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Unified Holomorphic Embedding Power Flow for Hybrid AC-DC Systems

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Abstract — *The hybrid ac-dc systems topology merely enabling the incorporation of ac-dc energy sources, loads, and converters typically compose the promising configurations for future smart grids and sustainable transportation. However, the key challenge of intelligent design, planning, and efficient operation for such hybrid ac-dc systems traditionally rely on the powerful and efficient power flow algorithm, independent of iterative methods. The conventional sequential and unified approaches to solve power flow equations are initial choice dependent and fail to converge in case of the inappropriate choice. For this apparent reason, this paper proposes a novel and reliable, unified holomorphic embedding power flow method (unified-HEPM) to accurately evaluate the power flow solution. The verification and validation of the proposed unified-HEPM are sufficiently illustrated by considering modified four buses ac-dc hybrid system including converters.*

Keywords — *Converter modeling, holomorphic embedding, hybrid ac-dc systems, power flow, stable operable solution.*

I. INTRODUCTION

The electric energy generation and demand have changed appreciably over the past decade with renewable energy resources integration into the existing grid leading to hybrid ac-dc power system [1], [2], [3].On the one hand, the dc power from renewable distribution generators (DGs) as a solar photovoltaic panel [4], fuel cell [5], etc., is increasing with an excessive speed which reasonably requires interconnection with distribution systems for better reliability of DGs. On the other hand, the rapid growth of electric vehicles (EVs) [6] and their integration [7] is promptly going to dominate the load handling capacity of the existing grid. To accommodate the demand and the supply of dc power in present system progressively introduces the hybrid ac-dc systems which include ac and dc generators, ac and dc buses, ac and dc transmission lines [8] all connected together by means of ac-dc/dc-ac converters. Therefore, demanding the possible need for the development of reliable and accurate power flow (PF) solution algorithm which can be used for the future hybrid ac-dc system's efficient design and efficient operation.

For hybrid ac-dc systems, the power flow problem is solved typically using either an unified approach [9] or a sequential [10] approach. With a sequential approach presented in [11], the power flow equations (PFEs) inac-dc formulation seamlessly incorporating the converter models are solved iteratively and sequentially till the convergence is reached. This comprehensive method has the advantage of utilizing current ac and dc power flow algorithms; nevertheless, this formulation may not result in solution convergence [12]. The authors of [13], have proposed the unified approach to solve the ac and dc nonlinear equations simultaneously using a Newton trust-region method and employed it to the ac-dc hybrid microgrid.

The possible variants of these unified and sequential approaches are proposed and validated in the extensive literature which naturally focuses the hybrid systems having converters and high voltage dc (HVDC) transmission links. There exists significantly limited research for handling low voltage power distribution including hybrid systems in the literature due to the apparent complexity of including converter models in PFE. This gives rise to an essential need for proposing an efficient power flow problem solving technique which can take into account the converter impedance, modulation index in hybrid ac-dc PFE formulation.

The conventional iterative power flow methods and their suitable modifications based on Newton-Raphson (NR), dc load flow, fast decoupled load flow (FDLF), and many others methodsare dependent on (i) initial choice to start promptly an iterative algorithm (ii) non-existence of the power flow solution. To address these issues, Antonio Trias has developed a recursive method in [14] labeled as holomorphic embedding load flow method (HELM). Further, the research based on HELM has moved in multiple directions as [15], [16], [17], [12], [18], [19].The thorough study described in [20] suggests Thyristor–based FACTS controller modeling using the HELM. In addition, the holomorphic embedding is developed for the identification of saddle-node bifurcation point [21] nonlinear network reduction for distribution systems [22], voltage stability analysis with induction machine [23] and many others. Despite of many the HELM based methods for diverse power system applications, it focuses either on ac systems or on dc systems power flow individually. Only one article [24] deals with the hybrid ac-dc load flow, however the algorithm is constrained to the the HCDC system and fails to justify the general notion to other hybrid systems.

