International Journal of Engineering Trends and Technology
ISSN: 2231-5381/ https://doi.org/10.14445/22315381/IJETT-V74I10P120

Volume 74 Issue 10, 280-287, October 2026
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Original Article

Designing of Variable Distance Linear Block Codes with High Code Rate

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Received: 09 May 2026 Revised: 19 August 2026 Accepted: 29 August 2026 Published: 27 October 2026

Abstract - *Developing long codes with high levels of error correction capability has been one of the most difficult issues in coding theory. This research introduces an alternative way to create a new type of Increased Distance Linear Block Code (IDLBC). This code has a code rate greater than 1/2. The IDLBC starts with the creation of a generator matrix by using an Upper Triangular Matrix (UTM) and constructing one Linear Block Code (LBC) that has Distance-Preserving Mapping (DPM) property for any (n, k) combinations. Afterwards, an algorithm is used to change the UTM of the generator matrix to give a code with a Distance-Increasing Mapping (DIM) property. The performance of the code is then enhanced using code puncturing to produce a code having a code rate of more than 1/2, but maintaining the improved performance for Bit Error Rate (BER). It is clear that the codes being developed in this paper are superior to the traditional Hamming Codes. The transition from DPM to DIM gives codes with improved distance performance, leading to more reliable performance.*

Keywords - *Code rate, Bit Error Rate, Generator matrix, Parity check matrix, Block code, Decoding, Puncturing.*

1. Introduction

In recent times, research in the field of permutation codes has become popular because of its use in power line communication. The use of permutation codes in power line applications is discussed in [1]. In [2], decoding of the information is obtained with the help of a permutation code. A decoding scheme over power-imbalanced channels for LDPC codes is designed in [3]. The construction of a long binary permutation code is difficult and is a challenging problem. The design of permutation codes has long been viewed by researchers as a problem rooted in mathematics. Algebraic geometry is an integral part of linear block codes as discussed in [4]. The system-theoretic method for a fully controllable discrete-time system can be converted to algebraic mathematical structures that can be used to design linear block codes.

In this paper, an illustration of the construction of a Modified Linear Block Code (MLBC) is provided, where the algebraic formulation is used, and the code rate is greater than ½. To develop such an architecture, the procedure that has been explained below needs to be followed, where it is required to solve the fully controllable discrete time system, which can be seen in Equation (1):

$$x(k+1) = Ax(k) + bu(k) \quad (1)$$

Here, x is the state vector, and u is the input vector at time t, while A and b are the matrices of the system.

For the construction of linear block codes, Generator matrix (G) is used, which the combination of an Identity matrix (I) and a Parity check matrix (P). The solution space of above Equation (1) will form the basis vectors which can be used as rows of P.

When a code is simulated from such a matrix, it is observed that the minimum distance established is 2 for any (n, k) code. To construct an error-correcting code, we propose Distance Increasing Mapping (DIM). The technique used in constructing DIM is a permutation of codewords. It is observed that the code rate achieved for the DIM constructed code is, in general, 1/2. The technique of puncturing of codes is used in this work for constructing MLBC with a code rate more than 1/2. The smallest code established with this method is (7, 4) code and has a minimum distance equal to three. This smallest code, which is established through the puncturing of codes, has similar BER characteristics as that of (7, 4) Hamming code. It is well known that the original state of a discrete-time fully controllable system can be made to all zero vectors in "n" sampling states, governed by Kalman's principle. It is observed that the solution space of these 'n' sampling intervals of the mentioned system by Equation (1) turns out to be an Upper Triangular Matrix (UTM) with all

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entries below the main diagonal equal to zero, as shown below:

$$P = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & .. & .. & 1 & 0 & 0 \\ 1 & .. & .. & 1 & 0 & : & : \\ : & .. & .. & 0 & : & : & 0 \\ 1 & 0 & .. & .. & .. & 0 & 0 \end{bmatrix} \quad (2)$$

Making use of Equation (2) for the construction of the generator matrix, and the simulation work carried on G further gives a single-bit error detection code. If the binary vectors of these arrays of P are permuted by using some transformation, DIM can be constructed. Constructed DIM gives increased distance and multiple error correction capability to the constructed MLBC.

