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Research Article

A Stochastic Model for Performability and Sensitivity Analysis of a Self-Indicating System

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Abstract - *The advancement of numerous technical fields has resulted in the development of complex systems. A notable innovation in this landscape is the self-indicating system – a purposefully designed mechanism or device that autonomously provides feedback or information about its own state, condition, or operation. This autonomy eliminates the reliance on ext ernal signals or inputs. The present paper investigates the stochastic model of a single-unit self-indicating system. When the system senses disruptions in its functioning, it enters a self-indicating mode and uses its built-in processes to give signals of the disruptions. Subsequently, a repairman conducts an inspection for repair, replacement, or maintenance. Preventive/corrective maintenance is performed frequently to enhance system productivity. Regenerative and Markov processes are used to accomplish stochastic analysis of the system. Expressions for performance metrics, including system reliability, Mean Time To Failure (MTTF), availability and busy period, are derived in this article. Additionally, the profitability of the system is discussed through the development of a profit function. Sensitivity functions for the derived measures are also defined. All the time distributions used in the study are considered general. A numerical analysis is conducted to validate the developed model by assuming all time distributions are exponential with specified parameter values. Lower/upper bounds for system profitability and factors affecting the least/most the various performance measures are obtained.*

Keywords - *Self-Indicating system, Stochastic model, Regenerative process, Markov process, Profit function, Sensitivity analysis.*

1. Introduction

Recent technological advancements have ushered in a wide array of innovations, prompting manufacturers to offer consumers an extensive range of options. While these developments bring significant benefits, the systems' growing complexity and diversity pose challenges. Users express concerns about the security and reliability of these intricate technologies. Acknowledging user concerns, manufacturers underscore the importance of ensuring the reliability and security of their products. To address these apprehensions, companies are increasingly focusing on the development of self-indicating systems. A self-indicating system is a technological configuration or device with features that autonomously detect and communicate information about its status, performance, or potential malfunctions. This technology greatly aids in extending the life span of systems. Examples of self-indicating systems abound, including warning lights in vehicles, error notifications on electronic devices, and advanced applications in Smartphones, Refrigerators, and Car Tire Pressure Monitoring Systems, among others. By incorporating self-indicating features, companies seek to boost user confidence, proactively address potential issues, and improve their technological systems' overall maintenance and durability. As a result, system reliability characteristics like MTTF, availability, and economic factors like revenue and expenditures that impact its profitability have become compelling subjects for further research, particularly with regard to their practical industrial applications. Numerous researchers have investigated the reliability of various industrial systems under different operating conditions.

Singh et al. [1] examined the reliability of a complex variable system composed of three priority-based units using the Gumbel-Hougaard copula. The key reliability measures, such as MTTF and the profit function, are derived using the supplementary variable technique and Laplace transform. Hoseinie et al. [2] focused on reliability modelling of the drum shearer machine and its subsystems, essential components in coal cutting and production processes at mechanized longwall mines. Lal et al. [3] proposed two stochastic models and a computational algorithm to evaluate the performance of piston manufacturing plants. Availability was used as the primary performance metric to assess system effectiveness. Kumar et al. [4] explored a stochastic model of a concrete mixture plant, discussing preventive maintenance after maximum operation time using the supplementary variable technique. Kumar and Batra [5] proposed a stochastic model for the PCB manufacturing process, considering a hardware-software integrated system and two distinct types of repair personnel. Navas et al. [6] discussed the reliability of railway repairable systems, utilizing homogeneous, non-homogeneous Poisson, and renewal process models to characterize failure intensity. Tsarouhas [7] carried out a Reliability, Availability, Maintainability (RAM) analysis of a wine packaging line using failure data and statistical modeling techniques to optimize maintenance schedules.

Zeng et al. [8] proposed an analytical framework to assess the reliability of non-repairable hardware-software codesign systems, emphasizing the interactions between hardware and software components. Gupta [9] analyzed the performance and reliability of a power plant's condensate system using a Markov birth-death process framework. Gahlot et al. [10] investigated a series system composed of two subsystems arranged in a parallel configuration, incorporating the influence of human operator dependency. The study integrated both exponential and copula-based repair distributions to better model real-time repair scenarios. Tyagi et al. [11] developed a Markovian model for an IoT-based flood alerting system, evaluating system reliability measures, profitability, and sensitivity. Sharma and Kaur [12] evaluated the reliability performance of a boiler system in a steam generation plant, accounting for both major and minor failures, thereby enhancing maintenance scheduling and downtime reduction. Sanusi et al. [13] determined the automated teller machine's reliability attributes.

The analysis employed the Gumbel-Hougaard family copula repair policy. One of the researcher investigated the reliability and economic performance of a membrane biofilm fuel cell system that includes a proton exchange membrane along with anode and cathode electrodes. They also conducted a sensitivity analysis for system availability and profit function. John et al. [14] formulated a mathematical model for a multi-component hardware-software system, considering different types of failure interactions. Sensitivity analysis of derived reliability indices was included to support decision-making.

Monika and Chopra [15] studied a food industry system comprising two series-connected subsystems responsible for production and packaging. Balushi et al. [16] investigated the reliability of power transformers in a power distribution company using real operational data. They also estimated confidence intervals for failure rates, contributing to better asset management practices. Ibrahim et al. [17] conducted Reliability, Availability, Maintainability, and Dependability (RAMD) analysis of a series-parallel photovoltaic system consisting of four subsystems. The study involved solving a system of linear differential-difference equations for performance evaluation.

