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Possible Approach to Prediction of Harmonic Amplitudes in the Considered Signals

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Abstract: *This article describes the approach to the determination of harmonic amplitudes in the considered signals. The approach assumes multiplication of the considered signal containing harmonics by an auxiliary sinusoidal signal containing unit amplitude and harmonic frequencies of initial signal with the subsequent taking of definite integral of the generated product over time. This approach is validated by taking the exact integral of the initial signal presented by the sinusoidal constituent. It has followed from the exact solution that the integral value equals harmonic amplitude with the precision up to the coefficient and additive constituent. This has confirmed that the calculated integral is the amplitude-frequency spectrum. Then, for direct prediction of amplitude, the exact value of the integral without loss of correctness has been converted into the form suitable for parallel calculations using PC. The considered approach is valid, provided that the mutual influence of harmonics upon determination of amplitudes can be neglected.*

Keywords: *spectral analysis, current amplitude-frequency spectrum, harmonics, amplitude, prediction of amplitudes.*

I. INTRODUCTION

Spectral analysis is widely used in various fields of science and engineering. Spectral analysis is applied for speech recognition, for noise recognition in submarine navigation, for diagnostics of technical and biological objects, for recognition of chemical composition of substances [1] – [13]. Therefore, it is highly important to study the properties of spectral analysis as well as its methods.

One of the known approaches to generate signal spectra is based on the following equation:

$$\Phi(t,\omega) = \sqrt{\left[\int_0^t f(\tau)\sin(\omega\tau)\,d\tau\right]^2 + \left[\int_0^t f(\tau)\cos(\omega\tau)\,d\tau\right]^2} \qquad (1)$$

Where $f = f(t)$ is the considered signal, t is the current time, ω is the cyclic frequency, τ is the integration variable [14].

The Amplitude-frequency spectrum of the considered signal is predicted using this equation.

A drawback of this approach is comprised of difficulties upon the arrangement of parallel computations stipulated by taking the squaring and the square rooting of integrals.

The aim of this article is to estimate the possibility of obtaining signal spectrum as well as an approach to predict harmonic amplitudes using the following equation:

$$\Phi(t,\omega,\varphi) = \left|\int_0^t f(\tau)\sin(\omega\tau+\varphi)\,d\tau\right| \qquad (2)$$

With fewer operations and without taking the squaring and square rooting.

II. SOLUTION AND ITS SUBSTANTIATION

In order to solve this problem, let us consider the dynamics of generation of amplitude-frequency spectrum for the considered signal $f = f(t)$ at the frequency ω in time.

Let us assume that the signal $f = f(t)$ contains harmonic $A\sin(\omega\tau+\alpha)$. Let us estimate the possibility of generation of amplitude-frequency spectrum for this harmonic by the equation:

$$\Phi(t,\omega,\alpha,\varphi) = \left|\int_0^t A\cdot\sin(\omega\tau+\alpha)\sin(\omega\tau+\varphi)\,d\tau\right|, \qquad (3)$$

Let us calculate the exact value of Eq. (3).

$$\begin{aligned}
&\Phi(t,\omega,\alpha,\varphi) = \left|\int_0^t A\cdot\sin(\omega\tau+\alpha)\sin(\omega\tau+\varphi)\,d\tau\right| = \\
&\left|\int_0^t A\cdot(\sin\omega\tau\cdot\cos\alpha + \cos\omega\tau\cdot\sin\alpha)\cdot(\sin\omega\tau\cdot\cos\varphi + \cos\omega\tau\cdot\sin\varphi)d\tau\right| = \left|\int_0^t A\cdot(\sin^2\omega\tau\cdot\cos\alpha\cdot\cos\varphi + \right. \\
&= \cos^2\omega\tau\cdot\sin\alpha\cdot\sin\varphi + \\
&+\sin\omega\tau\cdot\cos\omega\tau\cdot\cos\alpha\cdot\sin\varphi + \cos\omega\tau\cdot\sin\omega\tau\cdot\sin\alpha\sin\alpha)d\tau\Big| = \\
&= \left|\frac{A}{2}\cdot\int_0^t(1-\cos 2\omega\tau)\cdot\cos\alpha\cdot\cos\varphi + (1+\cos 2\omega\tau)\sin\alpha\sin\varphi + +2\sin\omega\tau\cdot\cos\omega\tau\cdot(\cos\alpha\cdot\sin\varphi + \sin\alpha\cos\varphi)d\tau\right| =
\end{aligned} \qquad (4)$$