In this research work, for the odd and even nature of binary vectors of the constructed UITM, mapping algorithms are designed. These mappings are further proved to be as DIM with the help of different examples. A dedicated algorithm is proposed using the developed DIM for the construction of the IDLBC for any odd length of binary vectors. Using the concept of puncturing in this algorithm, the error-correcting codes with a code rate greater than ½ are designed. We also propose a lemma based on the established facts that we came across designing of IDLBCs from the constructed DIM.

In Section 2 of this paper, a literature survey is discussed. The methodology of transformation of the generator matrix to get a punctured code with a higher code rate is discussed in segmnet 3. In part 4 and 5, mapping algorithms for punctured IDLBC are constructed. Section 6 presents the main results in the form of a lemma based on detailed analytical research work carried on for designing ILDBC. Section 7 presents conclusion.

2. Literature Survey

In the field of reliable digital communication systems, error control coding plays a vital role as a backbone. Of various coding techniques, linear block codes have attracted considerable attention in terms of their design and effectiveness in detecting and correcting transmission errors. Various studies have been carried out on different aspects of block codes.

Mathematical ideas employed in various communication systems and signal processing techniques were influenced by theoretical developments in system theory. It is observed in the literature that the mathematical ideas developed by Kalman's research work on control systems is widely used in applications, including various engineering fields, and communication systems [5]. An important area of research in coding theory is related to the study of various kinds of mappings from binary vectors to permutations. Chang et al. have carried out a study on various kinds of mappings preserving Hamming distance when binary vectors are transformed into permutations. The effectiveness of the coding scheme in detecting transmission errors can be established by use of such mappings [6]. Further research carried out by Lee [7] introduced various kinds of distance-preserving maps, especially in the case of vectors of odd length.

Subsequent studies have concentrated on the design of mappings whose preservation of distance is accompanied by an increase in distance. Larger Hamming distance is governed by DIM developed by Chang [8]. This results in the improvement of the reliability of the communication system in a noisy environment. Subsequent studies by Chang [8] have proposed various schemes of mappings in which the resulting Hamming distance is increased by at least two units.

Permutation-based structures have been explored in the development of communication systems. Ferreira and Vinck [9] have proposed permutation trellis arrays and have shown the effectiveness of the proposed structure in the development of communication systems. The distance is preserved between the code words by the schemes proposed by Lin et al. [10].

Another significant contribution in this topic is the design of efficient decoding algorithms. Lin et al. [11] gives computationally faster algorithm to compute the syndrome required for decoding. The proposed algorithm will result in the minimization of the computational complexity of the decoding process. Algebraic decoding of the Calderbank-McGuire code has been proposed by Rong and his researchers [12].

Schmidt et al. came up with a new way to decode Reed-Solomon codes based on the computation of syndromes [13]. Algebraic methods of decoding have also been used for quadratic residue codes. Decoding of quadratic residue codes is discussed in [14, 15], which gives improved decoding with a computationally faster algorithm. [16] gives decoding of convolutional codes for reliable communication over erasure channels.

Circulant matrices-based Linear Network Coding (LNC) using permutation techniques has been developed in [17]. The Multicast Solvability LNC is developed in [18], which also incorporates permutation of vectors. In [19], attractive designing of variant codes is discussed with the help of parity-check matrices. Although significant research has been conducted regarding the analysis of mapping techniques, it is still important to explore different coding techniques that offer better distance characteristics along with a high code rate. The present work aims at constructing linear block codes with a higher code rate that exhibit a variable distance property. Such designed codes are useful in high-frequency communication systems.

