

Original Article

Hybrid Method for Wireless Channel Estimation

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Abstract - Channel estimation involves the process of estimating the characteristics of the communication channel, particularly the channel's impulse response or frequency response. This information is essential for the receiver to compensate for the effects of the channel and properly decode the transmitted signal. The novelty of this work is integrating the deep learning framework with the compressive sensing approach for channel estimation. Combining Compressive Sensing (CS) with Convolutional Neural Networks (CNNs) for channel estimation leverages the strengths of both approaches: the ability of CS to exploit sparsity and the powerful feature extraction and learning capabilities of CNNs. The output is then compared with the channel estimated using the pilot-based channel estimation, least square estimation, and maximum likelihood estimation. It is found that the results obtained with the proposed fusion give a lower RMSE (0.11) and lower BER (1.82×10^{-6}) compared with the other methods. This indicates the effectiveness of the proposed method for channel estimation.

Keywords - Channel Estimation, CNN, Compressive Sensing, RMSE, BER.

1. Introduction

Channel estimation is crucial in wireless communication systems, especially in scenarios where the transmitted signal undergoes distortion and attenuation as it travels through the communication channel. The communication channel introduces various impairments, such as multipath fading, interference, and noise, which can affect the quality of the received signal. Channel estimation is needed in wireless communication systems for several important reasons:

1. **Mitigating Channel Effects:** The wireless communication channel introduces various impairments, such as multipath fading, attenuation, and interference. These effects can distort the transmitted signal, leading to errors and degradation in signal quality at the receiver. Channel estimation allows the receiver to understand and compensate for these effects, improving the accuracy of signal decoding.
2. **Equalization:** In scenarios where the channel introduces frequency-selective fading, the received signal may experience different attenuations at different frequencies. Channel estimation helps determine the channel's frequency response, enabling the receiver to apply equalization techniques to compensate for the frequency-selective fading.
3. **Adaptive Modulation and Coding:** Channel conditions can vary over time due to factors such as the movement of mobile devices, changes in the environment, and interference. Accurate channel estimation provides

information about the channel's current state, allowing the system to adaptively adjust modulation and coding schemes to optimize data transmission under varying conditions.

4. **Spatial Diversity and MIMO Systems:** Channel estimation becomes crucial for exploiting spatial diversity in multiple antenna systems, such as Multiple Input Multiple Output (MIMO) systems. The system can employ spatial processing techniques to enhance signal quality and increase data rates by estimating the channels associated with each antenna.
5. **Pilot Symbol Assisted Modulation (PSAM):** Many communication systems use pilot symbols, known symbols inserted into the transmitted signal, for channel estimation. These symbols help the receiver estimate the channel response at specific points, allowing for more accurate compensation of the channel effects.
6. **Link Adaptation:** Channel estimation is essential for link adaptation strategies, where the system dynamically adjusts transmission parameters such as modulation scheme and coding rate based on the estimated channel conditions. This adaptive approach improves overall system performance and efficiency.
7. **Improved Error Performance:** Accurate channel estimation leads to improved demodulation accuracy and reduced bit error rates. This is crucial for maintaining reliable communication, especially in wireless networks where the channel conditions can be challenging and dynamic.



To address these challenges and meet the needs of wireless channel estimation, researchers and engineers are developing advanced algorithms, machine learning approaches, and adaptive techniques that can efficiently estimate channel characteristics in diverse and dynamic 5G environments. These advancements aim to improve the overall performance, reliability, and efficiency of 5G communication systems. The details of the related works are discussed in the literature survey.

