2}_{f}}{E}(2N_{f}^{\,2b}) + \sigma'_{f}.\varepsilon'_{f}(2N_{f}^{\,b+c}) + \frac{\tau'^{2}_{f}}{G}\left(2N_{f}^{\,2b_{o}}\right) + \frac{\tau'_{f}.\gamma'_{f}}{2}\left(2N_{f}^{\,b_{o}+c_{o}}\right) \quad (21)$$

The proposed hybrid approach given by equation (21) stands out as a sophisticated and comprehensive method for assessing fatigue damage assessment in components subjected to multiaxial loading. This equation is used to find the critical plane and predict the fatigue life. The maximum value of the proposed damage assessment parameter indicates the place where the critical plane occurs.

## 4. Cumulative Fatigue Damage Assessment

### *4.1. Miner's Rule*

Predicting fatigue damage assessment in structures under variable loading is complex. The primary model, linear damage assessment (Miner's rule), can lead to non-conservative life predictions. It overlooks the impact of load sequence on cyclic fatigue damage assessment accumulation. In fatigue analysis, the cumulative damage assessment due to cyclic loading can be calculated using the Miner's rule. The rule is a fundamental concept in cumulative fatigue analysis.

It assumes that each stress repetition at a specific stress level results in equal damage assessment. The consumed fatigue life is directly related to the total number of cycles endured. Generally, it states that the fatigue life of a component is reached when the cumulative damage assessment from all the applied stress cycles reaches a value of 1. By applying Miner's Rule, researchers can estimate the cumulative damage assessment caused by various stress/load cycles, aiding in the prediction of a component's fatigue life. Under Miner's Rule in cumulative fatigue analysis, the order of stress/load cycles does not impact the results.

However, in reality, changing the loading sequence, especially in the presence of plasticity, can affect the fatigue life of a component. This discrepancy highlights a limitation of Miner's Rule and emphasizes the importance of considering the actual loading sequence in certain scenarios. Miner's rule in cumulative fatigue analysis offers simplicity and ease of application, making it a valuable tool for estimating fatigue life based on stress cycles. It provides a straightforward method to predict component durability and assess potential failure risks.

However, a limitation of Miner's Rule is its assumption of equal damage assessment from stress repetitions, which may not always reflect real-world conditions accurately. Additionally, the rule does not account for the varying effects of different stress levels on fatigue life, potentially leading to conservative or optimistic estimations of component longevity.

By summing up the ratio of the number of cycles at each stress level to the corresponding fatigue life at that stress level, we can determine if the cumulative damage assessment exceeds 1, indicating failure. This analysis helps in predicting the fatigue life of a component subjected to cyclic loading and is crucial in designing components to withstand repeated stress cycles without failure.

### *4.2. Carpinteri and Spagnoli Criteria*

The authors proposed the following nonlinear relationship to be estimated on the critical plane, which is given by equations 22-24 [37-38].

$$\left(\frac{Shear\ stress\ amplitude\ over\ one\ cycle\ (\tau_{\alpha})}{Shear\ stress\ amplitude\ endurance\ limit\ for\ fully\ reversed\ torque\ (\tau_{\alpha t})}\right)^{2} + \left(\frac{Maximum\ normal\ stress\ to\ shear\ stress\ over\ one\ cycle\ plane\ (\sigma^{n}_{max})}{unaxial\ amplitude\ of\ endurance\ limit\ for\ fully\ reversed\ tension\ (\sigma_{\alpha e})}\right)^{2} \leq 1 \quad (22)$$

As per the Carpinteri and Spagnoli, the critical plane is to be found ϕ0 equation (23) from the final fracture plane in such a way that

$$\phi^{0} = 45^{0} * \frac{3}{2}\left[1 - \left(\frac{\tau_{at}}{\sigma_{ae}}\right)^{2}\right] \quad (23)$$

It is possible to describe the criteria with a damage assessment value to damage assessment the criteria equal to a damage assessment value (D) as expressed in equation (24) [38].

$$\left[\left(\frac{\tau_{a}}{\tau_{at}}\right)^{2} + \left(\frac{\sigma^{n}_{max}}{\sigma_{ae}}\right)^{2}\right]^{\frac{1}{2}} \quad (24)$$

The Carpinteri and Spagnoli criterion is not restricted to analysis on the surface but can be applied at points anywhere within the bulk of the structure.

