*Original Article*

# Influence of Cross Bore Geometry on the Behaviour of a Pressurized Thick Compound Cylinder

Naftali Kiplagat<sup>1</sup>, Leonard Masu<sup>2</sup>, Patrick Nziu<sup>3</sup>

*1,2,3Vaal University of Technology, Faculty of Engineering and Technology, Department of Industrial Engineering, Operations Management and Mechanical Engineering, Vanderbijlpark, Andries Potgieter Blvd, South Africa*

*<sup>1</sup>Corresponding Author : nkiplagat@seku.ac.ke*

Received: 25 March 2024 Revised: 07 May 2024 Accepted: 14 June 2024 Published: 26 July 2024

*Abstract - The aim of this study was to investigate the impact of shape on cross-bored compound cylinders, particularly circular and elliptical configurations. The analysis was conducted with a uniform pressure of 88.494 MPa. It was found that a radial circular cross-bore with a cross-bore size ratio of 0.1 resulted in the lowest Stress Concentration Factor (SCF) of 2.66. Subsequently, a thorough examination of 12 different diameter ratios for the elliptical-shaped cross bore, ranging from 0.5 to 10, identified the minimum SCF value of 1.33, which occurred at a diameter ratio that was then used for further analysis. The mentioned SCF value represented a 24.81% reduction in the pressure-carrying capacity of the compound cylinder compared to a similar plain compound cylinder*. *In a broader comparison between circular and elliptical cross bores yielded lower hoop stresses than their circular counterparts*.

*Keywords - Finite element analysis, Compound cylinder, Cross bore size, Stress concentration factor.*

# **1. Introduction**

Cylinders are critical components in numerous industries, serving the purpose of containing and maintaining fluids under pressure. They are widely used in nuclear and chemical plants, as well as in gas storage and high-pressure vessels [1]. As raw material availability decreases and manufacturing costs increase, researchers have expanded their focus beyond traditional elastic techniques. Instead, they have explored elastic-plastic techniques, which promise more efficient utilization of materials. As a result, specific elastic-plastic methods, such as autofrettage and the use of compound cylinders, have been developed to improve the pressurecarrying capacity of thick-walled cylinders. [2]. This study particularly focuses on compound cylinders, which are typically formed by fitting two or more cylinders of different diameters into each other with a certain degree of interference. [1]. This process generates a residual hoop stress distribution within the walls of these compound cylinders, thereby enhancing their performance when exposed to operational pressure loads [3]. Pressure vessels may incorporate openings, commonly known as cross bores[4]. These openings, when positioned diametrically, are often referred to as radial cross bores. These openings serve the purpose of accommodating instrumentation accessories required for critical operations. The instrumentation accessories encompass temperature sensors, safety and relief valves, bursting discs, flow circuit meters, and lubrication systems. The presence of a cross bore significantly contributes to potential flaws in a pressure vessel

due to the elevated stress concentration it induces. As a result, these imperfections lead to changes in stress distribution and the formation of localized areas with high-stress concentrations. The stress concentration degree resulting from a sudden section change is quantified as a Stress Concentration Factor (SCF). Additionally, stress distribution assessment within pressure vessels can be conducted through experimental methods[8], analytical approaches, or numerical simulations[9]. In this study, the Stress Concentration Factor (SCF) was defined as the ratio of localized critical stresses within a cross-bore cylinder to the corresponding stresses in a similar plain cylinder without a cross-bore. In materials strength and engineering applications, it's crucial to emphasize that peak stresses play a pivotal role in determining material strength. When compound cylinders are subjected to internal pressure, they develop hoop, radial, and longitudinal stresses. Hoop stresses, being the maximum, is the main cause of failure in pressure vessels. Moreover, it's worth noting that fatigue failure and the initiation of cracks often occur in regions with high stress concentrations [11]. The failure of these vessels is particularly critical when they contain toxic, explosive, flammable, or reactive fluids, as it can result in catastrophic accidents, loss of life, loss damage of property, and displacement of populations. Hence, this study focused on analyzing the Stress Concentration Factor (SCF) to identify locations with the highest hoop stress values within the compound cylinder for various cross-bore ratios. The stress concentration in thick cross-bored compound cylinders is

influenced by several crucial design parameters, including the cross bore's shrinkage pressure, size, shape, location, and obliquity angle. This study specifically delves into the importance of shape as a key geometric factor in the design of compound cylinders with cross bores. Consequently, the impact of cross-bore shape on pressure vessels has been documented. Early studies were conducted to present the consequences of either elliptical holes or circular side holes [13], among others [4][14]. Therefore, the objective of this research was to determine the optimal size for a radial circular cross-bore in a compound cylinder that minimizes the Stress Concentration Factor (SCF).