The aim of the present study is to develop a discrete state-space continuous-time stochastic model for self-indicating systems, a class of systems that autonomously signal their operational status or failures. Despite their widespread presence in daily life-such as in refrigerators, water purifiers, and similar appliances-such systems have not been rigorously modelled in existing reliability literature. The proposed model integrates system-specific characteristics to evaluate key reliability attributes, formulate a profit function, and perform a sensitivity analysis on both the reliability measures and the economic performance.

The approach offers a novel framework for analyzing and optimizing the reliability and profitability of self-indicating systems, contributing valuable insights for both theoretical modelling and practical applications. The following is an outline of the paper. Section 1 presents an introduction to the work done. Section 2 describes the system and its underlying assumptions. Section 3 presents a list of the notations utilized in this study, and Section 4 proposes a stochastic model of the system. Sections 5 and 6 provide mathematical expressions for system reliability, availability and measures influencing system profitability. On the basis of the obtained measures, Sections 7 and 8 concentrate on developing profit and sensitivity functions for the given system. Numerical illustrations for the exponential situation are examined in Section 9. In Section 10, concluding remarks are given.

2. System Description and Assumptions

- The system under investigation comprises a single operational unit.
- Upon detection of any disruption in its functioning, the system transitions into a self-indicating mode, thereby signaling the presence of faults. During this mode, a repairman conducts an inspection to determine the necessity for repair, replacement, or maintenance.
- The model incorporates the possibility of system failure from both the initial operational and self-indicating states.
- In the event of failure-whether originating from the initial state or the self-indicating mode-the failed system is subjected to inspection, followed by appropriate repair, replacement, or maintenance procedures.
- Preventive /corrective maintenance strategies are employed with the objective of enhancing the system's overall reliability and productivity.
- The system is supported by a single repair facility that performs all maintenance and repair activities.
- It is assumed that the system does not experience failure during the inspection process.
- The time distributions used in the study are considered to be general, allowing for a flexible and comprehensive representation of system behavior.

Figure 1 depicts the described system. A stochastic model of the described system is developed using Markov and regenerative process frameworks. Laplace-Stieltjes transforms and convolution are used to derive expressions for various reliability characteristics, including system reliability, MTTF, and availability. Additionally, analytical expressions are obtained for key performance indicators that influence system profitability, such as the estimated time the repair personnel are engaged in inspection, repair, and maintenance activities and the total system downtime. Based on these results, system profitability and associated sensitivity functions are formulated. Numerical calculations are performed by assuming that all relevant time distributions follow an exponential distribution.

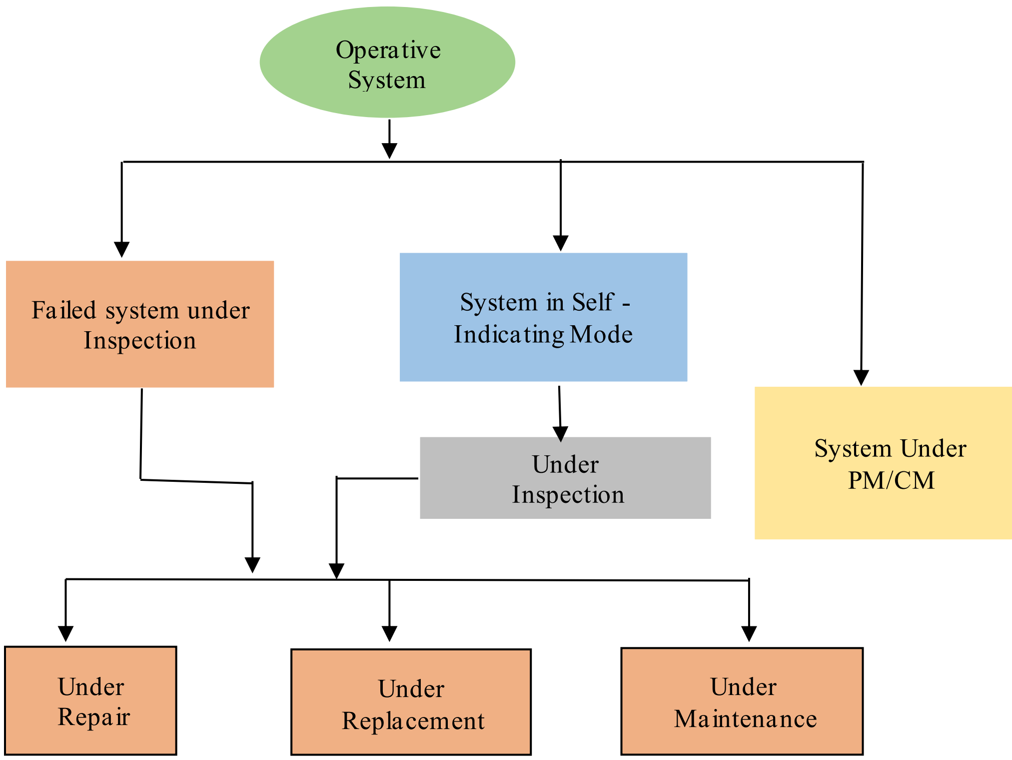

Fig. 1 System description

3. Nomenclature

The notations for different probabilities and transition densities are:

- $p_1/p_2/p_3$: probability of detection of repair/ replacement/ maintenance when the system is operative and is in self-indicating mode.
- $q_1/q_2/q_3$: probability of detection of repair/replacement maintenance on failure.
- $f_1(t)$, $f_2(t)$, $f_3(t)$ /$F_1(t)$, $F_2(t)$, $F_3(t)$:pdf/cdf of time taken to transit in self-indicating mode, inspection mode When the system is in self-indicating mode, failure mode from self-indicating mode.
- $i(t)$, $i_1(t)$ / $I(t)$, $I_1(t)$: pdf/cdf of inspection time from self indicating, failed state.
- $g(t)$, $g_1(t)$, $g_2(t)$/$G(t)$, $G_1(t)$, $G_2(t)$: pdf /cdf of repair, replacement, maintenance time from self indicating mode.
- $g_3(t)$, $g_4(t)$, $g_5(t)$/$G_3(t)$, $G_4(t)$, $G_5(t)$: pdf /cdf of repair, replacement, maintenance time from the failed state.
- ©/®: Laplace / Laplace- Steiljes convolution.
- $q_{ij}(t)/Q_{ij}(t)$: pdf /cdf representing transition time from state i to j.
- I_0: Initial state of the system.