2. Literature Survey

There are different techniques for channel estimation, and the choice of method depends on the specific characteristics of the communication system. Some common methods include:

2.1. Pilot-based Channel Estimation

In this approach, the transmitter inserts known pilot symbols into the transmitted signal. The receiver uses these pilot symbols to estimate the channel response at those specific locations. The estimated channel response is then used to interpolate the channel characteristics for the entire signal. Xu et al. [1] have discussed the pilot-based channel estimation for covert wireless channels. They have assumed a Rayleigh fading channel with AWGN. It is derived from a relationship between the covertness of the channel and the number of channel uses. Liu et al. [2] have compared least squares (LS), least mean square error (MMSE) and linear minimum error (LMMSE) algorithms for pilot-based channel estimation, and it is concluded that the MMSE algorithm is better than the LS algorithm. Raghunathrao et al. [3] have used a rider grey optimization technique to find the optimal number of pilots for insertion in cognitive radio channels. Anand et al. [4] have used Huffman sequences as pilot clusters for the channel estimation of doubly selective channels. They have shown an improvement in the channel estimation performance. Karn et al. [5] have used the LS technique for the channel estimation in the time domain for a Digital Video Broadcasting Terrestrial (DVB-T) system. Pilot-based channel estimation is particularly useful in scenarios where the channel conditions change over time or where there is a need to adapt to varying channel characteristics. It helps improve the reliability and performance of the communication system by providing the receiver with information about the channel state.

2.2. Least Squares Estimation

This method involves minimizing the squared error between the received signal and the estimated channel response. It provides a simple and computationally efficient solution but may be sensitive to noise. Sekokotoana et al. [6] have used two Least mean squares-based channel estimation approaches for a two-user downlink single-input-single-output non-orthogonal multiple access (SISO-NOMA) system. The comparative results show that the MSE behaviour and the SNR performance depend on the step size in each case.

Liao et al. [7] have exploited the channel sparsity for the wideband mm-wave MIMO systems channel estimation. LSE variations, including ordinary least squares (OLS) and weighted least squares (WLS), account for different considerations, such as the heteroscedasticity of errors or the presence of outliers in the data.

2.3. Maximum Likelihood Estimation

This technique aims to find the channel estimate that maximizes the likelihood of the received signal given the estimated channel response. It provides a statistically optimal solution but can be computationally more demanding. Maximum Likelihood Estimation (MLE) is used for pilot symbol-assisted modulation for underwater acoustic communication (Kumar & Kumar) [8]. ML is used in the time domain to improve the BER. For a multi-panel MIMO system, Zhao et al. [9] have proposed a method in which the MIMO channel is modelled as a block-sparse signal recovery problem. Then, ML based estimation technique is used to detect the support of the sparse channel response vector and then perform least square estimation. Elvira & Santamaria [10] have proposed a multiple-importance sampling method for Symbol Error Rate estimation in MIMO detection using ML. The Monte Carlo simulation technique has been used for the same, and the method has been shown to outperform the traditional methods employed. The work by Ming-Wei Wu et al. [11] compares Maximum Likelihood Estimation with Least Squares (LS) methods for channel estimation in OFDM systems. It evaluates the performance of MLE in terms of accuracy and complexity. The study investigate the use of Maximum Likelihood Estimation for time-varying channel estimation, addressing the challenges of dynamic channels and presenting efficient MLE algorithms. MLE can be applied to estimate parameters such as channel coefficients or noise characteristics. MLE provides estimates that are asymptotically unbiased, efficient and have desirable statistical properties under certain conditions.

2.4. Kalman Filtering

Kalman filters are recursive algorithms that can adaptively estimate the channel state over time. They are particularly useful in dynamic channel environments. Arya et al. [12] and Zhang et al. [13] present adaptive Kalman Filter techniques for channel estimation, focusing on how these techniques can adjust to dynamic wireless environments and improve estimation accuracy. Siebert et al. [14] and Zhu et al. [15] explore the use of recursive Kalman Filtering for channel estimation in OFDM systems with high mobility. It highlights the challenges and solutions for estimating channels in rapidly changing environments. Singh et al. [16] and Sadr et al. [17] discuss adaptive Kalman Filter techniques for channel estimation in wireless networks, presenting methodologies to adapt the filter parameters based on network conditions. Pourkabirian and Anisi [18] have used the Tobit Kalman filter (TKF) method to estimate the hidden state vectors of wireless channels. It finds its use in the Multiuser Downlink 5G

Systems under channel uncertainties. The Kalman filter is known for its optimality under certain conditions and efficiency in real-time applications.