### **2. Materials and Methods**

## *2.1. Geometry of the Pressure Vessel and the Cross-Bore Shape*

The choice of vessel size was based on its common usage in compound cylinder design. The inner and outer sleeves had respective inner and outer radii of 0.05 m and 0.075 m. Figure 1 illustrates the configuration of the cross-bore shapes. The radii of the cross-bores were then determined using the crossbore ratios provided in Table 1.

## *2.2. Part Modelling using Abaqus Finite Element Analysis Tool*

The creation of a deformable three-dimensional solid involved sketching and forming a quarter profile of a compound cylinder, resulting in a solid body composed of two cylinders: the inner and outer sleeves. To specify the thickness of the thick compound cylinder, it was extruded from the model's face. The thickness of the thick compound cylinder

was established as three times the outer diameter of the compound cylinder. This decision aimed to mitigate the propagation of effects from close-end enclosures of the compound cylinder to other remote sections of the cylinder. At this stage, the cut revolve technique was utilized, resulting in one of the models depicted in Figure 2.

During this study, an elastic steel model was used, exhibiting the following material properties: a Poisson's ratio of 0.29, Young's modulus of elasticity of 207 GPa, and a density of 7800 kg/m<sup>3</sup>. The material selection was aligned with previous studies on compound pressure vessels [12]. Both the inner and outer steel sections were characterized by solid and homogeneous properties. They were subsequently merged to form a unified assembly representing the compound cylinder. This assembly integrated distinct mesh types for the inner and outer sleeves, encompassing all geometries within the finite element model.

The modeling process consisted of two distinct steps: an interference step and a pressure step, intended for deployment during the loading phase. The interference step was utilized to examine the shrinkage pressure affecting both the inner and outer sleeves, while the pressure step was dedicated to analyzing the internal pressure within the compound cylinder. These steps were executed independently, and their outcomes were later combined. Each analysis step was coupled with specific boundary conditions to constrain any body movement. Symmetrical boundary conditions were applied to various planes of the cylinder, covering regions along the X, Y, and Z axes for both sleeves.

**Table 1. Circular cross-bore diameters** Cross bore ratio 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 Cross bore Diameter (m)  $\begin{array}{|c|c|c|c|c|c|c|c|c|c|c|c|} \hline 0.01 & 0.02 & 0.03 & 0.04 & 0.04 & 0.06 & 0.07 & 0.08 \ \hline \end{array}$ 



**Fig. 1 Cross bore shape configuration where a is the radius/ major radius and b is the minor radius**



**Fig. 2 Quarter profile model used in abaqus analysis**



**Fig. 3 Biased meshed density**

The model underwent internal pressure in both the cross and main bores, and shrinkage pressure was introduced at the interference fit between the inner and outer sleeves of the thick compound cylinder. These pressures were applied in two stages: the internal pressure of 1 MPa was applied during the pressure step, while the shrinkage pressure of 89.464 MPa was exclusively applied during the interference step and deactivated in the pressure step. |During the loading phase, the different load cases were applied to the models: (1) Positive internal pressure on the inner surface of the entire model, resulting in tension; (2) Negative shrinkage pressure on the outer surface of the inner sleeve, leading to compression (considered negative); (3) Positive shrinkage pressure on the

inner surface of the outer sleeve, resulting in tension (considered positive).

To comply with Abaqus software documentation recommendations for stress analysis, second-order tetrahedral and hexahedral elements were utilized. Specifically, secondorder C3D10 tetrahedral elements with 10 nodes were used in this study. Additionally, the C3D20R, a 20-node quadratic brick element, was selected to enhance integration. Hexahedral elements were employed for cylinders with small cross-bores, while tetrahedral elements were used for models with larger cross-bores. The mesh density was biased, with higher element density concentrated around specific areas of

interest, such as the cross bore. The mesh density ranged between 0.003-0.004 m, following the approach employed in a previous study by Nziu [10], as depicted in Figure 3. This biased mesh density strategy enabled a more accurate capture of localized stress concentration around the cross-bore region.