4. Stochastic Model

Using a probabilistic approach, the different states of the system and notations used in describing them are.

State 0: Op; Operative state
State 1: $Op^{(SIM)}$; Operative state in Self-Indicating Mode (SIM)
State 2: U_i; Failed system under inspection.
State 3: $Op^{(PM/CM)}$; Operative system under preventive/ corrective maintenance.
State 4: $Op_{ui}^{(SPM)}$; Operative system under inspection during self-indicating mode.
State 5/State 6/State7: D_r/ D_{rep}/ D_M; System in downstate for repair replacement/maintenance.
State 8/State 9/State10: U_r / U_{rep}/ U_M; Failed system under repair /replacement /maintenance.

By employing Markov and regenerative processes, a stochastic model showing transitions between various states is depicted in Figure 2. The state space is made up of the regenerative states S={0,1, 2,3,4,5,6,7,8,9,10} where O={0, 1, 3,4} is the operative state space and F={2,8,9, 10}is the failed state space, and D={5,6,7} is the downstate space, respectively.

The transition densities between various states are

$q_{01}(t) = f_1(t)\overline{H_1}(t)\overline{F}(t)$, $q_{02}(t) = \overline{F_1}(t)f(t)\overline{H_1}(t)$
$q_{03}(t) = \overline{F_1}(t)h_1(t)\overline{F}(t)$, $q_{12}(t) = \overline{F_2}(t)f_3(t)$
$q_{14}(t) = \overline{F_3}(t)f_2(t)$, $q_{28}(t) = q_1 i_1(t)$, $q_{29}(t) = q_2 i_1(t)$
$q_{2,10}(t) = q_3 i_1(t)$, $q_{30}(t) = h_2(t)$, $q_{45}(t) = p_1 i(t)$
$q_{46}(t) = p_2 i(t)$, $q_{47}(t) = p_3 i(t)$, $q_{50}(t) = g(t)$
$q_{60}(t) = g_1(t)$, $q_{70}(t) = g_2(t)$,
$q_{80}(t) = g_3(t)$ $q_{90}(t) = g_4(t)$, $q_{10,0}(t) = g_5(t)$

Expected time spent in a particular state i (μ_i) is given as:

$\mu_0 = \int_0^\infty \overline{F_1}(t)\overline{H_1}(t)\overline{F}(t)\,dt$, $\mu_1 = \int_0^\infty \overline{F_2}(t)\overline{F_3}(t)\,dt$
$\mu_2 = \int_0^\infty \overline{I_1}(t)dt$, $\mu_3 = \int_0^\infty \overline{H_2}(t)dt$, $\mu_4 = \int_0^\infty \overline{I}(t)\,dt$,
$\mu_5 = \int_0^\infty \overline{G}(t)\,dt$, $\mu_6 = \int_0^\infty \overline{G_1}(t)dt$, $\mu_7 = \int_0^\infty \overline{G_2}(t)\,dt$
$\mu_8 = \int_0^\infty \overline{G_3}(t)dt$, $\mu_9 = \int_0^\infty \overline{G_4}(t)dt$, $\mu_{10} = \int_0^\infty \overline{G_5}(t)dt$