### 4.3. Mc Diarmid Criteria

The author proposed the linear relationship as shown in equation (25) [39].

$$\left(\frac{Shear\ stress\ amplitude\ over\ one\ cycle\ (\tau_\alpha)}{Shear\ stress\ amplitude\ endurance\ limit\ for\ fully\ reversed\ torque\ (\tau_{\alpha t})}\right)^2 + \left(\frac{Maximum\ normal\ stress\ to\ shear\ stress\ over\ one\ cycle\ plane\ (\sigma_{max}^n)}{2(ultimate\ stress\ (\sigma_u))}\right)^2 = 1 \quad (25)$$

### 4.4. The Proposed Damage Assessment Model

Calculating the multiaxial fatigue loading damage assessment value is also proposed, which considers two criteria through the introduction of a weighting parameter (λ) and the material constant (n), as described in equation (26). This parameter is instrumental in identifying the damage assessment value (Dprop), signifying the cumulative effect of both stress and strain on the material.

The inclusion of a mathematical derivation and solved numerical examples not only enhances the clarity of the model but also provides researchers with practical insights into its application. The weighting parameter (λ) adds a layer of adaptability, allowing for a nuanced evaluation of the combined effects of stress and strain, making the model a valuable tool in optimizing the design and performance of critical engineering components.

$$D^{prop} = \lambda(\tau max - \tau f')^n + (1-\lambda)\,(\gamma max - \gamma f')^n \quad (26)$$

τ'f-Shear stress fatigue limit, γ'f –shear strain fatigue limits and n is being the material constant. It is taken as the ratio of fatigue strength exponent to fatigue ductility exponent. The establishment of the novel proposed damage assessment model occurs by varying the weighting parameter (λ) from 0 to 1. This dynamic hybrid model is designed to incorporate both stress and strain considerations within the range of $0 < \lambda < 1$. When the damage assessment amounts to 1, the material breaks down, and the index can be between 0 and 1, which makes it possible to figure out the life of different load amplitudes. The incorporation of both stress and strain parameters, represented by the variable λ, contributes to the model's versatility and applicability in scenarios involving complex loading conditions.

As a result, this hybrid model presents a robust and effective approach for enhancing fatigue life predictions in engineering applications. Additionally, this considers the linear and nonlinear effects of mean stress that cause damage assessment. In general, the novel hybrid critical plane model stands as an effective and versatile tool for indicating the most likely initiation of cracks or critical plane, predicting fatigue life and assessing damage assessment in components subjected to multiaxial variable amplitude loading conditions than models based on stress or strain alone.

## 5. Validation of the Hybrid Critical Plane Approach

Certainly, the validation process for the new hybrid approach involves gathering and evaluating experimental data from various sources. In this case, data from kinds of literature [15], [19], [22], [27] were considered, and simulations were performed using the Math Lab simulation during the validation process. Additionally, two numerical cases were simulated and the results were compared with the gathered experimental data. Table 1 provides the weight percentage of the chemical composition present in 316-stainless steel, and Table 2 outlines its properties [17]. These tables provide essential information about the material properties and composition that are crucial for the simulation and validation process. The use of both experimental data and numerical simulations, along with the comparison of results, adds rigor to the validation of the new hybrid approach, ensuring its accuracy and reliability in predicting the behavior of 316-stainless steel under various conditions.