#### *2.3. Validation*

In this study, the validity of the finite element model was confirmed through analytical validation. This involved examining and comparing principal stresses in plain compound cylinders using both analytical and Finite Element Analysis (FEA) methods. The study further verified its results by comparing the FEA hoop stresses, which were located at a considerable distance from the cross bore, with their corresponding theoretical values[7].

Moreover, this validation process adhered to the Saint Venant principle, which posits that stress far from the cross bore should yield results that are approximately equal to the stresses in compound cylinders without cross bores, i.e., plain compound cylinders.

#### *2.4. Radial Circular Cross-Bore Shape*

Finite element modeling of the compound cylinder with radial circular cross bores was conducted using the commercial engineering software Abaqus. The study aimed to analyze the influence of size on stress concentrations in the compound cylinder, considering eight different radial circular cross-bore sizes. The modeling process employed an optimized shrinkage pressure of 89.564 MPa and was subjected to an internal pressure of 1 MPa. After analyzing the eight-part models, it was determined that the radial circular cross bore with a size ratio of 0.1 exhibited the minimum Stress Concentration Factor (SCF). Consequently, this crossbore size with a shrinkage pressure of 89.464 MPa was selected as the optimized circular cross-bore size. Subsequently, this cross-bore was chosen for further analysis.

#### *2.5. Elliptical Cross Bore Shape*

The initial step focused on determining the ideal diameter ratio of a cross bore within a compound cylinder and determining the size or dimensions of the elliptical cross bore. This elliptical cross bore should be equal to the circular optimum cross bore to achieve an equal discharge flow. The following subheadings provide a detailed description of this process.

#### *2.6. Determination of Optimal Diameter Size Ratio*

The task involved the investigation of various diameter ratio(s) of elliptical-shaped cross bores to establish optimum diameter ratio, i.e.  $\frac{major}{minor}$  that gave minimum SCF. Hence, preliminary investigations were carried out to establish the optimum diameter ratio. Different minor diameters were established using various diameter ratios based on an arbitrary major diameter of 0.005 m, as depicted in Table 2.



**Table 2. Major and minor diameters evaluated**

Further, this work adopted diameter ratios  $\binom{a}{b}$  the configuration that was similar to that shown in fig. 4. Additionally, the optimal shrinkage pressure of 88.464 MPa was applied. These models were designed to match the ratios utilized in thick cylinder analysis [10]. Based on the findings and discussion outlined in Table 2, a diameter ratio of 5 was identified as yielding a minimum SCF of 1.34. Consequently, this ratio was selected as the optimal diameter ratio for a compound cylinder.

#### *2.7. Determination of Elliptical Cross Bore Sizes*

To ensure a meaningful comparison of the effects between elliptical and circular cross-bores, their crosssectional areas must be equivalent, resulting in equal discharge capabilities. Thus, the cross-sectional area of the optimal circular cross-bore shape, as determined, was calculated and equated to the cross-sectional area of an elliptical cross-bore with a diameter ratio identified in Table 2. This process involved using Equations 1 to 4.

Area of a circle = 
$$
\frac{\pi D^2}{4}
$$
 (1)

Area of an ellipse = 
$$
\pi ab
$$
 (2)

 $\pi D^2$  $\frac{b}{4} = \pi ab$  (3)

$$
ab = \frac{D^2}{4} \tag{4}
$$

Therefore, the equivalent dimensions of the elliptical cross bore that gave the same discharge flow as that of the optimum circular cross bore selected were:

(i) Major diameter  $a = 0.01118$  m

(ii) Minor diameter  $b = 0.002236$  m

Further, the elliptical-shaped cross bore identified in this section was created and analyzed. The modeling procedure of the radial elliptical cross bore was similar to the one that was performed on the radial circular cross bore. The noticeable difference is that the shape of the cross bore changed from circular to elliptical, as shown in Figure 4.