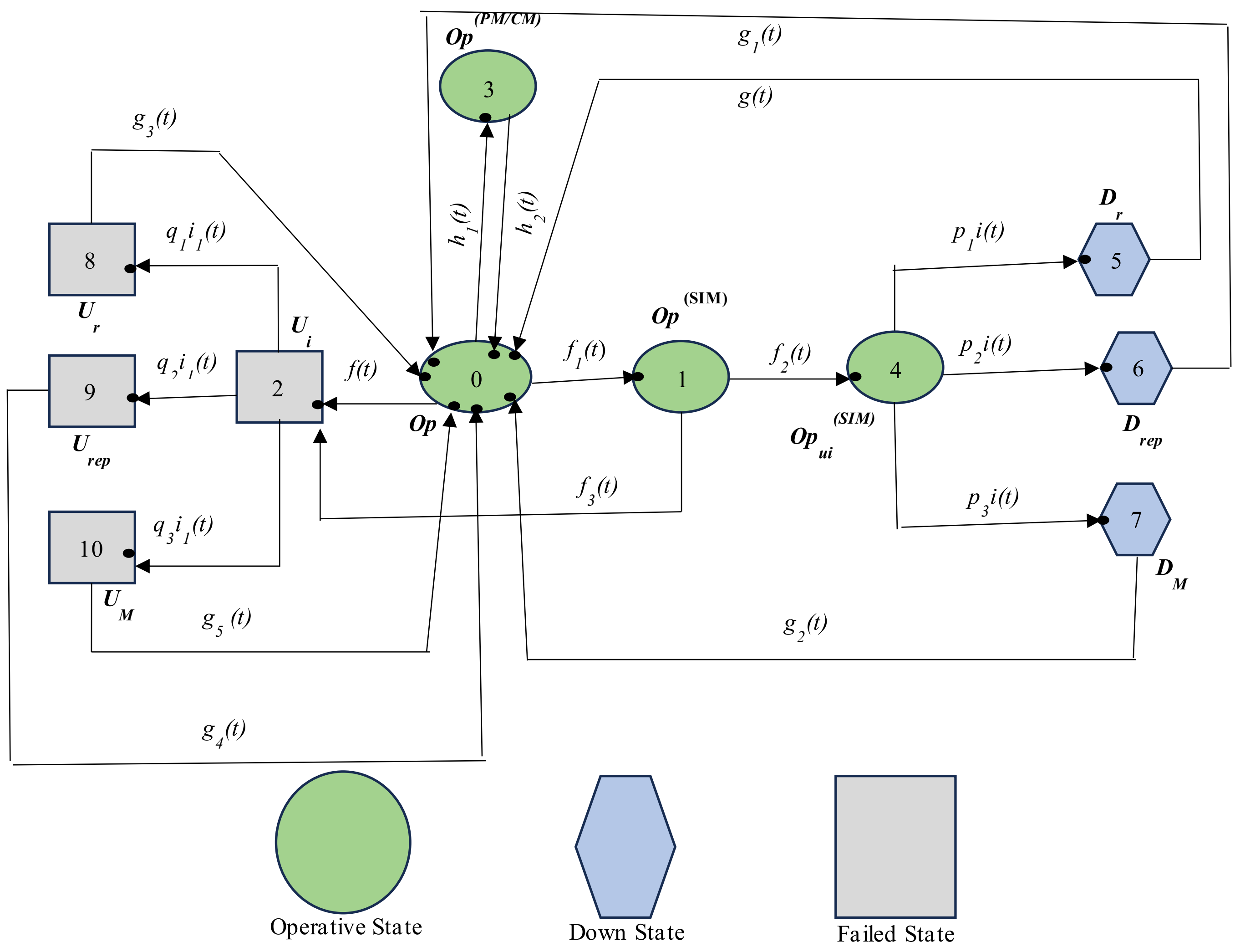

Fig. 2 Transition diagram

5. System Reliability and MTTF

Let $\xi_i(t)$ = Pr(system fails in time t | I_0=i). From the transition diagram, the system may move from state 0 to failed state 2 in time t with c.d.f. Q_{02} (t). Further, it can also move from state 0 to either state 1 or 3 in time u < t and then move to the failed state in time t with probability $\int_0^t Q_{01}(u)\ \xi_1(t-u)$ du and $\int_0^t Q_{03}(u)\ \xi_3(t-u)$ du respectively. Considering the different possible transitions from state 0 and using the addition law of probability, we have

$$\xi_0(t) = Q_{01}(t)\circledR\xi_1(t) + Q_{02}(t) + Q_{03}(t)\circledR\xi_3(t)$$

Similarly,

$\xi_1(t) = Q_{14}(t) Ⓢ \xi_4(t) + Q_{12}(t)$

$\xi_3(t) = Q_{30}(t) Ⓢ \xi_0(t)$

$\xi_4(t) = Q_{45}(t) Ⓢ \xi_5(t) + Q_{46}(t) Ⓢ \xi_6(t) + Q_{47}(t) Ⓢ \xi_7(t)$

$\xi_5(t) = Q_{50}(t) Ⓢ \xi_0(t)$

$\xi_6(t) = Q_{60}(t) Ⓢ \xi_0(t)$

$\xi_7(t) = Q_{70}(t) Ⓢ \xi_0(t)$

Using the Laplace-Stieltjes transformation and solving the above equations for $\xi_0^{**}(s)$, we have $\xi_0^{**}(s) = N(s)/L(s)$ Now, the system's reliability $\{R(t)\} = L^{-1}[\{1 - \xi_0^{**}(s)\}/s]$ and

$$\text{MTTF} = \int_0^\infty R(t)\,dt = N/L$$

Where,

$$L(s) = \begin{vmatrix} 1 & -Q_{01}^{**}(s) & -Q_{03}^{**}(s) & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -Q_{14}^{**}(s) & 0 & 0 & 0 \\ -Q_{30}^{**}(s) & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -Q_{45}^{**}(s) & -Q_{46}^{**}(s) & -Q_{47}^{**}(s) \\ -Q_{50}^{**}(s) & 0 & 0 & 0 & 1 & 0 & 0 \\ -Q_{60}^{**}(s) & 0 & 0 & 0 & 0 & 1 & 0 \\ -Q_{70}^{**}(s) & 0 & 0 & 0 & 0 & 0 & 1 \end{vmatrix}$$

$$N(s) = \begin{vmatrix} Q_{02}^{**}(s) & -Q_{01}^{**}(s) & -Q_{03}^{**}(s) & 0 & 0 & 0 & 0 \\ Q_{12}(s) & 1 & 0 & -Q_{14}^{**}(s) & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -Q_{45}^{**}(s) & -Q_{46}^{**}(s) & -Q_{47}^{**}(s) \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{vmatrix}$$

$N = \mu_0 + p_{01}\mu_1 + p_{01}p_{14}\mu_4 + p_{01}p_{14}p_{45}\mu_5 + p_{01}p_{14}p_{46}\mu_6 + p_{01}p_{14}p_{47}\mu_7$
and $L = 1 - p_{01}p_{14} - p_{03}$

6. System Availability

Defining $AV_i(t)$ = Pr (system is in operative mode at time t | I_0=i) and $W_i(t)$= Pr (system is operative at time in state i.).The recursive relations for $AV_i(t)$ can be expressed as follows:

$AV_0(t) = W_0(t) + \sum_i q_{0i}(t) © AV_i(t)$, i=1,2,3

$AV_1(t) = W_1(t) + \sum_i q_{1i}(t) © AV_i(t)$, i=2,4

$AV_2(t) = \sum_i q_{2i}(t) © AV_i(t)$, i=8,9,10

$AV_3(t) = W_3(t) + q_{30}(t) © AV_0(t)$

$AV_4(t) = W_4(t) + \sum_i q_{4i}(t) © AV_i(t)$, i=5,6,7

$AV_5(t) = q_{50}(t) © AV_0(t)$

$AV_6(t) = q_{60}(t) © AV_0(t)$

$AV_7(t) = q_{70}(t) © AV_0(t)$

$AV_8(t) = q_{80}(t) © AV_0(t)$

$AV_9(t) = q_{90}(t) © AV_0(t)$

$AV_{10,0}(t) = q_{10,0}(t) © AV_0(t)$

Where, $W_0(t) = \overline{F_1}(t)\overline{H_1}(t)\bar{F}(t)$, $W_1(t) = \overline{F_2}(t)\overline{F_3}(t)$, $W_3(t) = \overline{H_2}(t)$, $W_4(t) = \bar{I}(t)$.

Using the Laplace transformation and solving the above equations for $AV_0^*(s)$, we have $AV_0^*(s) = N_1(s)/L_1(s)$

Where,

$$N_1(s) = \begin{vmatrix} W_0^*(s) & -q_{01}^*(s) & -q_{02}^*(s) & -q_{03}^*(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ W_1^*(s) & 1 & -q_{12}^*(s) & 0 & -q_{14}^*(s) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ W_3^*(s) & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ W_4^*(s) & 0 & 0 & 0 & 1 & -q_{45}^*(s) & -q_{46}^*(s) & -q_{47}^*(s) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{vmatrix}$$

$$L_1(s) = \begin{vmatrix} 1 & -q_{01}^*(s) & -q_{02}^*(s) & -q_{03}^*(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -q_{12}^*(s) & 0 & -q_{14}^*(s) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -q_{30}^*(s) & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -q_{45}^*(s) & -q_{46}^*(s) & -q_{47}^*(s) & 0 & 0 & 0 \\ -q_{50}^*(s) & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ -q_{60}^*(s) & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ -q_{70}^*(s) & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ -q_{80}^*(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ -q_{90}^*(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ -q_{10,0}^*(s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{vmatrix}$$

The system availability in the steady state is

$$AV_0 = \lim_{t\to\infty} AV_0(t) = \lim_{s\to 0} sAV_0^*(s) = N_1/L_1$$

Where,

$$L = \mu_0+\mu_1 p_{01}+\mu_2 p_{02}+\mu_2 p_{01} p_{12}+\mu_3 p_{03}+\mu_4 p_{01} p_{14}+p_{01} p_{14}(p_{45}\mu_5+\mu_6 p_{46} + \mu_7 p_{47}) + p_{01} p_{12}(p_{28}\mu_8+\mu_9 p_{29}+\mu_{10} p_{2,10})+p_{02}(p_{28}\mu_8 +\mu_9 p_{29}+\mu_{10} p_{2,10})$$

and $N_1 = \mu_0 + \mu_1 p_{01} + \mu_3 p_{03} + \mu_4 p_{01} p_{14}$

Proceeding as earlier, in steady state, the other parameters affecting system profitability are:

- Expected downtime of the system $(DT_0) = N_2/L_1$
- Expected time taken by the repairman for maintenance of the system $(MT_0) = N_3/L_1$
- Expected number of replacements $(ER_0) = N_4/L_1$
- Expected time taken by the repairman for inspecting the system $(IT_0) = N_5/L_1$
- Expected time taken by the repairman to repair the system $(BP_0) = N_6/L_1$
- Expected counts of visits by repairmen $(VR_0) = N_7/L_1$
- Expected counts of preventive/ corrective maintenance $(PC_0) = N_8/L_1$

Where

$N_2 = p_{01} p_{14} (p_{45}\mu_5 + p_{46}\mu_6 + p_{47}\mu_7)$

$N_3 = p_{01} p_{14} p_{47}\mu_7 + p_{01} p_{12} p_{2,10}\mu_{10} + p_{02} p_{2,10}\mu_{10}$

$N_4 = p_{02} p_{29} + p_{01} p_{14} p_{46} + p_{01} p_{12} p_{29}$

$N_5 = p_{02}\mu_2 + p_{01} p_{14}\mu_4 + p_{01} p_{12}\mu_2$

$N_6 = p_{02} p_{28}\mu_8 + p_{01} p_{14} p_{45}\mu_5 + p_{01} p_{12} p_{28}\mu_8$

$N_7 = p_{02} + p_{01} p_{14} + p_{01} p_{12}$

$N_8 = p_{03}$.

7. Profit Function

The profit function is formulated in this section to assess the system's profitability. A profit function is a mathematical equation that describes the relation between the system's total revenue and total maintenance expenses. The profit function for the described system in a steady state is:

$$P = R_0 AV_0 - E_1 DT_0 - E_2 MT_0 - E_3 ER_0 - E_4 IT_0 - E_5 BP_0 - E_6 VR_0 - E_7 PC_0$$

- R_0: revenue generated
- E_1: loss incurred during system downtime
- $E_2/E_3/E_4/E_5/E_6/E_7$: expenditure incurred during maintenance/replacement/inspection/repair/ visits by repairman/preventive, corrective maintenance

Revenue/loss/expenditures are considered per unit time.