$= |\frac{A}{2} \cdot \int_0^t \cos\alpha \cdot \sin\varphi + sin\alpha \cdot sin\varphi - \cos 2\omega\tau \cdot (\cos\alpha\cos\varphi - \sin\alpha\sin\varphi) +$
$+ 2\sin\omega\tau\cos\omega\tau\sin(\alpha+\varphi)\ d\tau| =$
$= |\frac{A}{2} \cdot \int_0^t \cos\alpha \cdot \sin\varphi + sin\alpha \cdot sin\varphi - \cos 2\omega\tau \cdot \cos(\alpha+\varphi) +$
$+ \sin 2\omega\tau \cdot \sin(\alpha+\varphi)\ d\tau| =$
$= |\frac{A}{2} \cdot \int_0^t \cos(\alpha-\varphi) - \cos 2\omega\tau \cdot \cos(\alpha+\varphi) + \sin 2\omega\tau \cdot \sin(\alpha+\varphi)\ d\tau| =$
$= |\frac{A}{2} \cdot \int_0^t \cos(\alpha-\varphi) - \cos(2\omega\tau+\alpha+\varphi)\ d\tau| =$
$= |\frac{A \cdot t}{2} \cdot \cos(\alpha-\varphi) - \frac{A}{2} \cdot \int_0^t \cos(2\omega\tau+\alpha+\varphi) \cdot \frac{d\tau \cdot 2\omega}{2\omega}| =$
$= |\frac{A \cdot t}{2} \cdot \cos(\alpha-\varphi) - \frac{A}{4\omega} \cdot \sin(2\omega\tau+\alpha+\varphi) + \frac{A}{4\omega} \cdot \sin(\alpha+\varphi)| =$
$= |\frac{A \cdot t}{2} \cdot \cos(\alpha-\varphi) - \frac{A}{4\omega} \cdot [\sin(2\omega\tau+\alpha+\varphi) - \sin(\alpha+\varphi)]| =$
$= |\frac{A \cdot t}{2} \cdot \cos(\alpha-\varphi) - \frac{A}{4\omega} \cdot [\sin(2\omega\tau+\alpha+\varphi) - \sin(\alpha+\varphi)]|$

It follows from Eq. (4) that in the cases when the variable $\Delta\Phi = \frac{A}{4\omega} \cdot [\sin(2\omega\tau+\alpha+\varphi) - \sin(\alpha+\varphi)]$ pulsating with the frequency 2ω can be neglected, Eq. (4) can be applied for prediction of amplitudes and for construction of amplitude-frequency spectra. It follows from the fact that in this case, Eq. (4) is rewritten as follows:

$$\Phi(t,\omega,\alpha,\varphi) = \frac{A \cdot t}{2} \cdot |\cos(\alpha-\varphi)| \qquad (5)$$

It follows from Eq. (5) that at all $\cos(\alpha-\varphi) \neq 0$ and at the current, t the harmonic amplitude is proportional to the current spectrum $\Phi(t,\omega,\alpha,\varphi) \sim \frac{A \cdot t}{2}$. Thus, using the known *(t, ω, α, φ), t*, and | cos(*α* - *φ*)| it is possible to predict the amplitude A.

In order to exclude $\Delta\Phi = \frac{A}{4\omega} \cdot [\sin(2\omega\tau+\alpha+\varphi) - \sin(\alpha+\varphi)]$ from Eq. (4), it is sufficient to take the current spectrum $\Phi(t,\omega,\alpha,\varphi)$ at times $t_j = \frac{j\pi}{\omega}$; j = 1, 2, 3, . . . during the prediction of A. Herewith, $\Delta\Phi = 0$.

It follows from Eq. (4) that the highest rate of spectrum generation is achieved in the cases of $|\cos(\alpha-\varphi)| \approx 1$, that is, when $(\alpha-\varphi) \approx 0$.

However, the initial phase α of the harmonic $A \cdot \sin(\omega\tau+\alpha)$ in the considered signal $f = f(t)$ is unknown, thus complicating the prediction of the amplitude A.

In order to eliminate this difficulty, it is assumed to apply the following approach convenient for parallel computations.

The values of initial phases α AND β in the considered harmonic $A \cdot \sin(\omega\tau+\alpha)$ and in the auxiliary sinusoidal signal $\sin(\omega\tau+\varphi)$ are in the range of [$-\frac{\pi}{2}, \frac{\pi}{2}$]. Then, applying n auxiliary sinusoidal signals with initial phases $\Phi_i = [\frac{\pi}{2} - \frac{\pi}{n}]$, where i = 1, 2, . . . n, we obtain, in accordance with Eq. (3), n spectra:

$$\Phi_i(t,\omega,\alpha,\varphi) = |\int_0^t A \cdot \sin(\omega\tau+\alpha)\sin(\omega\tau+\varphi_i)\,d\tau| \qquad (6)$$

And the most rapidly growing spectrum, as mentioned above, corresponds to φ_i being the closest to α.

Let us estimate the methodological error of the proposed approach.