2.5. Convolutional Neural Networks

The use of Convolutional Neural Networks (CNNs) for channel estimation in wireless communication systems is a relatively recent but rapidly growing area of research. Zhao et al., 2024 [19] and Lv & Luo, 2023 [20] provide a comprehensive overview of how deep learning, including CNNs, can be applied to channel estimation. It discusses the basic principles and advantages of using deep learning models compared to traditional methods. CNNs can also be used to reduce the overhead and improve the accuracy of channel estimation, as shown by Mashhadi and Gündüz, 2020 [21]. Shilpa et al., 2023 [22] specifically address using CNNs for channel estimation in MIMO-OFDM systems. The authors show that CNNs can outperform traditional estimation techniques regarding BER and computational efficiency. Huang et al., 2020 [23] have written a review paper that provides a detailed survey of various deep learning techniques, including CNNs, used for channel estimation in MIMO systems. It highlights the strengths and weaknesses of different approaches and suggests future research directions. Hasini & Reddy [24] and Wong et al. [25] explore various deep learning techniques, including CNNs, for both channel estimation and signal detection, providing comparative analysis with traditional methods.

2.6. Compressive Sensing

In the context of channel estimation, CS can effectively estimate channel state information (CSI) in scenarios where the channel is sparse or has a limited number of dominant paths. Munshi & Unnikrishnan [26] and Albataineh et al. [27] explore the application of compressive sensing for sparse channel estimation and provide algorithms for efficient channel recovery. Manur & Ali [28] and Nair & Menon [29] discuss using compressive sensing for channel estimation in MIMO-OFDM systems, highlighting improvements in estimation accuracy and system performance. Dhanasekaran and Ramesh [30] present efficient algorithms for channel estimation using compressive sensing, focusing on wireless networks with sparse channel conditions. A review paper (Li et al., [31]) provides an overview of compressive sensing techniques for channel estimation and discusses various

applications in wireless communication. The work by Baranidharan et al. [32] focuses on using compressive sensing for channel estimation in sparse MIMO channels, presenting experimental results and performance analysis.

2.7. Combination Algorithms

Combination algorithms for channel estimation often involve integrating multiple techniques to leverage their individual strengths and improve overall performance. The work by Srivastava et al. [33] explores hybrid channel estimation methods combining Least Squares (LS) and Minimum Mean Square Error (MMSE) techniques for MIMO-OFDM systems. It provides insights into how combining these approaches can improve estimation accuracy. The work by Arora and Chawla [34] presents adaptive hybrid algorithms that combine different channel estimation techniques, such as Kalman filtering and MMSE, for improved performance in OFDM systems. The paper by Pradheep et al. [35] discusses a combination of pilot-based and data-based channel estimation methods, offering a hybrid approach to enhance estimation accuracy in MIMO systems. De et al. [36] present a multi-stage approach that combines compressive sensing with Bayesian inference techniques for improved channel estimation.

2.8. Research Gaps

Although many algorithms have been implemented separately, a combination of the algorithms for pilot-based channel estimation has not been attempted. This work aims to combine the Compressive Sensing and the Convolutional Neural Network algorithms to reduce the RMSE error (hence the BER).

3. Proposed Methodology

The novelty of this work is combining the Convolutional Neural Networks and the Compressive sensing techniques for channel estimation. This work utilizes the Compressive Sensing technique to get the initial Channel Estimate. Further refinement is done using CNN. The main objective of using a CS for the initial estimation is that the number of meaningful parameters to be fed to CNN is drastically reduced. This enhances the performance of the model. The parameters of the simulation are given in Table 2. The methodology for the proposed work is shown in Figure 1. A brief explanation of each of the blocks is also given below.