8. Sensitivity Functions

Sensitivity analysis is a method used to evaluate how variations in parameters affect a system's performance, especially when those parameters vary widely in value. It identifies the most influential parameters, supporting informed decision-making, effective risk management, enhanced system reliability, and ensuring cost-effective resource allocation. A standardized version of this analysis is the relative sensitivity function. The following equations are devised to determine the sensitivity and relative sensitivity functions for the system MTTF, availability, and profit function, respectively.

$$\chi_k = \frac{\partial MTTF}{\partial k}, \quad \omega_k = \chi_k(k/MTTF)$$

$$\lambda_k = \frac{\partial AV_0}{\partial k}, \quad \tau_k = \lambda_k(k/AV_0)$$

$$\epsilon_k = \frac{\partial P}{\partial k}, \quad \delta_k = \epsilon_k(k/P),$$

Where k is parameter

9. Numerical Results and Discussions

For numerical results and discussion, consider that all distributions are exponentially distributed,

$$f(t) = \alpha e^{-\alpha t}, \quad f_1(t) = \alpha_1 e^{-\alpha_1 t}, f_2(t) = \alpha_2 e^{-\alpha_2 t}$$
$$f_3(t) = \alpha_3 e^{-\alpha_3 t}, i(t) = \alpha_4 e^{-\alpha_4 t}, \quad i_1(t) = \alpha_5 e^{-\alpha_5 t}$$
$$g(t) = \beta e^{-\beta t}, \quad g_1(t) = \beta_1 e^{-\beta_1 t}, g_2(t) = \beta_2 e^{-\beta_2 t}$$
$$g_3(t) = \gamma_1 e^{-\gamma_1 t}, g_4(t) = \gamma_2 e^{-\gamma_2 t}, g_5(t) = \gamma_3 e^{-\gamma_3 t}$$
$$h_1(t) = \beta_3 e^{-\beta_3 t}, h_2(t) = \beta_4 e^{-\beta_4 t}$$

And the values involved are as follows
$\alpha = 0.01$, $\alpha_1 = 0.01$, $\alpha_2 = 0.003$, $\alpha_3 = 0.28$, $\alpha_4 = 0.037$,
$\alpha_5 = 0.043, \beta = 0.27$, $\beta_1 = 0.7, \beta_2 = 0.35, \beta_3 = 0.29$, $\beta_4 = 0.32, \gamma_1 = 0.4, \gamma_2 = 0.417, \gamma_3 = 0.5, p_1 = 0.5$, $p_2 = 0.3$, $p_3 = 0.2$, $q_1 = 0.5, q_2 = 0.3, q_3 = 0.2$, $R_0=200$, $E_1=275$, $E_2=300$, $E_3=300$, $E_4=250$, $E_5=130$, $E_6=225$, $E_7=240$.

9.1. Impact of Various Rates on Reliability, MTTF and Availability

Maintaining constant values for the remaining parameters, the reliability function R(t) for various values of rate α is given as

For $\alpha = 0.01$,

$$R(t) = -2.38436 \times 10^{-15} + 3.38398 \times 10^{-7} e^{-0.700004t} + 0.0075585\, e^{-0.619669t} + 2.1764 \times 10^{-9} e^{-0.35t}\ 2.66761 \times 10^{-8} e^{-0.270001t} + 1.15386 e^{-0.00935t} + e^{(0.0344843)t}(-0.0807097) - (0.0807097)$$

For $\alpha = 0.02$,

$$R(t) = -3.18718 \times 10^{-15} + 7.97173 \times 10^{-7} e^{-0.700004t} + 0.015631\, e^{-0.624628t} + 4.53077 \times 10^{-9} e^{-0.35t} + 5.7524 \times 10^{-8} e^{-0.270001t} + 1.13907\, e^{-0.0139019t} + e^{(-0.0347327)t}(-0.0773523) - (0.0773523)$$

For $\alpha = 0.03$,

$$R(t) = -2.03808 \times 10^{-15} + 1.40333 \times 10^{-16} e^{-0.700004t} + 0.0238339\, e^{-0.629668t} + 6.85485 \times 10^{-9} e^{-0.35t} + 8.90598 \times 10^{-8} e^{-0.270001t} + 1.08286 e^{-0.0179911t} + e^{(-0.0351676)t}(-0.0533494) - (0.0533494)$$

The impact of different rates on reliability (R(t)), MTTF, and availability (AV_0) is presented in Tables 1, 2, and 3, respectively. It is evident that:

- The system's reliability (R(t)) declines as time (t) and failure rate (α) rise.
- MTTF decreases with a rise in failure rate (α) and replacement rate (β_1).
- Availability increases as the repair rate (α_5) and inspection rate (γ_1) increase.