**Fig. 4 Part and mesh profiles of radial elliptical shaped cross bore**



**Fig. 5 Hoop stress vs cross-bore radius**



**Fig. 6 Hoop stress concentration factor vs cross bore size**

## **3. Results and Discussion**

#### *3.1. Determination of Optimal Circular Cross-bore that Offers the Least SCF*

A comparison was conducted among a range of compound cylinders featuring different cross-bore dimensions, resulting from cross-bore ratios spanning from 0.1-0.8. This comparison was made against a standard plain compound cylinder. Figure 5 illustrates the impact of hoop stress on the transverse edge length of a cross-bored compound cylinder. In this comparative study, it is examined how the stress patterns in cross-bored compounds differ from those in plain cylinders. The introduction of a cross bore at a specific point led to elevated hoop stress values. The analysis demonstrated that the maximum hoop stress in compound cylinders consistently exceeded that of plain cylinders without cross bores.

#### *3.2. Optimization of the Radial Circular Cross Bore*

As the cross-bore size varied, it became apparent that the most noteworthy increase in hoop stress within a thick cylinder occurred when a circular radial cross-bore was introduced. Consequently, the authors computed the Stress Concentration Factors (SCF) by comparing the maximum hoop stress location in the compound cylinder with the corresponding location in the plain thick compound cylinder. The SCFs were calculated using Equation 3.1, and the resulting hoop stresses were thoroughly examined below.

## *3.2.1. Cross Bore Size Effects on Hoop Stress Concentration Factor*

The behavior of the hoop SCF on the several cross-bore sizes is illustrated in Fig. 6. The SCFs of the compound cylinders with distinct cross-bore sizes cylinder were computed in relation to the locations with the highest amounts of hoop stress in the compound cylinder.

As depicted in Figure 6, it becomes evident that increases in the size of the cross-bore result in higher magnitudes of hoop SCF. Notably, a cross-bore ratio yielded the lowest hoop SCF when compared to all the other cross-bore ratios under study. Conversely, a compound cylinder with a cross-bore ratio of 0.5 exhibited the highest hoop SCF at 6.74. This observation suggests that a compound cylinder with a small cross-bore ratio of 0.1 can withstand pressure up to twice that of its counterpart with a cross-bore ratio of 0.8. In general, it was concluded that the cross-bore ratio, which represents the cross-bore size, exerts a notable influence on the hoop SCF. Therefore, an increase in the cross-bore size leads to a corresponding increase in the magnitude of the hoop stress concentration factor. This implies that the structural rigidity of the compound cylinder diminishes as the cross-bore size grows, resulting in higher hoop stresses and, subsequently, elevated stress concentration factors.In summary, it is evident that a compound cylinder with a cross-bore ratio of 0.1 exhibits the lowest hoop Stress Concentration Factor (SCF) and the least hoop stress per unit pressure.

Typically, larger cross bores involve the removal of excess material subject to load. Consequently, when significant material is removed within a cross bore, it leads to an increased magnitude of hoop stress, which can potentially lead to the failure of the compound cylinder. These findings align with prior research on thick cylinders conducted by Nziu and Masu., in which the authors concluded that, for the thick cylinders, the maximum hoop stress increases as the size of the cross bore grows.

#### *3.3. Elliptical Cross Bore Effects in Compound Cylinders 3.3.1. Determination of Diameter Ratio*

After analyzing data from 11 different diameter ratios, Figure 7 shows the minimum hoop stress concentration factor. Therefore, it can be observed that the SCF reduces with changes in diameter ratio. The highest magnitude of SCF occurred at the diameter ratio of 0.5, with a magnitude of 4.72. Further, the shape of the cross bore becomes circular at the diameter size ratio of 1. The stress concentration became a constant of 1.33 once the diameter ratio reached 5, 6, 8, and 10. Hence, the diameter size ratio of 5 was then selected as the optimal diameter size ratio of elliptically shaped cross bores. Therefore, this optimized diameter ratio was considered for further analyses in the current study. The behavior of the magnitude of SCFs can be attributed to the changing shape of the cross bore when viewed at the intersection of the main bore, as shown in Figure 7.