**Table 1. Weight percentage of the chemical composition**

| Mo | Ni | C | Si | Mn | P | S | Cr |
|---|---|---|---|---|---|---|---|
| 2.03 | 10.15 | 0.06 | 0.46 | 1.33 | 0.32 | 0.25 | 16.97 |

**Table 2. Properties of 316-stainless steel** [17]

| Mechanical Property | Value |
|---|---|
| Yield stress, $\sigma_y$ | 240 Mpa |
| Ultimate stress, $\sigma_u$ | 297 Mpa |
| Modulus of Elasticity, E | 193 Gpa |
| Shear Modulus, G | 75 Gpa |
| Elastic Poisson's ratio, $\nu_e$ | 0.3 |
| Plastic Poisson's ratio, $\nu_p$ | 0.5 |
| Fatigue strength coefficients, $\sigma'_f$ | 663.25 Mpa |
| Fatigue strength exponent, b | -0.093 |
| Fatigue ductility coefficients, $\varepsilon'_f$ | 0.1895 |
| Fatigue ductility exponent, c | -0.4657 |

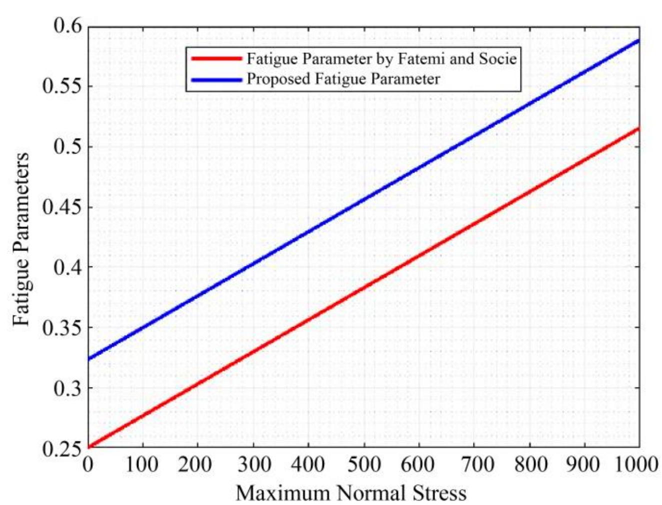


**Fig. 1 Proposed Fatigue damage assessment Parameter Vs Fatigue Parameter (FP) by Fatemi and Socie**

The model demonstrated reasonable accuracy in predicting the critical plane under different multiaxial loading spectrums. The proposed Fatigue damage assessment Parameter Vs Fatigue Parameter (FP) by Fatemi and Socie is shown in Figure 1.

The proposed hybrid model stands out as the most reliable approach for calculating the fatigue damage assessment parameter when contrasted with the Fatigue Parameter (FP) introduced by Fatemi and Socie, as depicted in Figure 1. In the assessment of accuracy, the error calculation relies on equation (27) [17].

This comparison suggests that the hybrid model surpasses the Fatigue Parameter (FP) methodology in terms of reliability and precision in calculating fatigue damage assessment. Figure 1 likely illustrates a visual representation of the comparative performance, emphasizing the superiority of the proposed hybrid model. The utilization of equation (27) [17] for error calculation provides a quantitative measure of the disparity between the predictions of the hybrid model and the Fatigue Parameter (FP).

This analytical approach aids in precisely assessing the efficacy of the proposed hybrid model, establishing its superiority as a more dependable method for fatigue damage assessment parameter determination. In essence, this assertion underscores the significance of the proposed hybrid model, presenting it as a preferable and more accurate tool for researchers engaged in fatigue analysis when compared to the existing Fatigue Parameter (FP) proposed by Fatemi and Socie.

$$Error = Log_{10}(CalculatedValue) - Log_{10}(ExprimentalValue) \qquad (27)$$

### *5.1. Solved Numerical Cases by the Hybrid Critical Plane Model*

*5.1.1. Case 1. Thick-Walled Cylinder under Axial and Torsional Loading*

Consider the thick-walled with inner radii of 60mm and the outer radii of 120mm, which is subject to a constant axial stress ($\sigma_a$) and a cyclic torsional shear stress ($\tau$).Material properties are E=20Gpa, v = 0.25, σy = 530 MPa, σu = 750 MPa. At the inner surface, the axial and shear stresses are estimated to be 70 MPa and ±190 MPa, respectively, while at the outer inner surface, the axial stress is 45 Mpa and shear stress ±90 MPa. The hybrid model is effectively applied to calculate the critical plane and its associated component life. The estimated life of the critical plane, identified at an angle of 45 degrees based on crack initiation criteria, is determined to be 14,530 cycles.