This paper proposes the unified holomorphic embedding power flow method (unified-HEPM) to calculate the solution toPFEs and solve recursively for hybrid ac-dc systems. In unified-HEPM the ac and dc PFEs are solved simultaneously. In summary, the key contributions of proposed novel unified-HEPM strategy for hybrid ac-dc systems are listed below:

- To address issue of converter inclusion in ac-dc power flow equation, a unique bus power injection model is developed.
- In order to remove the dependency of power flow solution on initial choice when it exists and undoubtedly notify its non-existence, the holomorphic embedding of power flow equations is performed.
- Design of the unique algorithm to find the stable unique high voltage power flow solution with a non-iterative method which none of the existing hybrid ac-dc power flow methods address.

The remainder of the paper is arranged as follows: Section II provides the mathematical formulation of the hybrid ac-dc system power flow using line flow equations for different bus interconnection scenarios. Section III develops the unified-HEPM algorithm for precisely calculating the power flow solution. In Section IV, the case study of hybrid ac-dc four bus systems is considered for validation of proposed unified-HEPM strategy. In the end, usage and advantages of unified-HEPM in hybrid ac-dc system power flow problem are wisely concluded in Section V.

II. HYBRID AC-DC SYSTEM POWER FLOW

Conventional power flow methods based on the Newton-Raphson (NR), Fast Decoupled Load Flow, and dc load flow, in common are used methods for calculating the unique PFE solution of interconnected ac systems. However, in distribution systems where penetration of renewable energy sources increases in a few years, these NR based methods fail to converge [9]. In the hybrid ac-dc system converter invariably plays an important role in apparent power flow.

A. Converter Modeling

In this paper, ac-dc hybrid system is considered by installing a converter in the lines which give ac-dc power conversion as shown in Fig. 1.

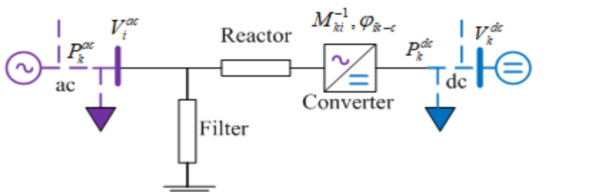

Fig. 1: Converter configuration connected between ac-dc hybrid system

The filter impedance in Fig. 1 depends on the switching method used to converter control. In the steady state converter, ac and dc side voltages are related by (1), where converter constant $K_c$depends on type and pulse width modulation (PWM) technique utilized for a converter.

$$V_i^{ac} = K_c\, M\, V_k^{dc} \quad (1)$$

With choice of accurate base value, (1) results in $V_i^{ac} = M\,V_k^{dc}$, where, $V_i^{ac}$and $V_k^{dc}$are in per unit. The relation between ac and dc power is function of converter efficiency $\eta_c$depicted in (2) and (3). The reactive power of converter at ac side is described by (4) which also can be calculated by direct set point control.

$$P^{ac} = \frac{P^{dc}}{\eta_c} (2)$$

$$P^{ac} = \frac{G^{dc}}{\eta_c}\left(M^{-2}(V_i^{ac})^2 - M^{-1}V_i^{ac}V_k^{dc}\right) \quad (3)$$

$$Q_c = P_c\ \tan\phi_c \quad (4)$$

To reduce the computational complexity in the formulation of ac-dc system, negligible converter losses are considered.