3. Methodology

3.1. Smallest Fourth-Order UTM for Designing Block Codes with Code Rate 1/2

Consider a fourth-order UTM represented as below, which is a part of the generator matrix:

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \end{bmatrix} \quad (3)$$

After simulation of G, we get the following code words:

```
C = [ 0   0   0   0   0   0   0   0;
      0   0   0   1   1   0   0   0;
      0   0   1   0   1   1   0   0;
      0   0   1   1   0   1   0   0;
      0   1   0   0   1   1   1   0;
      0   1   0   1   0   1   1   0;
      0   1   1   0   0   0   1   0;
      0   1   1   1   1   0   1   0;
      1   0   0   0   1   1   1   1;
      1   0   0   1   0   1   1   1;
      1   0   1   0   0   0   1   1;
      1   0   1   1   1   0   1   1;
      1   1   0   0   0   0   0   1;
      1   1   0   1   1   0   0   1;
      1   1   1   0   1   1   0   1;
      1   1   1   1   0   1   0   1]
```

Above code has GHT = 0 and HCT = 0, which are the properties of a block code and hold the minimum distance equal to 2.

The order of the UTM discussed in the above example is of the order of 4x4. The result can be generalised for nth order UTM system, and MLBCs can be constructed from such a UTM.

It is observed that these MLBCs are always single-error detecting and exhibit Distance Preserving Mapping (DPM). Equation (4) shows nth order matrix for such a transformation.

$$G = \begin{bmatrix} 1 & 0 & 0 & . & .. & .. & \dots & .. & . & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & .. & .. & .. & .. & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & .. & .. & .. & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & .. & .. & .. & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & .. & .. & .. & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & .. & .. & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & .. & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & .. & .. & .. & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & .. & .. & 0 \\ 0 & 0 & 0 & .. & .. & .. & .. & .. & .. & 1 & 1 & 0 & 0 & .. & .. & .. & .. & .. & .. & 0 \end{bmatrix} \quad (4)$$

3.2. Methodology of Transformation of Generator Matrix to get Punctured Code with Enhanced Error Correction Ability

In this section, in order to increase d min for the designed MLBC, following two steps can be carried out. Depending upon whether the binary vector of UTM is an odd or even length, two different transformation techniques for transforming the first row of established UTM, which will be needed to execute step one below, are proposed in the further section of this correspondence.

Step 1: Mapping algorithms discussed in section 4 can be applied to construct the first row of modified P part of G, depending upon odd or even length of UTM. Then, one-bit rotate right operation will get you the second row of the transformed P matrix in G. Repeat rotation for the remaining (n - 2) rows.

In case there exists an odd or even number of rows of UTM of the generator matrix, get the first row of the transformed P matrix in G as per the mapping algorithm. The one-bit right shift will get you the second row of the transformed P matrix in G.

Step 2: Puncture the generator matrix by removing any one of the columns.

4. Mapping Algorithms for Punctured IDLBC with a Code Rate Greater than Half

Step 1's Implementation

Even Order UTM Transformation: The first row of the newly constructed ULTP (C) Modified Upper Triangular Matrix (Modified UTM) will be made by adding together each row of the Upper Triangular Matrix (UTM) and eliminating the last row from the sum, as shown in Table 1.

Table 1. Transformed ULTP(C) matrix

P(C)	ULTP(C)
1111	1 1 0 1
1110	1 1 1 0
1100	0 1 1 1
1000	1 0 1 1

Transformation for Odd Length UTM: Add the given rows of the considered upper triangular matrix and get the initial row.