Table1. R(t) for varied (t, α)

t (time in hrs)	R(t)		
	α=0.01	α=0.02	α=0.03
0	1	1	1
10	0.935948	0.883665	0.834839
20	0.875325	0.787923	0.7104
30	0.813561	0.698951	0.602196
40	0.752602	0.617527	0.508896
50	0.693714	0.54386	0.42899
60	0.637674	0.4777783	0.360919
70	0.58492	0.418904	0.30317

Table 2. MTTF for varied (α_1, β_1)

α	MTTF		
	β_1=0.02	β_1=0.04	β_1=0.08
0.001	136.1206	135.3971	135.0354
0.002	123.7822	123.1243	122.7953
0.003	113.4947	112.8914	112.5898
0.004	104.7859	104.2290	103.9505
0.005	97.3184	96.8012	96.5425
0.006	90.8444	90.3616	90.1202
0.007	85.1781	84.7254	84.4990

Table 3. AV_0 for varied (α_5, γ_1)

α_5	Availability (AV_0)		
	γ_1=0.3	γ_1=0.6	γ_1=0.9
0.015	0.6300	0.6328	0.6337
0.017	0.6574	0.6604	0.6614
0.019	0.6808	0.6840	0.6851
0.021	0.7009	0.7044	0.7056
0.023	0.7185	0.7222	0.7234
0.027	0.7340	0.7378	0.7391
0.029	0.7478	0.7517	0.7530

9.2. Effects of Different Rates and Expenses on the Profit Function

Keeping the other parameters constant, the variations in profit function (P) are examined for (R_0, E_5), (R_0, α), (E_6, γ_1) and (E_7, β_3) in Figures 3, 4, 5 and 6, respectively. The results are summarised as follows:

- Profit increases with a rise in R_0 and is higher for lower values of E_5 and α, respectively.
- Profit declines with rising values of E_6, E_7 and β_3, respectively.
- Profit rises for a rise in γ_1.

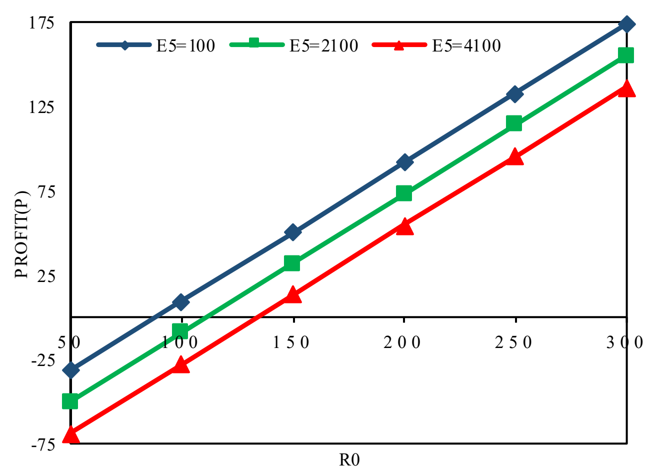

Fig. 3 P for varied (R_0, E_5)

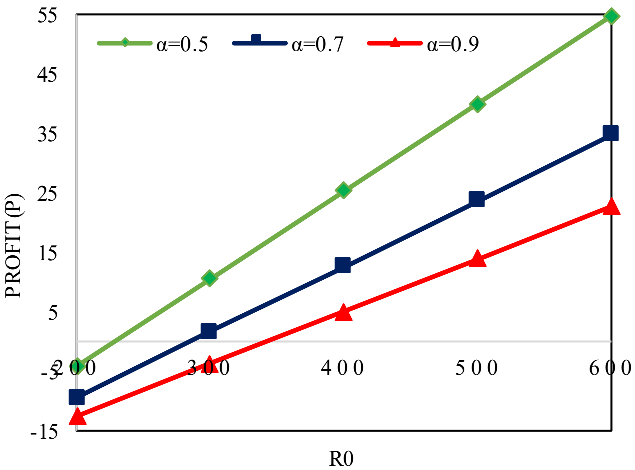

Fig. 4 P for varied (R_0, α)

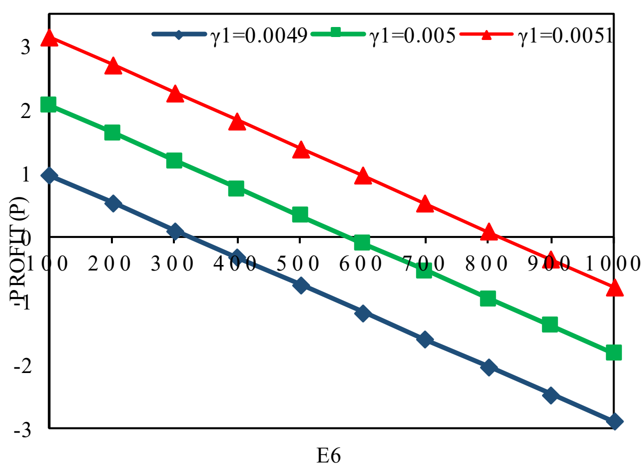

Fig. 5 P for varied (E_6, γ_1)

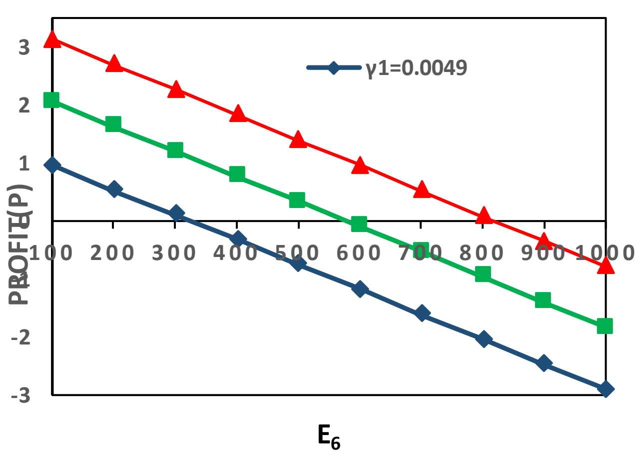

Fig. 6 P for varied (E_7, β_3)

Table 4. Upper/lower bounds for system profitability

Source	Parameter Value	Profit function(P)≥0, if
Figure 3	E_5=100 E_5=2100 E_5=4100	$R_0 \geq 88.21$ $R_0 \geq 110.86$ $R_0 \geq 133.54$
Figure 4	α=0.5 α=0.7 α=0.9	$R_0 \geq 228.13$ $R_0 \geq 285.94$ $R_0 \geq 342.97$
Figure 5	γ_1 =0.0049 γ_1=0.0050 γ_1 =0.0051	$E_6 \leq 303.13$ $E_6 \leq 576.56$ $E_6 \leq 820.31$
Figure 6	β_3 =0.6 β_3=0.75 β_3 =0.99	$E_7 \leq 859.34$ $E_7 \leq 810.94$ $E_7 \leq 757.81$

Further bounds (upper/lower) pertaining to system profitability are listed, i.e.