This prediction is then compared with the result from experimental testing, which records a life of 11,985 cycles. Utilizing equation (27) for error analysis, the calculated error is found to be 0.0836. The observed error of 0.0836 signifies a relatively small discrepancy between the predicted and experimental fatigue life values. This finding underscores a strong correlation between the predictions made by the hybrid model and the actual outcomes from experimental tests.

The calculated error serves as a quantitative measure of the model's accuracy, demonstrating its capability to reliably estimate the fatigue life of the critical plane under consideration. In this case, the hybrid model showcases its effectiveness by providing a predicted fatigue life that closely aligns with experimental results. The small error indicates a robust correlation, emphasizing the model's accuracy and reliability in estimating component life under specified loading conditions. This validation supports the practical applicability of the hybrid model in fatigue life prediction, reinforcing its value in engineering analyses and design optimizations.

*5.1.2. Case 2. Notched Plate under Tension-Compression Loading*

Consider a notched plate subjected to tension compression with a ratio of R = 0.1. Its material properties are comprised of E= 210Gpa, $\sigma_y$ = 460Mpa, $\sigma_u$ = 835 MPa. Additionally, its nominal stress range is $\Delta\sigma_{nom}$ = ±200 MPa. The hybrid model's application involved identifying the critical plane at the notch root, determining a 75-degree angle and predicting a fatigue life of 9,241 cycles. This prediction was compared to an experimental result of 7,865 cycles, yielding a calculated error of 0.0700 by using equation (27).

Despite a small discrepancy, the findings indicate a strong correlation between the model's predictions and experimental outcomes. The model effectively estimated critical plane angles and fatigue life, showcasing its ability to capture complex material interactions. Notably, the hybrid critical plane model demonstrated versatility, estimating life and orientation components across diverse dimensions and loading conditions. Case studies involving thick-walled cylinders and notched plates were presented and systematically compared with experimental data for comprehensive validation. The model's effectiveness in correlating with experimental results positions it as a valuable tool for researchers in fatigue life and damage assessment. Its adaptability to various scenarios further solidifies its role as a reliable asset in structural analysis and design in real-world engineering applications.

## 6. Validation Proposed Damage Assessment Model

To validate a proposed damage assessment model, two multiaxial fatigue criteria are used: the McDiarmid criterion and the Carpinteri and Spagnoli criterion [32-39]. The McDiarmid criterion is distinguished by its assessment of multiaxial loading conditions. Equation (24) most likely captures the mathematical expression of this criterion.

By comparing the Mc Diarmid criteria predictions to the proposed damage assessment equation (26), one can evaluate how well the proposed equation represents the intricacies of multiaxial fatigue behavior [29]. The Carpinteri and Spagnoli criterion takes into account multiaxial stress states, and equation (25) is most likely the mathematical representation of this criterion.

By comparing this criterion's predictions to the proposed damage assessment model, one can assess the proposed equation's capacity to account for mean stress effects and multiaxial stress situations. The proposed fatigue damage assessment criterion, represented by equation (26), is the focus of the investigation. This criterion most likely introduces new approach criteria for forecasting fatigue damage assessment under multiaxial loads. Researchers may validate the novel equation's effectiveness and accuracy in capturing multiaxial fatigue behavior by comparing its predictions to the existing McDiarmid and Carpinteri-Spagnoli criteria.