B. Modeling of hybrid ac-dc configurations

According to [25], the hybrid ac-dc systems can be modeled in five possible configurations based on the type of bus, line converter. Following cases represents the possible combination of hybrid ac-dc connections:

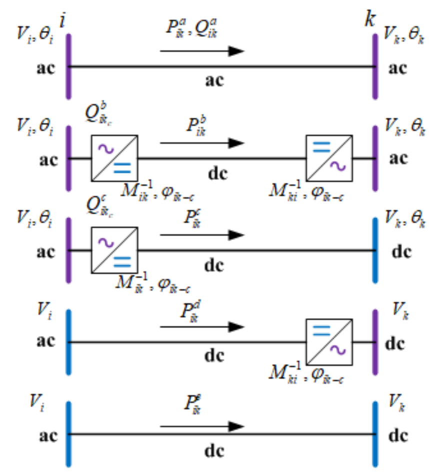

Fig. 2: All possible combination of ac-dc connection

Configuration I (ac buses with ac link) - It represents the connection of ac buses through ac line as shown in Fig. 2 (a). The power flow through the line is given by,

$$S_{ik}^{a\,*} = V_i^*(V_i - V_k)y_{ik} + V_i^*V_i y_{ik0} \quad (5)$$

Configuration II (ac buses with dc link) - In this case,two ac-dc converters are connected via a dc line between two ac buses as depicted in Fig. 2(b). The line flow equations corresponding to active and reactive power are given by (6) and (7) respectively. Based on the power flowdirection, converter acts either in rectifier or in inverter mode. The total power flowing through the line is obtained as $S_{ik}^{b\,*} = P_{ik}^b - jQ_{ik}^b$.

$$P_{ik}{}^{b} = g_{ik}{}^{dc}\eta_{ik-c}\left(M_{ik}{}^{-2}V_i^2 - M_{ik}^{-1}V_i M_{ki}{}^{-1}V_k\right)(6)$$

$$Q_{ik_c}^{b} = P_{ik}^{c} \tan\phi_{ik-c} \quad (7)$$

$$S_{ik}^{b\,*} = g_{ik}^{dc} V_i^* M_{ik}^{-1} \eta_{ik-c}\left(M_{ik}^{-1} V_i - M_{ki}^{-1} V_k\right)(1 - j\tan\phi_{ik-c})(y_{i0} V_i^* V_i) \quad (8)$$

where P_{ik}^{b} is the active power flowing through dc line, Q_{ik}^{b} is the reactive power at the converterac side, g_{ik}^{dc} represents the dc line conductance, M_{ik}, η, and $\tan\phi_{ik-c}$ are converter modulation index, efficiency, and power factor respectively. The shunt admittance at i^{th} bus is given by y_{ik0}.

Configuration III, and Configuration IV (dc link connecting ac and dc buses) - In configuration III, different type of buses are considered, one is ac and the other is dc. The line flows are developed in two possible ways for two buses. The total power flowing through line shown in Fig. 2(c) is given by (11) where line flow equations (9)-(10) for active and reactive power flow is developed in same way as in configuration II.

$$P_{ik}^{\ c} = g_{ik}^{\ dc} \eta_{ik-c}\left(M_{ik}^{\ -2} V_i^2 - M_{ik}^{-1} V_i M_{ki}^{\ -1} V_k\right) \quad (9)$$

$$Q_{ik_c}^{c} = P_{ik}^{c} \tan\phi_{ik-c} \quad (10)$$

$$S_{ik}^{c\,*} = g_{ik}^{dc} V_i^* M_{ik}^{-1} \eta_{ik-c}\left(M_{ik}^{-1} V_i - M_{ki}^{\ -1} V_k\right)(1 - j\tan\phi_{ik-c})(y_{i0} V_i^* V_i) \quad (11)$$

In configuration IV as depicted in Fig. 2 (d), (12) power line flow equation is expressed at dc node, giving the active power flow through dc line,

$$S_{\{ik\}}^{d\ *} = P_{ik}^{d} = g_{ik}^{dc} V_i^* (V_i - M_{ik}^{-1} V_k) + g_{i0} V_i^* V_i \quad (12)$$