From Figure 8, when  $a < b$ , at the ends of the major axes, higher stresses were recorded because a large area of major diameter, b, is perpendicular to the hoop stress direction. Hence, stress increases with the ratio, making it apparent that a very narrow hole perpendicular to the direction of tension produces a very high stress concentration. Because of this, cracks perpendicular to the direction of applied force tend to spread. For this reason, cracks in this direction should be avoided in the design of compound cylinders because they are prone to failure. Hence, Figure 8 illustrates the effects of changes in elliptical shape against hoop stress direction. In contrast, when  $b < a$ , lower stresses were experienced at the ends of the minor axes. Therefore, the cracks generated in this direction were less severe. In this situation, the maximum stress value reduces as the ellipse is very slender, thus, cracks parallel to the direction of tension are less prone to propagate than those perpendicular to it. Therefore, with this type of design of cross bores, the cracks are less prone to failure, hence encouraged during the design of compound cylinders.

By comparing compound and thick cylinder magnitudes of SCFs with different diameter ratios, the study noted that for a compound cylinder, the minimum SCF of 1.33 was a result of a diameter size ratio of 5. Whereas, for the thick cylinder the diameter size ratio of 2 gave the minimum SCF of 1.95. Therefore, a compound cylinder has a 46% percentage difference in SCF over a thick cylinder. Therefore, SCF for an elliptical hole will always be more than that of a circular hole

unless the minor axis is made perpendicular to the hoop direction. This is of great practical importance and is the reason that elliptical openings in cylindrical vessels should be placed with their minor axis perpendicular to the hoop direction, thereby obtaining lower maximum stress than with a circular opening. Therefore, this could be attributed to the compound cylinder being stronger than the thick cylinder and, hence requires more pressure before it yields. The results of this study are consistent with studies by [7], where the book stated that cross-bore configuration of elliptical cross bores where  $a < b$  results in high magnitudes of hoop stress in the cylinder.







**Fig. 10 Hoop stress vs radius along the transverse edge**



**Fig. 11 Hoop SCF vs cross-bore location**

## *3.3.2. Hoop Stress Distribution Along the Transverse Edge of the Optimum Elliptical Shape Cross-Bore*

Figure 10 shows the behavior of hoop stress along the transverse edge of a cross-bore for an optimum elliptical cross-bore shape. Hoop stresses of both compound cylinders with optimum elliptically shaped cross bore were compared against stresses of a plain compound cylinder. The stress behavior trend was the same; however, elliptical-shaped cross bore developed higher stresses. The introduction of a cross causes these higher stresses to bore into the compound cylinder. These cross-bores cause geometric discontinuities and cracks, which cause higher hoop stresses. These results also gave observations similar to studies in thick cylinders by [6], where the authors stated that offsetting elliptically shaped cross bores increased the magnitude of SCFs in a thick cylinder.

### *3.3.3. Effects of Elliptical Cross Bore Location on Hoop Stress Concentration Factor*

Figure 11 shows the variation of the hoop stress concentration factor with offset location. Therefore, it was deduced that the lowest magnitudes of hoop stress concentration factor in a compound cylinder with an elliptical cross bore occurred radial position with a magnitude of 1.33. This offset position gave a lower hoop SCF than the cross bore located at the radial position, which gave the highest concentration factor of 1.85. It was observed that there was a gradual increase in hoop stress SCF as the offset position changed away from the radial position. Generally, it was observed that the magnitude of the hoop stress concentration factor was highest when the cross bore was at position 0.0225. These changes in the magnitude of hoop SCF are related to the changing shape of the cross bore, i.e. variation of the minor and major diameters of a and b, respectively. This phenomenon has been explained in a Section and reaffirmed further by the use of the FEA shape. This observation of the effect of hoop stress because of changes in shapes is consistent with the observation noted in this previous study.

### *3.4. Comparison of Effects from the Offset Circular and Elliptical Cross Bore in Compound Cylinders*

The study further did a comparison of stress profiles generated by the optimum circular and elliptically shaped cross bore at each offset position. The results are discussed in the subsequent sections.

#### *3.4.1. Maximum Hoop Stress*

Figure 12 illustrates the comparison of maximum hoop stress per unit pressure predicted by circular and elliptical cross bores together with a plain compound cylinder at each offset position. It was observed that the elliptical cross bore gave lower hoop stresses than circularly shaped ones ranging from position 0 to 0.0225 m. It was also noted that an offset position of 0.006 m gave the optimum location for circularly, while for the elliptically shaped cross bores, the minimum SCF was achieved at the radial position.