In summary, the Mc Diarmid, Carpinteri, and Spagnoli estimations are compared with the Proposed Fatigue Damage assessment criterion to validate the proposed damage assessment model (26). This comparison will aid in determining the suggested criterion's strengths and limits in capturing the intricacies of multiaxial fatigue, as well as its performance in contrast to other well-established field criteria. The validation method comprises applying several types of loads to determine the accuracy of the suggested fatigue damage assessment equation. These loads are Proportional Bending/Torsion, Non-proportional Bending/ Torsion, Non-proportional Axial/ Torsion, and Non-proportional Pressure/ Axial.

Each type of load poses distinct problems and stress distributions that can influence the fatigue behavior of the materials under consideration. The validation data for each load type is categorized and given in Table 3.This table is likely to include load magnitudes, loading conditions, and experimental results relating to fatigue performance. The experiment results are used to compare the fatigue criteria's predictions to the actual behavior observed during the tests. This comparison will assist in determining the proposed criterion's strengths and limits in capturing the complexities of multiaxial fatigue, as well as its performance in contrast to well-established field criteria.

$$\text{Error \%} = (1-D)\, 100\% \tag{28}$$

## 7. Result and Discussion

The proposed damage assessment parameter is precisely applied to a series of case studies, with the primary objective of establishing correlations with distinct sets of experimental data. The intention is to assess the predictive efficacy of the damage assessment parameters across a spectrum of loading conditions.

The fundamental focus lies in comparing fatigue life estimation derived from these damage parameters with the corresponding experimental fatigue data sets. This comparative analysis serves as a critical evaluation, shedding light on the predictive accuracy and reliability of the proposed damage assessment parameters under diverse modes of loading. These case studies are carefully designed to encompass a variety of loading scenarios, ensuring a comprehensive examination of the damage assessment parameter's performance across different conditions. The evaluation involves scrutinizing the correspondence between predicted fatigue life values and the actual fatigue life data obtained from rigorous experimental tests. Through this systematic comparison, the researchers aim to gauge the robustness and versatility of the proposed damage assessment parameter.

The scrutiny includes assessments under various modes of loading, encompassing tension, compression, torsion, or combinations thereof. By subjecting the damage assessment parameter to such a rigorous examination against experimental data, researchers aim to explain its ability to accurately capture the intricate nuances of material behavior and fatigue response. The outcome of this comprehensive evaluation is pivotal for establishing the credibility and applicability of the proposed damage assessment parameter in real-world engineering scenarios. Any gaps or concordances observed between predicted and experimental fatigue life values contribute valuable insights into the parameter's strengths and limitations, paving the way for potential refinements and enhancements in the realm of fatigue life prediction methodologies.

Tables 4, 5, 6, and 7 present different materials used in the validation process, such as 6082-T6 Al, 76S-T61Al, 30NCD16 Steel, and 0.35% C. Each material may exhibit unique fatigue characteristics under different loading conditions, and understanding these properties is crucial for accurate fatigue life predictions. The accuracy of the fatigue damage assessment predictions is assessed by calculating the error for each load type based on Equation (28).

The error calculation likely involves comparing the predicted fatigue life using the proposed damage assessment equation with the experimental results obtained from the validation tests. The error metric helps quantify the inconsistency between the predicted and observed fatigue behavior. Table 4-7 shows the predicted value of all models. In each table, the least error value for each load type and the mean error value indicates the most accurate result. This result is highlighted by being bold and underlined, indicating that it represents the closest match between the predicted value and the actual experimental data. Selecting the least error as the accurate result helps to identify the performance of the proposed fatigue damage assessment model under different loading conditions and material types.

**Table 3. Load cases summary and all stresses are described in Mpa**

| Author | Material Used | Type of Test | $\tau_{at}$ | $\sigma_{ae}$ | $\sigma_u$ |
|---|---|---|---|---|---|
| Findley WN. [33] | Criteria for 76S-T61 Al | Proportional Bending/ torsion | 110 | 171 | 500 |
| Froustery C and Lasserre .S [34] | 30NCD16 Steel | Non-proportional Bending/ torsion | 411 | 662 | 1161 |
| Rotvel F [35] | 0.35%C Steel | Non-proportional Axial/torsion | 137 | 216 | 560 |
| Petron and Susmel [36] | 6082-T6 Al | Non-proportional Pressure/axial | 88 | 146 | 344 |