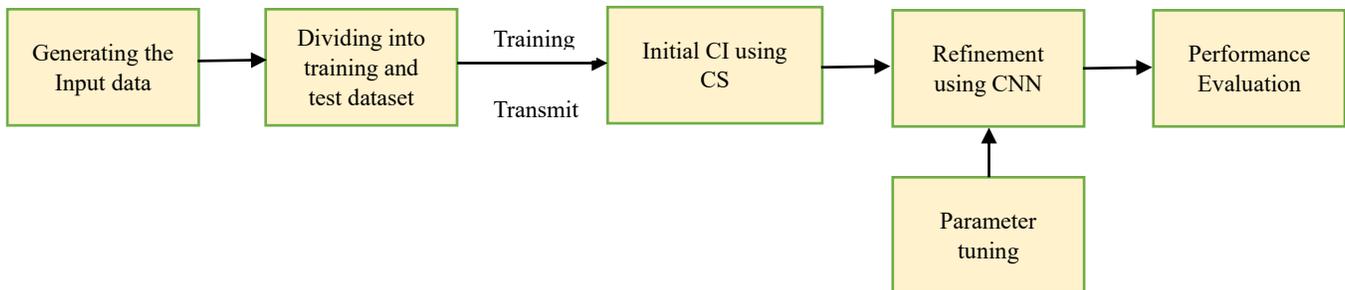


Fig. 1 Methodology for the proposed work

3.1. Generating the Input Data

The typical method for channel estimation involves sending a known pilot symbol through the channel. As shown in Figure 1, in this work, the pilot symbols are generated synthetically using Python's random number generation function. Also, care is taken to have both real and imaginary numbers in the pilot symbols. In all, 1000 symbols are generated for the present work.

3.2. Dividing the Generated Symbols into Training and Testing Sets

Out of the generated symbols, 70% is kept for training, and the remaining 30% is kept for testing purposes.

3.3. Compressive Sensing

Compressive Sensing (CS) is a technique used in signal processing to reconstruct a signal from a small number of measurements, exploiting the signal's sparsity. In the context of channel estimation, CS can be particularly effective in scenarios where the channel impulse response is sparse, such as in millimeter-wave (mmWave) communication systems or channels with significant multipath components.

The parameters of CS, which are typically used in channel estimation, are

1. *Sparsity Level (k):* The number of non-zero elements in the sparse representation of the channel. It is a critical parameter because CS algorithms leverage this sparsity to reconstruct the channel with fewer measurements.
2. *Measurement Matrix (Φ):* This matrix determines how the compressed measurements are taken. It should satisfy certain properties like the Restricted Isometry Property (RIP) to ensure accurate reconstruction.

3. *Number of Measurements (M):* This is the number of compressed measurements taken, which should be greater than the sparsity level but much less than the original signal length.
4. *Reconstruction Error (ϵ epsilon):* This parameter quantifies the error tolerance in the reconstruction process. Lower values of ϵ epsilon indicate more accurate reconstruction.
5. *Algorithm Parameters:* Parameters specific to the chosen reconstruction algorithm, such as the OMP threshold or the LASSO regularisation parameter.

The process generally involves the following steps:

1. *Modelling the Sparse Channel:* The channel is assumed to be sparse in a certain domain (e.g., delay domain for multipath channels).
2. *Designing the Measurement Matrix:* This involves selecting pilot symbols and their positions to form the measurement matrix that satisfies the RIP.
3. *Collecting Measurements:* Transmit the pilot symbols through the channel and collect the received signals, which form the compressed measurements.
4. *Reconstructing the Channel:* Use a CS algorithm to reconstruct the sparse channel from the compressed measurements.

3.4. Convolutional Neural Network

The Sequential CNN architecture with 7 layers is implemented as shown in Figure 2. The real parts and the imaginary parts are input separately to the CNN. For this, the training data is converted from a complex matrix to 2 real matrices so that the CNN considers the real and imaginary parts separately as 2D images.