Configuration V(dc line between two dc buses) - In configuration V, as shown in Fig. 2 (e) dc line is connected between two dc buses. The dc power line flow equation is represented as,

$$S_{ik}^{e\ *} = P_{ik}^{e} = g_{ik}^{dc} V_i^* (V_i - V_k) + g_{i0} V_i^* V_i \quad (13)$$

C. Power flow equation formulation of hybrid ac-dc systems

The power flow equation (PFE) is obtained by summing up all the current injection with assumption of zero losses. The line flow equations (5), (8), (11), (12) and (13)add together to provide the injected power at i^{th} bus and is expressed as

$$S_i^* = \sum_{k=1}^{N} S_{ik}^* \quad (14)$$

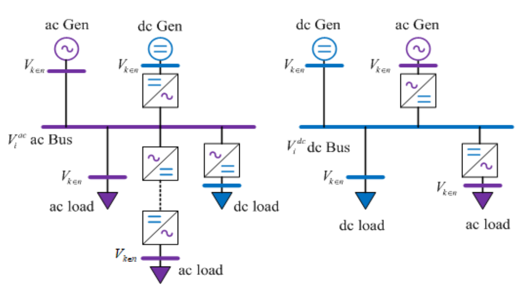

Fig. 3: Possible connections to ac bus and dc bus

In hybrid ac-dc systems there are two categories of buses as ac and dc. In addition Fig. 3, represent the maximum possible bus connection either to ac or dc bus. Hence, the power injection at i^{th} ac bus and dc bus considering all possible bus type connections is given by (15) and (16).

Ac bus power injection:

$$\frac{S_i^*}{V_i^*} = \sum_{k=1}^{N}\Big[(V_i - V_k)Y_{ik} + G_{ik}^{dc} M_{ik}^{-1} \eta_{ik-c}(M_{ik}^{-1} V_i - M_{ki}^{-1} V_k)(1 - j\tan\phi_{ik-c}) + G_{ik}^{dc} M_{ik}^{-1} \eta_{ik-c}(M_{ik}^{-1} V_i - M_{ki}^{-1} V_k)(1 - j\tan\phi_{ik-c})\Big] + (V_i Y_{i0}) \quad (15)$$

Dc bus power injection:

$$\frac{S_i^*}{V_i^*} = \sum_{k=1}^{N}\left[G_{ik}^{dc}(V_i - M_{ik}^{-1} V_k) + G_{ik}^{dc}(V_i - V_k)\right] + (V_i G_{i0}) \quad (16)$$

where $Y_{ik} = -y_{ik}$, i.e, negative of total admittance connected between i^{th} and k^{th} bus in case of ac bus and $G_{ik} = -g_{ik}$ in case of dc bus. In addition to this, $Y_{i0} = \sum_{k=1}^{N} y_{ik0}$ and $G_{i0} = \sum_{k=1}^{N} g_{ik0}$ is shunt admittance for ac and dc, respectively.

III. HOLOMORPHIC EMBEDDING POWER FLOW METHOD FOR HYBRID AC-DC

The solution of an algebraic power flow equation (PFE) embedded in a larger problem is indeed the goal of HEPM. Fig. 4 represents procedural steps of traditional holomorphic embedding algorithm [14]. The key concepts involved in the HEPM solution method involve the algebraic PFEs, embedded set of equations, germ solutions, projective invariance, and the recursive relation developments. The power series is developed using the germ solution, and its Pad\'e approximant is utilized that achieves the maximal analytic continuation and Stahl's theorem.