Construction 1 Algorithm: Odd length binary vector systems

$Input$:
1) $P(C)$
2)$P(c_1,c_2,\ldots\ldots c_n) \in Z_2^n$
$Output$:
$A)(\xi_1,\xi_2,\ldots\ldots\xi_{n-1}) = ULTP(C)\ and$
$B)GeneratorMatrix(G)$
$Begin$:
$ULTP(C_1) = (\psi_1,\psi_2,\ldots..\psi_n)$
$\leftarrow Get\ this\ first\ row\ by$
$adding\ all\ rows\ of\ given\ matrix\ P(C)$
$Perform\ rotate\ one\ bit\ right\ operatiom$
$for\ j\ from\ 0\ to\ (n-2)$
$Rotate\ one\ bit\ right\ of\ every$
$ULTP(C_{1+j})\ to\ get\ ULTP(C_{1+j_{+1}})$
end
$Get\ ULTP(C)$
$Re\ m\ ove\ any\ one\ column\ of\ ULTP(C) \rightarrow\circ P(C)$
$G = [I:\circ P(C)]$
end

Construction 2 Algorithm: ULT: Even length binary vector systems

$Input$:
1) $P(C)$
2)$P(c_1,c_2,\ldots\ldots c_n) \in Z_2^n$
$Output$:
$A)(\xi_1,\xi_2,\ldots\ldots\xi_{n-1}) = ULTP(C)\ and$
$B)GeneratorMatrix(G)$
$Begin$:
$ULTP(C_1) = (\psi_1,\psi_2,\ldots..\psi_n)$
$\leftarrow Get\ this\ first\ row\ by\ adding$
$all\ rows\ of\ given\ matrix\ P(C)\ except\ the\ last\ row$
$for\ j\ from\ 0\ to\ (n-2)$
$Rotate\ one\ bit\ right\ of\ every$
$ULTP(C_{1+j})\ to\ get\ ULTP(C_{1+j_{+1}})$
end
$Get\ ULTP(C)$
$Re\ m\ ove\ any\ one\ column\ of\ ULTP(C) \rightarrow\circ P(C)$
$G = [I:\circ P(C)]$
end

Here odd and even length UTM is P(C), P (c1, c2, …, cn), stands as the first row of given UTM, ULTP(C1) indicates the row 1 of transformed matrix, (ξ1, ξ2, ……ξn) is established DIM, obtained through ULT mapping and oP(C) is the punctured matrix obtained from ULTP(C).

5. A Method for Constructing Block Codes with Rate Greater than 1/2

An algorithm for designing of IDLBC is designed in this section. With the proposed methodology, it is observed that designed code will have a code rate above 0.5. Also, error correction capability is enhanced with the proposed methodology. Such designed codes are useful for noisy environment communication channels.

From the flowchart depicted in Figure 1, it is evident that dmin can be enhanced using transformations aided by permutation arrays, as shown in Constructions 1 and 2. Table 2 gives the list of various designed IDLBC developed by using DIM "ULT".

Table 2. List of designed IDLBC

n	k	Designed Code	Designed Code	k/n code rate ratio	Modulation Scheme
7	4	(7, 4)	(7, 4)	0.571	BPSK
9	5	(9, 5)	(9, 5)	0.555	BPSK
11	6	(11, 6)	(11, 6)	0.545	BPSK
13	7	(13, 7)	(13, 7)	0.538	BPSK
17	9	(17, 9)	(17, 9)	0.529	BPSK
21	11	(21, 11)	(21, 11)	0.524	BPSK
27	14	(27, 14)	(27, 14)	0.518	BPSK