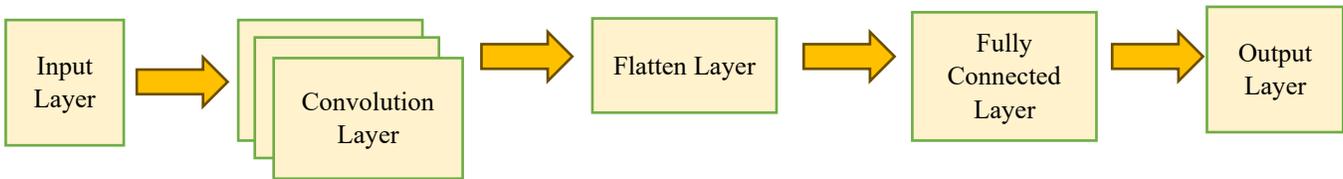


Fig. 2 Architecture of the Sequential CNN model used

3.5. Explanation of the Architecture

1. *Input Shape:* The input shape is defined as $(1000, 1)$ since we have 1000 symbols, and each symbol is represented as a single feature.
2. *Convolutional Layers:* Three 1D convolutional layers are used with 64 filters each and a kernel size 3. The padding='same' ensures the output has the same length as the input. Batch normalization and ReLU activation are applied after each convolutional layer.
3. *Flatten Layer:* The output from the last convolutional layer is flattened to feed into the fully connected layer.
4. *Fully Connected Layer:* A dense layer with 256 units and ReLU activation captures more complex relationships.

5. *Output Layer:* The output layer has the same number of units as the input symbols (1000) with a linear activation function, which is suitable for regression tasks such as channel estimation.

3.6. Steps to Combine CNN and CS for Channel Estimation

3.6.1. Sparsity-Based Channel Estimation (Compressive Sensing)

- *Sparse Representation:* Represent the channel as a sparse vector in a suitable domain (e.g., time, frequency, or spatial domain).
- *Measurement Matrix Design:* Design the measurement matrix Φ to collect compressed measurements. This is typically done using pilot symbols in the transmission.

- *Compressive Measurements:* Transmit pilot symbols and receive compressed measurements $y = \Phi h + n$, where h is the sparse channel vector, and n is noise.

3.6.2. *Initial Channel Estimation Using CS Algorithm*

- *Sparse Reconstruction:* Apply a CS algorithm (in this work, Basis Pursuit) to the compressed measurements to obtain an initial estimate of the sparse channel vector \hat{h}_{CS} . This initial estimate might be noisy or incomplete due to the limited number of measurements and the presence of noise.

3.6.3. *Refinement Using CNN*

- *CNN Design:* Design a CNN architecture suitable for denoising and refining the initial channel estimate. The CNN takes the initial CS-based estimate \hat{h}_{CS} as input and outputs, a refined channel estimate \hat{h}_{CNN} .

3.7. *Performance evaluation*

In this work, the performance of the algorithms is evaluated using the Root Mean Square Error RMSE, and the accuracy of data transmission is measured using the Bit Error Rate (BER). RMSE is a standard metric in signal processing and wireless channel estimation because it aligns well with error variance in noisy signals.

RMSE is a commonly used measure to evaluate the accuracy of channel estimation in communication systems. It quantifies the difference between the estimated channel parameters and the actual channel parameters. Lower RMSE values indicate better estimation accuracy. RMSE is widely used because it provides a single value that summarizes the estimation accuracy, making it easy to compare different estimation methods or system configurations.

The mathematical formula for RMSE is given by Equation 1.

$$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^N (\hat{h}_i - h_i)^2} \tag{1}$$

Where h_i = true channel value at the $i - th$ observation.

\hat{h}_i = estimated channel value at the $i - th$ observation.

N = number of observations (or time samples).