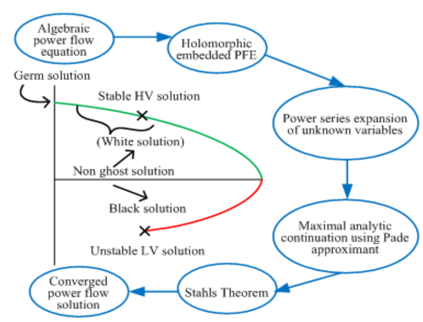

Fig. 4: Holomorphic embedding power flowin nutshell

The current injection at i^{th} bus for a given power system the PFA can be written as,

$$\sum_k Y_{ik} V_k = \frac{S_i^*}{V_i^*} \quad (17)$$

Let the embedding parameter denoted by μ is introduced in (17), then the the holomorphically embedded PFE is obtained as

$$\sum_k Y_{ik} V_k(\mu) = \frac{\mu S_i^*}{V_i^*(\mu)} \quad (18)$$

In this case, if the scaled voltage $V_i' = \lambda V_i$ is obtained by the projective invariance then the embedding parameter μ is given by $\mu = |\lambda|^2$which belongs to a complex domain. In order to satisfy the reflection condition (21), the voltage V_i^* is embedded as $V_i^*(\mu^*)$ instead of embedding$V_i^*(\mu)$. Denoting V_i^* (μ^*) by $\hat{V}_i(\mu)$ in (18) yields the equation (19). In addition, the mirror image of (19) is obtained as (20).

$$\sum_k Y_{ik} V_k(\mu) = \frac{\mu S_i^*}{\hat{V}_i^*(\mu)} \quad (19)$$

$$\sum_m Y_{ik}^* \hat{V}_k(\mu) = \frac{\mu S_i}{V_i(\mu)} \quad (20)$$

$$\hat{V}_i(\mu) = V_i^*(\mu^*) \quad (21)$$

This demonstrates that (19) and (20) are the complex conjugates, and that the embedded PFE (18) can be restored. If $\mu = 0$is substituted in the embedded equation (18) the solution of algebraic equation is termed as the germ solution.This germ solution corresponds the absense of the generationandload, and the power is supplied only by the slack bus.To calculate the coefficients of voltage power series where $\mu > 0$, the recursive relation is developed starting at the reference solution (i.e., germ solution).

The set of equations that fail to satisfy the reflection condition (21) given by (18) are referred to as the ghost solutions. These ghost solutions can notexists in real physical systems. In contrast, the solution satisfying the reflection condition and achieved by the embedded equations which are algebraic and physically existing solutions. The non-ghost solutions are categorised according to their operability condition as white (stable operable) and black (unstable nonoperable) solution. For example, if a PV curve is considered then the upper half curve till voltage collapse corresponds to white solution which is stable operable high voltage (SOHV) solution. On the other had the lower part of PV curve is unstable operable low-voltage (USOLV) solution i.e., the black solution.

A. Variables and Parameters for various bus types

In order to establish the HEPM based formulation it is necessary to categorize the buses with respect to known and unknown parameters belonging toP_i, Q_i, V_i, δ. Table I is created as an illustrationhaving five categories. The ×, √represents the unknown parameter to be determined, known quantity at the bus used in solving the PFE. Since, the dc bus has only active power and voltage associated with it, the reactive power Q_i and δ is not applicable and is represented by NA in Table I.

Table I: Known and unknown variables in different buses

Bus Type	□□	□□	□□	□□
Slack bus	√	√	×	×
PV Bus	√	×	×	×
PQ Bus	×	×	√	√
P node	×	NA	√	NA
V node	√	NA	×	NA

B. Holomorphic embedding for various bus types

This section deals with the holomorphic embedding based on the bus type. As shown in Fig. 3 the unknown variables associated with bus change according to its type resulting in the following set of embedded equations:

*Slack bus:*The original PFE at slack bus is represented as magnitude condition $V_{sl} = V_i^{sl}$ and is embedded in (22).

$$V_{sl}(\mu) = 1 + (V_i^{sl} - 1)\mu \quad (22)$$

*ac - PV bus:*According to Table I the unknown variables for ac PV buses are reactive power Q_i and angle δ_i. Thus, along with the voltages, the reactive power is also embedded with an embedding parameter μ resulting in a algebraic function.