**Table 4. Considering the material 6082-T6 Al [32]**

| | | | **Mc Diarmid** | | | **Carpinteri and spagnoli** | | | **Proposed fatigue Damage assessment** | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\sigma_y$ | $\tau_{xy}$ | $\varphi$ | $\tau_a$ | $\sigma_{max}^n$ | Error (%) | $\tau_a$ | $\sigma_{max}^n$ | Error (%) | $\tau_{max}$ | $\gamma_{max}$ | $\lambda$ | n | Error (%) |
| 19.4 | 82.3 | 0 | 82.3 | 17 | **2.13** | 88.5 | 16.78 | **2.612** | 78.6 | 0.035 | 0.3 | 0.19 | **1.85** |
| 55.2 | 87.9 | -2 | 92.05 | 28.64 | **4.6** | 88.6 | 33.48 | **3.34** | 105.1 | 0.057 | 0.3 | 0.19 | **3.14** |
| 66 | 98 | 124 | 96.2 | 57.2 | **11.5** | 96.4 | 65.52 | **8.7** | 86.5 | 0.064 | 0.3 | 0.19 | **4.27** |
| 122 | 75 | 0 | 94.5 | 67.8 | **9.26** | 98.2 | 67.4 | **5.62** | 92.4 | 0.081 | 0.3 | 0.19 | **3.89** |
| Mean error | | | | | 6.873 | Mean error | | 5.068 | Mean error | | | | 3.288 |

**Table 5. Considering the material 76S-T61Al [33]**

| | | | **Mc Diarmid** | | | **Carpinteri and spagnoli** | | | **Proposed fatigue Damage assessment** | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\sigma_y$ | $\tau_{xy}$ | $\varphi$ | $\tau_a$ | $\sigma_{max}^n$ | Error (%) | $\tau_a$ | $\sigma_{max}^n$ | Error (%) | $\tau_{max}$ | $\gamma_{max}$ | $\lambda$ | n | Error (%) |
| 174 | 0.0 | - | 82 | 86 | **10.2** | 84 | 102 | **3.3** | 580.2 | 0.035 | 0.3 | 0.19 | **2.218** |
| 162 | 34 | 0 | 90 | 81 | **12.5** | 86 | 80 | **12.4** | 689.7 | 0.062 | 0.3 | 0.19 | **5.306** |
| 125 | 62 | 0 | 88 | 104 | **9.6** | 102 | 59 | **13.2** | 790.4 | 0.078 | 0.3 | 0.19 | **7.041** |
| 80.4 | 95.7 | 0 | 105.2 | 41.5 | **6.4** | 100.9 | 64 | **6.8** | 863.5 | 0.092 | 0.3 | 0.19 | **4.340** |
| Mean error | | | | | 8.075 | Mean error | | 8.925 | Mean error | | | | 4.726 |

**Table 6. Considering the material 30NCD16 Steel [34]**

| | | | **Mc Diarmid** | | | **Carpinteri and spagnoli** | | | **Proposed fatigue Damage assessment** | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\sigma_y$ | $\tau_{xy}$ | $\varphi$ | $\tau_a$ | $\sigma_{max}^n$ | Error (%) | $\tau_a$ | $\sigma_{max}^n$ | Error (%) | $\tau_{max}$ | $\gamma_{max}$ | $\lambda$ | n | Error (%) |
| 220 | 365 | 0 | 380 | 401 | **10.02** | 374.2 | 310.3 | **1.3** | 250.8 | 0.028 | 0.3 | 0.19 | **1.142** |
| 230 | 385 | 90 | 387 | 530 | **14.3** | 336.5 | 574.3 | **10.6** | 462.7 | 0.041 | 0.3 | 0.19 | **4.155** |
| 363.8 | 142 | 45 | 312 | 516 | **4.53** | 312 | 523.5 | **5.79** | 289.6 | 0.053 | 0.3 | 0.19 | **3.231** |
| 480 | 291 | 60 | 564 | 1.6 | **6.4** | 298.5 | 548.5 | **9.52** | 301.4 | 0.077 | 0.3 | 0.19 | **2.115** |
| Mean error | | | | | 8.813 | Mean error | | 6.805 | Mean error | | | | 2.661 |