Table 3 shows a summary of stress variation between two shapes at different locations position, considering an elliptical shape as the reference. It was observed that the percentage difference of hoop stresses at different locations varies from a low of 39.6% to a high of 115%. However, the percentage difference was more pronounced when the cross-bore shapes were at the radial position. The difference in hoop stresses is attributed to the shape of the cross bore. Circularly shaped cross-bores gave higher hoop stresses than elliptically shaped cross-bores. This is because the radius of the circular cross bore is smaller than the minor diameter of the elliptical-shaped cross bore; hence, more hoop stress is subjected to the radius direction of the circular cross bore. The pronounced difference in hoop stresses of the different shapes of cross bores at the radial position is attributed to the large difference in the size of the radius of the circle and the minor diameter of the ellipse. These results were consistent with the comparison done by Nziu and Masu (2019d), where the authors stated that elliptical-shaped cross bores lower hoop stresses at a radial position than circular ones.







**Fig. 12 Circular and elliptical cross bore maximum hoop stress**





## *3.4.2. Hoop Stress Concentration Factor*

Figure 13 shows the comparison of stress concentration factors predicted by circular and elliptical cross bores at each offset position.As observed in Figure 13, the hoop stress concentration factors of elliptically shaped cross bores were lower than that of circularly shaped ones in all positions. Further, as shown in Table 4, the study could deduce a clear pattern for each position. For example, both elliptically and circularly shaped cross bores the offset 0.0225 position generated the highest SCF.

Location(m)	<b>Elliptical Shape</b>	<b>Circular Shape</b>	<b>Percentage Difference %</b>
	1.4	2.63	87.9
0.003	l.34	2.633	96.5
0.006	1.33	2.51	88.7
0.009	1.36	2.521	85.4
0.014	1.32	2.53	91.7
0.01725	1.35	2.62	94.1
0.019	1.37	2.71	97.8
0.0225	1.925	2.64	37.1

**Table 4. Summary of variation in hoop SCFs for both circularly and elliptically shaped cross bores**

The highest SCF due to an elliptically shaped cross bore was recorded offset position 0.0225 m with a magnitude of 1.85, while for the circularly shaped cross bore, the minimum SCF was 2.72; on the other hand, the minimum SCF due to a circularly shaped cross bore occurred at offset position of 0.006 m with a magnitude of 2.50.With this available information, the optimum location of the elliptical-shaped cross bore can reduce SCF magnitudes. This is attributed to the shapes of the cross bore when viewed from the intersection of the main bore and cross bore.

## **4. Conclusion**

As a result of this study, it was concluded that among 8 examined circular radial cross-bores introduced into the compound cylinder, the cross-bore ratio of 0.1 yielded the lowest SCF of 2.66. This translates to a 62.4% reduction in the pressure-carrying of the compound cylinder when compared to a similar compound cylinder without a cross bore. Furthermore, the assessment of 12 different diameter ratios for

# elliptical-shaped cross-bores, ranging from 0.5 to 10, at the radial position, identified the minimum SCF value of 1.33, occurring at a diameter ratio of 5. This optimal diameter ratio resulted in a 40% reduction in the pressure-carrying capacity of the compound cylinder compared to a plain compound cylinder without a cross bore. In summary, elliptical-shaped cross bores, in general, tend to produce lower hoop stresses and SCFs when compared to circular cross bores.

#### **Funding Statement**

The author(s) received no financial support for the research, authorship, and publication of this article.

#### **Acknowledgements**

The Vaal University of Technology supports this research work. The authors wish to thank the Department of Industrial Engineering, Operation Management & Mechanical Engineering at the Vaal University of Technology for facilitating this work.