- For E_5=100, $P \geq 0$ iff $R_0 \geq 88.21$.
- For α=0.5, $P \geq 0$ iff $R_0 \geq 228.13$.
- For γ_1 =0.0049, $P \geq 0$ iff $E_6 \leq 303.13$.
- For β_3 =0.6, $P \geq 0$ iff $E_7 \leq 859.34$.

Bounds for the other values of E_5, α, γ_1 and β_3 are mentioned in Table 4.

9.3. Computations for Sensitivity Functions

Tables 5, 6, and 7 include the numerical values for the relative sensitivity and sensitivity functions, as described in Section 8.

Table 5. Values of sensitivity and relative sensitivity functions for MTTF

Parameter(k)	$\xi_k = \frac{\partial\, MTTF}{\partial k}$	$\sigma_k = \xi_k(k/MTTF)$
α	-3731.1	-0.5254
α_1	-1523.2	-0.2145
α_2	997.7642	0.0422
α_3	-712.2302	-0.2808
α_4	-37.1420	-0.0194
β	-0.3487	-0.0013
β_1	-0.0311	-0.00030657
β_2	-0.0830	-0.00040909
β_3	7.3896 x 10^{-13}	3.0178 x 10^{-15}

Table 6. Values of sensitivity and relative sensitivity functions for AV_0

Parameter(k)	$\rho_k = \frac{\partial AV_0}{\partial k}$	$\tau_k = \rho_k\left(\frac{k}{AV_0}\right)$
α	-7.6663	-0.0933
α_1	-4.7377	-0.0577
α_2	1.8204	0.0066
α_3	-0.9447	-0.0322
α_4	-0.0460	-0.0021
α_5	3.0794	0.1612
β	0.0020	0.00065741
β_1	0.00017726	0.00015106
β_2	0.00047268	0.00020141
β_3	0.2033	0.0718
β_4	-0.1843	-0.0718

γ_1	0.0178	0.0087
γ_2	0.0098	0.0050
γ_3	0.0046	0.0028

Table 7. Values of sensitivity and relative sensitivity functions for P

Parameter (k)	$\omega_k = \frac{\partial P}{\partial k}$	$\delta_k = \omega_k(k/P)$
α	-3189.9	-0.3484
α_1	-1804.4	-0.1971
α_2	-75.7391	-0.0025
α_3	-446.9353	-0.1367
α_4	36.4485	0.0147
α_5	1280.6	0.6014
β	1.2005	0.0035
β_1	0.0791	0.00060475
β_2	0.3836	0.0015
β_3	36.0157	0.1141
β_4	-111.8608	-0.3910
γ_1	4.7997	0.0210
γ_2	1.0950	0.0050
γ_3	2.1715	0.0119
R_0	0.8214	1.7942
E_1	-0.0010	-0.0030
E_2	-0.0093	-0.0132
E_3	-0.0073	-0.0179
E_4	-0.1056	-0.2768
E_5	-0.1707	-0.4661
E_6	-0.0022	-0.0072
E_7	-0.0030	-0.0098

Upon considering the absolute values of these functions, it becomes apparent that:

- MTTF is most affected by α and least affected by β_3
- Availability is predominantly influenced by α, with β_1 having the least impact.
- System profitability is primarily impacted by α and least by E_6.

The sequence in which parameters affect the measured outcomes in descending order is given as follows:

MTTF: $\alpha; \alpha_3; \alpha_1; \alpha_2; \alpha_4; \beta; \beta_2; \beta_1; \beta_3$

Availability:

$\alpha_5; \alpha; \beta_3; \beta_4; \alpha_1; \alpha_3; \gamma_1; \alpha_2; \gamma_2; \gamma_3; \alpha_4; \beta; \beta_2; \beta_1$.

Profitability:

$R_0; \alpha_5; E_4; \beta_4; \alpha; E_7; \alpha_1; \alpha_3; \beta_3; \gamma_1; E_6; \alpha_4; E_5; \gamma_3; E_2; E_3; \gamma_2; \beta; E_1; \alpha_2; \beta_2; \beta_1$

10. Conclusion

This article presents a stochastic model designed for self-indicating systems with arbitrary time distributions. It delves into the reliability characteristics and identifies factors influencing system profitability. Utilizing these findings, profit and sensitivity functions are formulated, with numerical results specifically explored for the exponential case. Additionally, bounds concerning various expenses related to system profitability are established.

The parameters exerting the greatest influence on both reliability measures and system profitability are highlighted through sensitivity analysis. The developed model demonstrates significant economic promise for industries leveraging such technologies. The limitation of this study is the lack of real-time data on various rates and costs, which would enhance the practical relevance of the results. However, since general time distributions are employed in the analysis, any time distribution that aligns with real-time data (if available) can be incorporated.

10.1. Future Work

The current study focuses on a single-unit self-indicating system. Future research could extend this model to a two-unit standby self-indicating system incorporating the concept of warranty.

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