BER is defined as a measure of the number of bit errors that occur in a communication system, divided by the total number of bits transmitted. The formula for BER is given by Equation 2.

$$BER = \frac{N_e}{N_t} \tag{2}$$

Where N_e = number of bit errors.

N_t = total number of bits transmitted.

4. **Results and Discussion**

The channel estimation for 1000 symbols is implemented using Python with the tensorflow and keras framework. To begin with, the implementation was done with only CNN and only CS (separately), and the RMSE error was evaluated. Then, a combination of these two was implemented, and the results were tabulated in Table 1. An experiment was also carried out to evaluate the effect of the number of symbols generated on the RMSE. This experiment was implemented using the LS, MMSE, CNN, CS and CS+CNN algorithms. The simulation parameters are shown in Table 2. The results are tabulated in Figure 2.

Table 1. Performance evaluation of the implementation

No. of symbols →	100	250	500	1000	1500	2000
Algorithms Used	RMSE					
Least Squares	0.66	0.61	0.53	0.41	0.39	0.35
Minimum Mean Square Error	0.78	0.71	0.63	0.45	0.42	0.41
Convolutional Neural Network	0.48	0.36	0.25	0.23	0.21	0.21
Compressive Sensing	0.45	0.40	0.33	0.29	0.25	0.24
Compressive Sensing + Convolutional Neural Networks	0.32	0.27	0.16	0.12	0.11	0.11

As shown in Table 1, it is seen that the RMSE error is the least for the combination of CS and CNN algorithms. To find out the effect of the number of symbols on the performance of the algorithms, experiments are conducted by varying the number of pilot symbols used for the training. It is seen from Table 1 that as the number of pilot symbols increases, the RMSE error decreases, but after a certain point (1000 symbols), the RMSE error does not vary much. Hence, for the present work, the number of pilot symbols = 1000 can be considered as an optimal value. The changes in the RMSE with the changes in the number of pilot symbols are gradual

and not very drastic. This applies to all the algorithms used here. It can also be noted that the decrease in the RMSE in the case of Compressive Sensing is very gradual. This is because compressive sensing is very computationally intensive, and hence, changes in the number of symbols do not contribute much to the change in RMSE. This is precisely the reason we need to combine CS with the CNN so that the decrease in the RMSE is remarkable. Increasing the number of pilot symbols also increases the overhead, reducing the effective data rate because more bandwidth is used for pilot transmission instead of actual data. Fewer pilot symbols might lead to less accurate

channel estimation, increasing the BER as the receiver might not correctly compensate for the channel effects. The effectiveness of pilot symbols also depends on the channel estimation algorithm used.

Advanced algorithms might extract better channel estimates with fewer pilot symbols than simpler ones. One more metric used to evaluate the performance of communication systems is the Bit Error Rate (BER). RMSE and BER measure different aspects of the communication system. RMSE measures the accuracy of channel estimation, while BER measures the accuracy of data transmission. Accurate channel estimation (low RMSE) generally leads to better detection and decoding of transmitted symbols, which in turn can lower the BER. Conversely, poor channel

estimation (high RMSE) can result in higher BER because the receiver may not correctly compensate for the channel effects. The change in the BER with the algorithms used is tabulated in Table 3. More pilot symbols generally provide a more accurate channel estimation because they offer more reference points for the receiver to characterize the channel variations. This improved accuracy can lead to a lower BER. As can be seen from Table 3, there is a trade-off between the number of pilot symbols and the BER of the system. Excessive pilot symbols can reduce the overall throughput. The least BER is achieved with the combination algorithm (CS + CNN). It is also seen that after the pilot symbols = 1500, the decrease in BER is very gradual because of the pilot overhead, as explained above. The graphs obtained for the BER with different algorithms are shown in Figure 4.