The initial phases φ_i vary at the step of $\frac{\pi}{n}$. Then, the deviations of the initial phase φ_i in the auxiliary sinusoidal signal, which provides the most rapid generation of the spectrum, will not exceed the value:

$$\alpha - \varphi_i| \leq \frac{\pi}{2n} \qquad (7)$$

With consideration for Eq. (7), the amplitude A will be in the range of:

$$\frac{\Phi_i(t,\omega,\alpha\varphi)}{t} \leq A \leq \frac{\Phi_i(t,\omega,\alpha\varphi)}{t \cdot \cos\frac{\pi}{2n}} \qquad (8)$$

Therefore, the absolute error ΔA during the prediction of the amplitude A will not exceed the value:

$$\Delta A \leq \frac{\Phi_i(t,\omega,\alpha,\varphi)}{t\cdot\cos\frac{\pi}{2n}} - \frac{\Phi_i(t,\omega,\alpha,\varphi)}{t} = \frac{\Phi_i(t,\omega,\alpha,\varphi)}{t}\cdot\left(\frac{1}{\cos\frac{\pi}{2n}} - 1\right) \quad (9)$$

Respectively, the relative error δA will not exceed the value:

$$\delta A = \frac{\Delta A}{A} \leq \left(\frac{1}{\cos\frac{\pi}{2n}} - 1\right) \quad (10)$$

Therefore, according to the proposed approach, while measuring amplitude-frequency spectrum at the frequency ω in the times $t_j = \frac{j\pi}{\omega}$, j = 1, 2, 3, . . . , and it's dividing by the current time, it is possible to estimate harmonic amplitudes at the frequency ω with acceptable accuracy provided that the mutual influence of harmonics on the generated spectrum can be neglected.

III. RESULTS AND DISCUSSION

The main result of the work is the proposed approach to the prediction of harmonic amplitudes in the considered signals.

The validity of the approach is stipulated by the fact that the approach is based on the exact value of the integral (2), where the considered signal is the harmonic $A\cdot\sin(\omega\tau+\alpha)$, the amplitude A of which should be determined. It follows from the exact value of Eq. (4) that it coincides with the amplitude A with the accuracy of up to the coefficient $\frac{A\cdot t}{2}\cdot|\cos(\alpha-\varphi)|$ and additive constituent $\frac{A}{4\omega}\cdot[\sin(2\omega\tau+\alpha+\varphi)-\sin(\alpha+\varphi)]$. Thus, it can be concluded that Eq. (4) defines the amplitude-frequency spectrum and can be used for the prediction of harmonic amplitudes in the considered signals.

Then, the obtained exact solution without violation of correctness can be converted into a form convenient for the prediction of amplitudes. The constituent $\frac{A}{4\omega}\cdot[\sin(2\omega\tau+\alpha+\varphi)-\sin(\alpha+\varphi)]$ was excluded from the exact solution by a selection of times $t_j = \frac{j\pi}{\omega}$; j = 0, 1, 2, . . . of calculation of the integral (2), thus decreasing the error of calculations. In order to eliminate uncertainty in the calculations, stipulated by the uncertainty of the initial phase α, the value $\Phi_i(t,\omega,\alpha,\varphi)$ was calculated n times with the step of the initial phase φ equaling to $\Delta\varphi = \frac{\pi}{n}$.

With the mentioned corrections, the considered approach became suitable for practical implementation.

IV. CONCLUSION

The aim of this article is an analysis of the approach to determine harmonic amplitudes in the considered signals using the following equation:

$$\Phi(t,\omega,\alpha,\varphi) = \left|\int_0^t f(\tau)\sin(\omega\tau+\varphi)\,d\tau\right|,$$

where $f = f(t)$ is the considered signal containing harmonics, the amplitudes of which should be determined?

In order to solve the formulated problem, the work calculates the exact value of the integral $\Phi(t,\omega,\alpha,\varphi)$ at $f = f(t) = A\cdot\sin(\omega t+\alpha)$ and demonstrates that $\Phi(t,\omega,\alpha,\varphi)$ it is the amplitude-frequency spectrum with the precision up to additive constituent ΔΦ $= \frac{A}{4\omega}\cdot[\sin(2\omega\tau+\alpha+\varphi)-\sin(\alpha+\varphi)]$ and can be used for determination of A.

Then, with the aim of its use for direct predictions, the obtained exact value $\Phi(t,\omega,\alpha,\varphi)$ is reduced to the form suitable for the determination of the amplitudes A. The constituent ΔΦ is eliminated from the exact value. This is achieved by prediction of $\Phi(t,\omega,\alpha,\varphi)$ with the selection of times $t_j = \frac{j\pi}{\omega}$; j = 0, 1, 2, . . . , at which ΔΦ = 0. Uncertainty upon predictions, stipulated by the uncertainty of the initial phase α, is eliminated by prediction of $\Phi(t,\omega,\alpha,\varphi)$ with the values of the initial phase φ in auxiliary sinusoidal constituent equaling to $\varphi_i = [\frac{\pi}{2} - \frac{\pi}{n}]$, where i = 1, 2, . . . n. Then, in order to determine A, the spectrum $\Phi_i(t,\omega,\alpha,\varphi)$ is applied characterized by the most rapid growth. Herewith, the equation for prediction A is as follows $A = \frac{2\Phi_i(t,\omega,\alpha,\varphi)}{t_j}$.

The considered approach is valid, provided that the mutual influence of harmonics on the generated spectrum can be neglected.

The feature of this approach is convenient parallel computations upon its implementation.

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