#### **References**

- [1] Kaoutar Bahoum, Mohammed Diany, and Mustapha Mabrouki, "Stress Analysis of Compound Cylinders Subjected to Thermo-Mechanical Loads," *Journal of Mechanical Science and Technology*, vol. 31, pp. 1805-1811, 2017. [\[CrossRef\]](https://doi.org/10.1007/s12206-017-0328-5) [\[Google Scholar\]](https://scholar.google.com/scholar?hl=en&as_sdt=0%2C5&q=Stress+analysis+of+compound+cylinders+subjected+to+thermo-mechanical+loads&btnG=) [\[Publisher Link\]](https://link.springer.com/article/10.1007/s12206-017-0328-5)
- [2] G.H. Majzoobi et al., "Experimental and Finite Element Prediction of Bursting Pressure in Compound Cylinders," *International Journal of Pressure Vessels and Piping*, vol. 81, no. 12, pp. 889-896, 2004. [\[CrossRef\]](https://doi.org/10.1016/j.ijpvp.2004.06.011) [\[Google Scholar\]](https://scholar.google.com/scholar?hl=en&as_sdt=0%2C5&q=Experimental+and+finite+element+prediction+of+bursting+pressure+in+compound+cylinders&btnG=) [\[Publisher Link\]](https://www.sciencedirect.com/science/article/abs/pii/S0308016104001553)
- [3] M. Kumaresan, and K. Chockalingam, "Stress Analysis in Compound Cylinders and Autofrettaged Cylinders," *International Journal of Engineering Research and Technology*, vol. 7, no. 4, pp. 169-174, 2018. [\[Google Scholar\]](https://scholar.google.com/scholar?hl=en&as_sdt=0%2C5&q=M.+Kumaresan+and+Chocklingam%2C+Stress+Analysis+in+Compound+Cylinder+and+Autofrettaged+Cylinders&btnG=) [\[Publisher Link\]](https://www.ijert.org/stress-analysis-in-compound-cylinders-and-autofrettaged-cylinders)
- [4] P. Makulsawatudom, D. Mackenzie, and R. Hamilton, "Stress Concentration at Crossholes in Thick Cylindrical Vessels," *The Journal of Strain Analysis for Engineering Design*, vol. 39, no. 5, pp. 471-481, 2004. [\[CrossRef\]](https://doi.org/10.1243/0309324041896506) [\[Google Scholar\]](https://scholar.google.com/scholar?hl=en&as_sdt=0%2C5&q=Stress+concentration+at+crossholes+in+thick+cylindrical+vessels&btnG=) [\[Publisher Link\]](https://journals.sagepub.com/doi/abs/10.1243/0309324041896506)
- [5] Gérard C. Nihous, Christopher K. Kinoshita, and Stephen M. Masutani, "Stress Concentration Factors for Oblique Holes in Pressurized Thick-Walled Cylinders," *Journal of Pressure Vessel Technology*, vol. 130, no. 2, pp. 0212021-0212027, 2008. [\[CrossRef\]](https://doi.org/10.1115/1.2891915) [\[Google](https://scholar.google.com/scholar?hl=en&as_sdt=0%2C5&q=Stress+concentration+factors+for+oblique+holes+in+pressurized+thick-walled+cylinders&btnG=)  [Scholar\]](https://scholar.google.com/scholar?hl=en&as_sdt=0%2C5&q=Stress+concentration+factors+for+oblique+holes+in+pressurized+thick-walled+cylinders&btnG=) [\[Publisher Link\]](https://asmedigitalcollection.asme.org/pressurevesseltech/article-abstract/130/2/021202/442778/Stress-Concentration-Factors-for-Oblique-Holes-in?redirectedFrom=fulltext)
- [6] John Harvey, and J.H. Harvey, "Theory and Design of Pressure Vessels (Third Edition)," *Journal of Pressure Vessel Technology*, vol. 108, no. 2, pp. 244-245, 1986. [\[CrossRef\]](https://doi.org/10.1115/1.3264775) [\[Google Scholar\]](https://scholar.google.com/scholar?hl=en&as_sdt=0%2C5&q=J+Harvey%2C+JH+Harvey%2C+Theory+and+Design+of+Pressure+Vessels+%28Third+Edition%29&btnG=) [\[Publisher Link\]](https://asmedigitalcollection.asme.org/pressurevesseltech/article/108/2/244/437056/Theory-and-Design-of-Pressure-Vessels-Third)