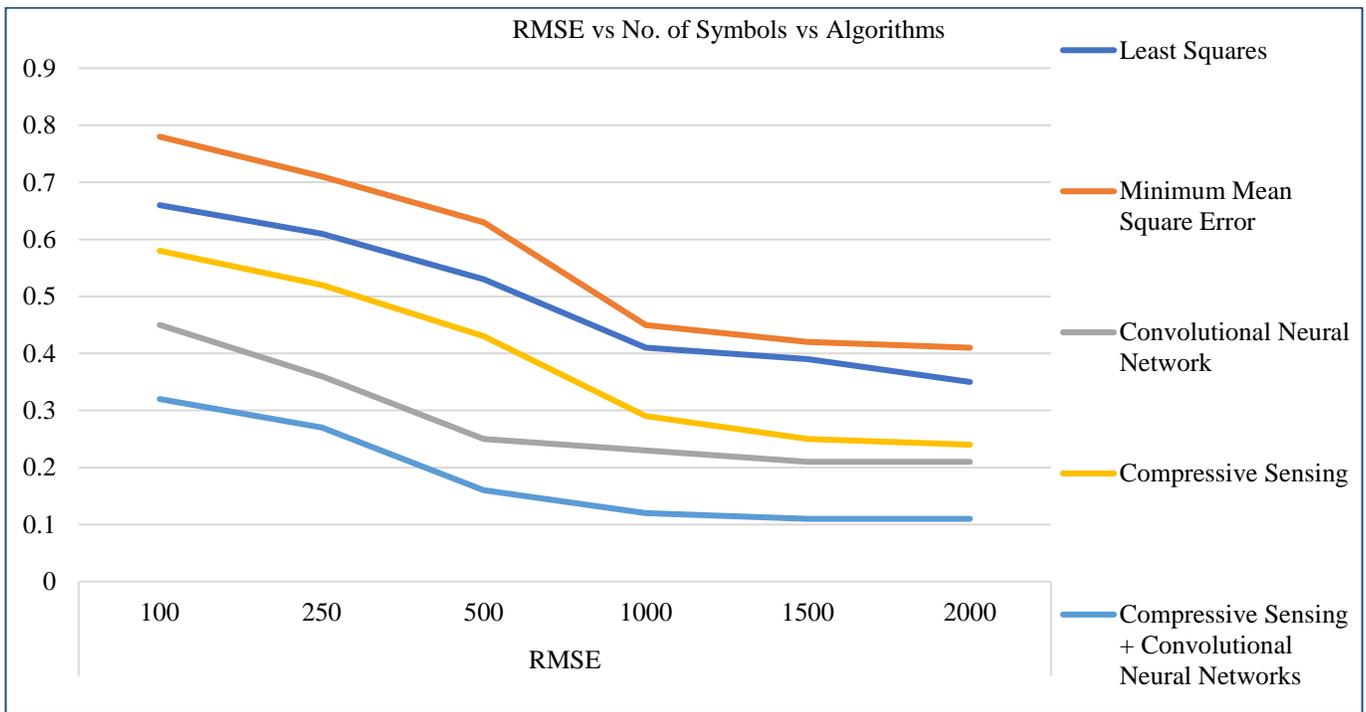


Fig. 3 Performance Evaluation vs No. of Symbols used in different Algorithms

Table 2. Simulation parameters for the proposed work

Parameters	Values
Modulation	16 QAM
Channel	Rayleigh Fading Channel
Multiplexing	Spatial Multiplexing
SNR	10 dB

Table 3. BER vs the Algorithm used (SNR = 10 dB)

No. of symbols →	100	250	500	1000	1500	2000
Algorithms Used	BER (x 10⁻⁶)					
Least Squares	7.6	6.2	5.8	4.3	4.1	3.96
Minimum Mean Square Error	6.6	5.3	4.8	4.1	3.8	3.5
Convolutional Neural Network	5.12	4.6	4.18	3.7	3.5	3.4
Compressive Sensing	4.38	3.93	3.25	2.96	2.7	2.5
Compressive Sensing + Convolutional Neural Networks	3.22	2.98	2.65	2.21	1.94	1.82