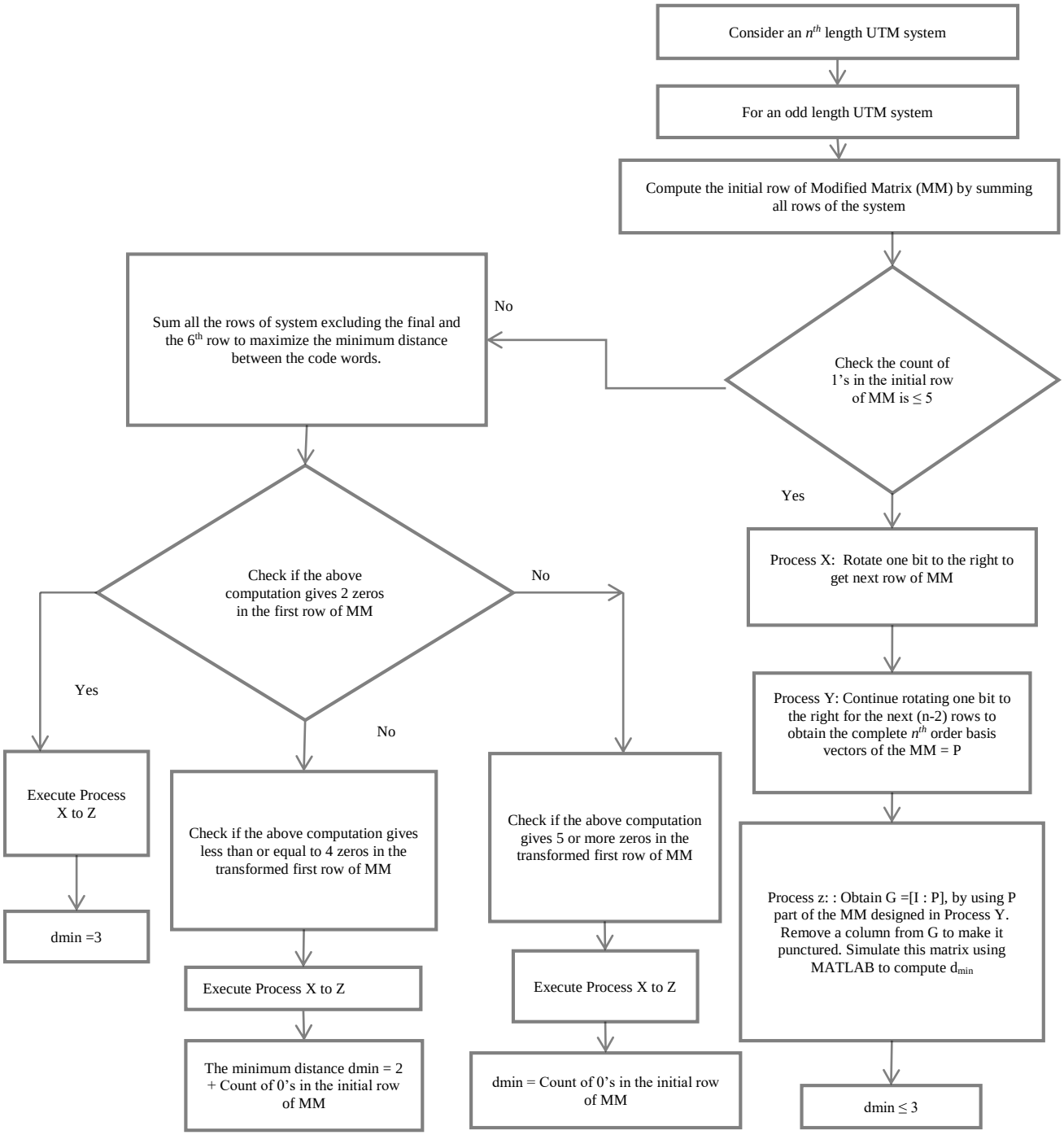


Fig. 1 An algorithm for enhancing error correction capability of designed code by increasing distance property

6. Simulation and Analytical Results

Figure 2 presents the simulation work done for the above-designed codes. Figure 3 shows the simulation results of the (35, 18) MLBC. In Figure 4, the comparison of 3-bit error correcting increased distance punctured MLBC with the other Hamming code is illustrated.