- [7] D.S. Mankar, and P.M. Khodake, "Residual Stress Produced after Machining in Mechanical Components and its effects on Fatigue Life: A State of Art," *International Journal of Mechanical and Production Engineering Research and Development (IJMPERD)*, vol. 5, pp. 1- 10, 2015. [\[Google Scholar\]](https://scholar.google.com/scholar?hl=en&as_sdt=0%2C5&q=residual+stress+produced+after+machining+in+mechanical+components+and+its+effects+on+fatigue+life%3A+a+state+of+art&btnG=)
- [8] Avinash Kharat, and V.V. Kulkarni, "Stress Concentration at Openings in Pressure Vessels A Review," *International Journal of Innovative Research in Science, Engineering and Technology*, vol. 2, no. 3, pp. 670-678, 2013. [\[Google Scholar\]](https://www.ijirset.com/volume-2-issue-3.html) [\[Publisher Link\]](https://scholar.google.com/scholar?hl=en&as_sdt=0%2C5&q=Stress+Concentration+at+Openings+in+Pressure+Vessels%E2%80%93A+Review%E2%80%99&btnG=)
- [9] Patel Dharmin, Panchal Khushbu, and Jadav Chetan, "A Review on Stress Analysis of an Infinite Plate with Cut-Outs," *International Journal of Scientific and Research Publications*, vol. 2, no. 11, pp. 1-7, 2012. [\[Google Scholar\]](https://scholar.google.com/scholar?hl=en&as_sdt=0%2C5&q=A+review+on+stress+analysis+of+an+infinite+plate+with+cut-outs&btnG=) [\[Publisher Link\]](https://www.ijsrp.org/research-paper-1112.php?rp=P11424)
- [10] P.K. Nziu, "*Optimal Geometric Configuration of a Cross Bore in High Pressure Vessels*," Theses and Dissertations, Vaal University of Technology, pp. 1-223, 2018. [\[Google Scholar\]](https://scholar.google.com/scholar?hl=en&as_sdt=0%2C5&q=Optimal+Geometric+Configuration+of+a+Cross+Bore+in+High+Pressure+Vessels&btnG=) [\[Publisher Link\]](http://digiresearch.vut.ac.za/items/3820ef90-52ae-4ebb-a3ea-98660f8b8642)
- [11] B.N. Cole, G. Craggs, and I. Ficenec, "Strength of Cylinders Containing Radial or Offset Cross-Bores," *Journal of Mechanical Engineering Science*, vol. 18, no. 6, pp. 279-286, 1976. [\[CrossRef\]](https://doi.org/10.1243/JMES_JOUR_1976_018_045_02) [\[Google Scholar\]](https://scholar.google.com/scholar?hl=en&as_sdt=0%2C5&q=Strength+of+Cylinders+Containing+Radial+or+Offset+Cross-Bores&btnG=) [\[Publisher Link\]](https://journals.sagepub.com/doi/abs/10.1243/JMES_JOUR_1976_018_045_02)
- [12] Tanjin Amin et al., "Optimum Design of Autofrettaged Thick-Walled Cylinders," *Global Journal of Researches in Engineering Mechanical and Mechanics Engineering*, vol. 13, no. 8, pp. 1-9, 2013. [\[Google Scholar\]](https://scholar.google.com/scholar?hl=en&as_sdt=0%2C5&q=Optimum+Design+of+Autofrettaged+Thick-Walled+Cylinders%E2%80%99&btnG=) [\[Publisher Link\]](https://globaljournals.org/item/2349-optimum-design-of-autofrettaged-thick-walled-cylinders)
- [13] J.H. Faupel, and D.B. Harris, "Stress Concentration in Heavy-Walled Cylindrical Pressure Vessels Effect of Elliptic and Circular Side Holes," *Industrial & Engineering Chemistry*, vol. 49, no. 12, pp. 1979-1986, 1957. [\[CrossRef\]](https://doi.org/10.1021/ie50576a029) [\[Google Scholar\]](https://scholar.google.com/scholar?hl=en&as_sdt=0%2C5&q=Stress+Concentration+in+Heavy-Walled+Cylindrical+Pressure+Vessels+-+Effect+of+Elliptic+and+Circular+Side+Holes%E2%80%99&btnG=) [\[Publisher Link\]](https://pubs.acs.org/doi/pdf/10.1021/ie50576a029)
- [14] C.A. Adenya, and J.M. Kihiu, "*Stress Concentration Factors in Thick Walled Cylinders with Elliptical Cross-Bores*," Jomo Kenyatta University of Agriculture and Technology, pp. 181-200, 2010. [\[Google Scholar\]](https://scholar.google.com/scholar?hl=en&as_sdt=0%2C5&q=Stress+Concentration+Factors+in+Thick+Walled+Cylinders+with+Elliptical+Cross-Bores&btnG=) [\[Publisher Link\]](http://ir.jkuat.ac.ke/handle/123456789/707)