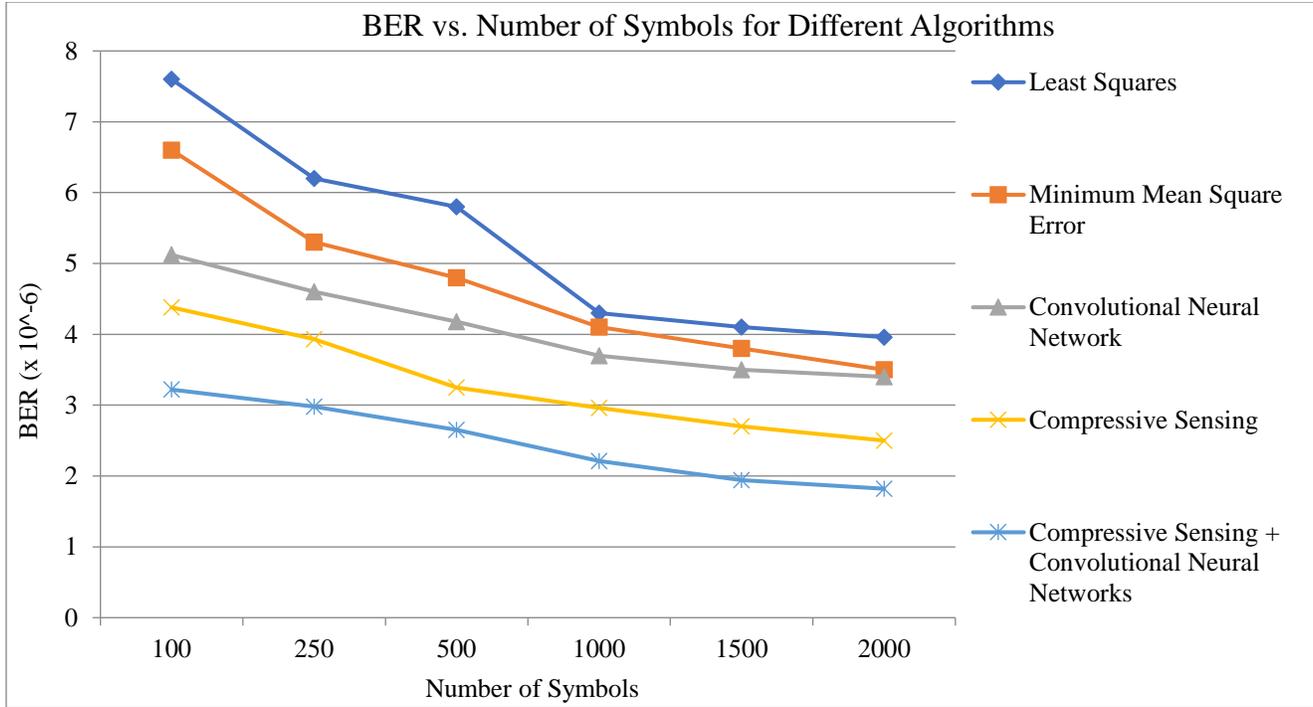


Fig. 4 BER vs the Algorithm used (SNR = 10 dB)

5. Conclusion

The main objective of this work is to improve the channel estimation (in turn the BER also) using a combination of algorithms. In this work, it is shown that the combination of Compressive Sensing and convolutional Neural Networks gives the least RMSE and the least BER. It is also shown that both these metrics depend on the number of pilot symbols used for the channel estimation.

The relation between these parameters has been experimented with and tabulated. It is shown that the CS + CNN gives an RMSE of 0.11 and BER of 1.82×10^{-6} (both with

pilot symbols = 2000). In conclusion, combining Compressive Sensing and Convolutional Neural Networks improves accuracy compared to the individual algorithms. As a future work, the same combination algorithm can be used to obtain the channel estimates of different channel models.

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