**Table 7. Considering the material 0.35%C Steel [35]**

| | | | **Mc Diarmid** | | | **Carpinteri and spagnoli** | | | **Proposed fatigue Damage assessment** | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\sigma_y$ | $\tau_{xy}$ | $\varphi$ | $\tau_a$ | $\sigma_{max}^n$ | Error (%) | $\tau_a$ | $\sigma_{max}^n$ | Error (%) | $\tau_{max}$ | $\gamma_{max}$ | $\lambda$ | n | Error (%) |
| 225.2 | 175.8 | 0 | 110.2 | 110.2 | **7.59** | 32.5 | 198.6 | **5.68** | 120.1 | 0.014 | 0.3 | 0.19 | **3.324** |
| 157 | 130.7 | 90 | 145.3 | 24.5 | **3.64** | 151.6 | 15.4 | **2.06** | 98.4 | 0.022 | 0.3 | 0.19 | **1.151** |
| 219.3 | 23.9 | 0 | 112.4 | 106.1 | **4.562** | 113.6 | 121.2 | **6.24** | 115.2 | 0.065 | 0.3 | 0.19 | **3.275** |
| 119.5 | 164.2 | 90 | 153 | 20.8 | **1.308** | 134.5 | 64.2 | **1.08** | 145.3 | 0.079 | 0.3 | 0.19 | **2.144** |
| Mean error | | | | | 4.275 | Mean error | | 3.765 | Mean error | | | | 2.474 |

In summary, the validation process involves using various loads, analyzing experimental data, considering material properties, calculating errors, and identifying the most accurate predictions for each load type. This rigorous evaluation helps validate the proposed fatigue damage assessment model and assess its effectiveness in predicting fatigue life under multiaxial loading conditions for different materials.

## 8. Conclusion

The development of a new hybrid critical plane model, incorporating both stress and strain parameters, marks a significant advancement in the field of fatigue life and damage assessment for components exposed to multiaxial cyclic loading conditions. This innovative approach enhances the accuracy of calculating dependability and durability, offering a more reliable method for identifying the critical plane most susceptible to crack initiation. The capability to estimate fatigue life and assess damage assessment across various dimensions and loading conditions enhances the applicability of this model in real-world engineering scenarios. To validate the effectiveness of the proposed model both proportional and non-proportional are considered. Simulation and case studies involving thick-walled cylinders and notched plates were conducted, with results compared against experimental data from the literature. In the case of a Thick-Walled Cylinder under Axial and Torsional Loading, the hybrid model identified the damage assessment critical plane at a 45-degree angle based on crack initiation criteria. The calculated fatigue life was 14,530 cycles, while the experimental result stood at 11,985 cycles. Despite a slight discrepancy, the recorded error of 0.0836 suggests a strong correlation with the experimental findings, affirming the model's reliability in predicting fatigue life. Similarly, for the Notched Plate under Tension-Compression Loading, the hybrid model determined a critical plane angle of 75 degrees at the notch root, resulting in a fatigue life of 9,241 cycles. The experimental test yielded a life of 7,865 cycles, with a recorded error of 0.0700. Once again, the findings indicate a robust correlation between the model predictions and experimental results, reinforcing the model's capability to estimate life and orientation components under diverse conditions. Overall, the hybrid critical plane model introduces a valuable tool for researchers seeking accurate fatigue life and damage assessment predictions. Its adaptability to various dimensions and loading conditions, coupled with its demonstrated correlation with experimental data, positions it as a valuable asset in the realm of structural analysis and design. This hybrid critical plane and damage assessment model will be integrated into commercial Finite Element Method (FEM) software through a user-defined system, providing a versatile tool for researchers.

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