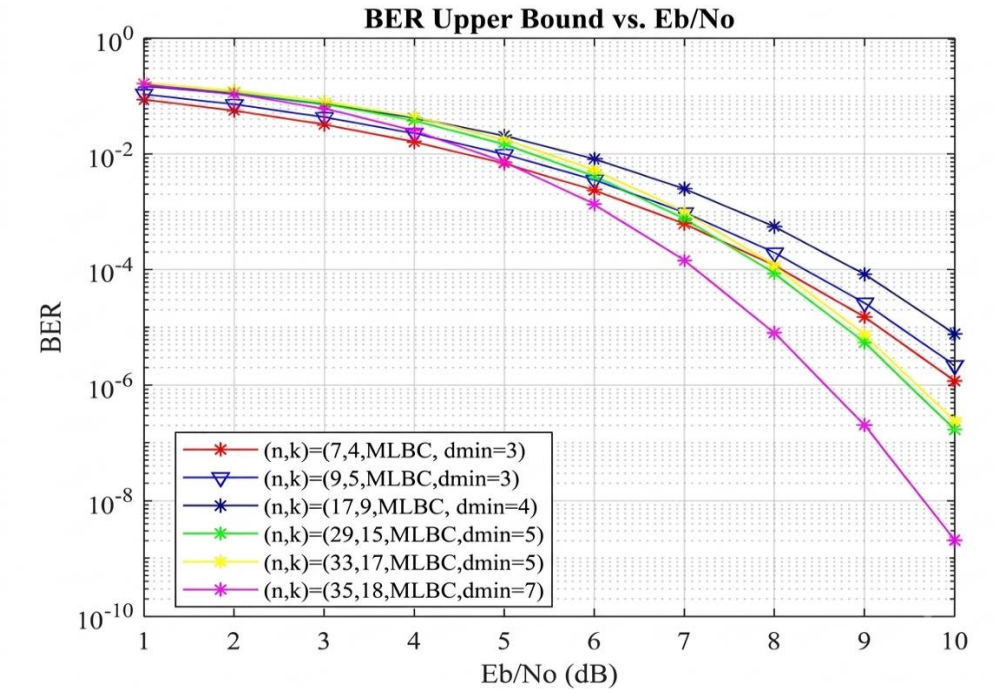


Fig. 2 BER performance for designed codes

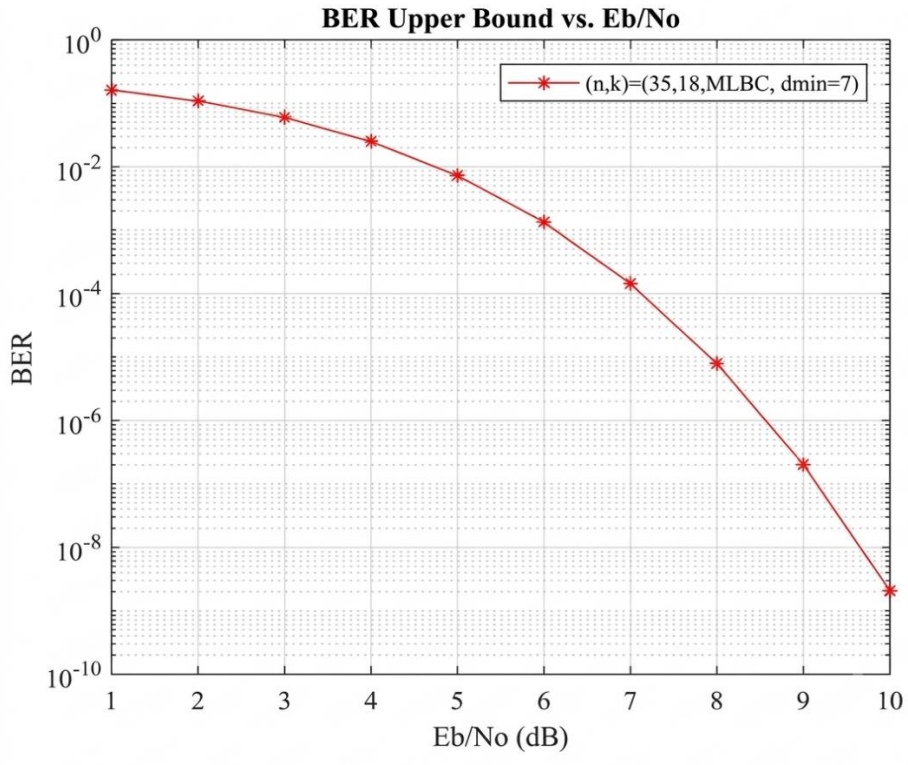


Fig. 3 BER performance for (35, 18) MLBC

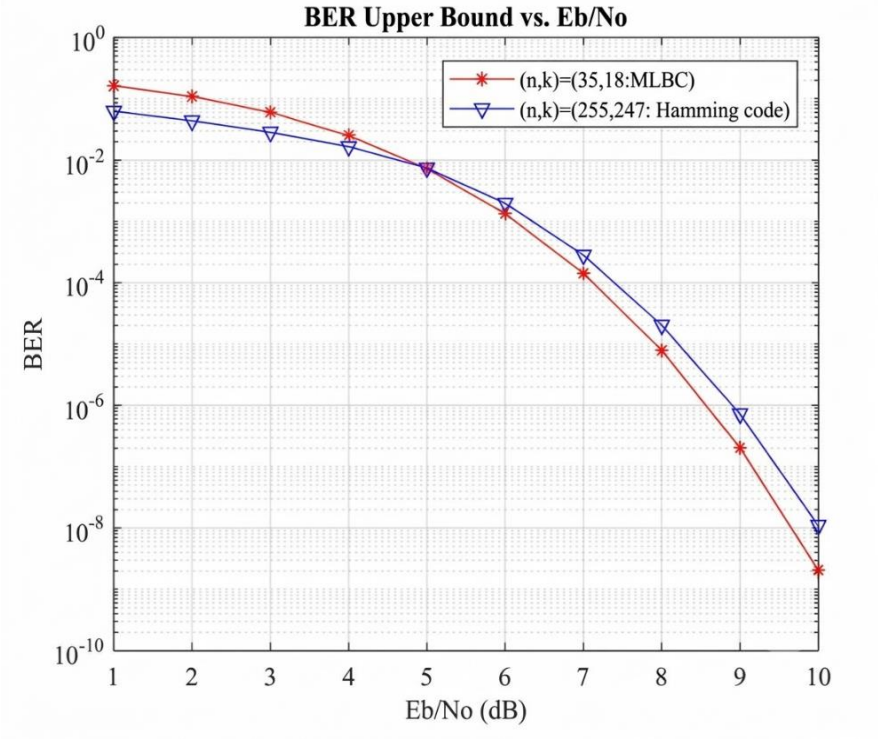


Fig. 4 BER comparison

The analytical result of this research work is presented with the help of following lemma:

Lemma: For an nth order UTM, P(C), mapping from P(C) to ULTP(C) can be achieved by permuting code words to devise the modified matrix. Applying a puncturing process to the generator matrix, which is designed through the modified matrix, results in DIM.

Observation: The resulting DIM is due to the permutation of the code words of the constructed UTM obtained systematically for odd and even length. As a result, there is an increase in the error correction capability of the designed MLBCs.

7. Conclusion

This work presents a new family of linear block codes, called Increased Distance Linear Block Codes (IDLBCs), which are created through the use of permutation arrays. The presented construction achieves a rate greater than 1/2 by applying a puncturing technique while still satisfying all of the fundamental characteristics of linear block codes. An efficient algorithm using ULTP (C) mapping was developed to produce codes with a further increased minimum distance between code words. Simulation results demonstrating improved performance include decreased probabilities of undetected errors, improved Bit Error Rates (BER), and improved error-correcting capabilities due to the increased minimum distance.

The IDLBC's included in this work exhibit better performance characteristics when comparing to traditional linear block codes. For example, the (35, 18) IDLBC shows good performance. Additionally, this ULTP(C) based framework facilitates the construction of longer (n, k) codes. Due to their robustness, these newly constructed codes are suitable for high-frequency communication systems, such as deep space missions, where reliable error correction is essential.

Conflicts of Interest

The authors state that this publication involves no conflicts of interest.

Funding Statement

This research was conducted without any external financial support.

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