$$\sum_{k=1}^{N}[(V_i(\mu) - V_k(\mu))Y_{ik} + G_{ik}^{dc} M_{ik}^{-1} \eta_{ik-c} (M_{ik}^{-1} V_i(\mu) M_{ki}^{-1} V_k(\mu)\ (1 - j \tan\phi_{ik-c}) + G_{ik}^{dc} M_{ik}^{-1} \eta_{ik-c} \left(M_{ik}^{-1} V_i(\mu) - M_{ki}^{-1} V_k(\mu)\right)(1 - j \tan\phi_{ik-c})] = \frac{\mu P_i}{V_i^*(\mu^*)} - j \frac{Q_i(\mu)}{V_i^*(\mu^*)} - \mu V_i(\mu) Y_{i0} \quad (23)$$

The conventional PV bus has the voltage magnitude condition as $|V_i| = V_i^{spec}$ where V_i^{spec} is the specified voltage at i^{th} bus. Consequently the embedding is performed on voltage magnitude condition resulting in (24).

$$V_i(\mu) V_i^*(\mu^*) = 1 + \mu \left(\left|V_i^{spec}\right|^2 - 1\right) \quad (24)$$

ac - PQ bus: Furthermore, the embedded PQ bus equation is developed in (25) where the quantities to be determined are bus voltages and angles. Hence, the voltages are represented as function of embedding parameter μ.

$$\sum_{k=1}^{N}[\ (V_i(\mu) - V_k(\mu))Y_{ik} + G_{ik}^{dc} M_{ik}^{-1} \eta_{ik-c} \Big(M_{ik}^{-1} V_i(\mu) - M_{ki}^{-1} V_k(\mu)\Big)(1 - j \tan\phi_{ik-c}) + G_{ik}^{dc} M_{ik}^{-1} \Big(M_{ik}^{-1} V_i(\mu) - M_{ki}^{-1} V_k(\mu)\Big) \eta_{ik-c}\ (1 - j \tan\phi_{ik-c}\ )\ ] = \frac{\mu S_i^*}{V_i^*(\mu^*)} - \mu V_i(\mu) \quad (25)$$

*dc - P node and V node:*Dc buses are categorized as P node and V node depending on its connections with loads and generators. If a dc bus has only dc loads connected to it then the voltage is the only parameter to be calculated and power is considered as known constant, resulting in the embedded equation (26) for P node where $S_{\{ik\}}^* = P_{\{ik\}}$.

$$\sum_{k=1}^{N}[G_{ik}^{dc}\ (V_i(\mu) - M_{ik}^{-1} V_k(\mu)) + G_{ik}^{dc}(V_i(\mu) - V_k(\mu))] = \frac{\mu S_i^*}{V_i^*(\mu^*)} - (\mu V_i(\mu) G_{i0}) \quad (26)$$

In similar manner (27) is developed for V node in order to find the unknown power at dc V node and is expressed as function of μ.

$$\sum_{k=1}^{N}[\,G_{ik}^{dc}\,(\,V_i(\mu)-\,M_{ik}^{-1}V_k(\mu))+\;G_{ik}^{dc}\,(V_i(\mu)-\,V_k(\mu))\,] = \frac{S_i^*(\mu)}{V_i^*(\mu^*)} - \,(\mu V_i(\mu)G_{i0}) \qquad (27)$$

Until now the holomorphically embedded set of PFE which are analogous to original PFE, formulated for hybrid ac-dc systems, at $\mu = 1$. In proposed unified-HEPM to get rid of iterative methods a non-iterative method is incorporated which uses the power series (30) expansion of unknown variables. Furthermore, to incorporate power series expansion of $1/(V_i^*(\mu^*)$ in PFEs $W_i^*(\mu^*) = 1/(V_i^*(\mu^*)$ is considered.

$$V_i(\mu)=\sum_n V_i[n]\mu^n\,,\quad Q_i(\mu)=\sum_n Q_i[n]\mu^n\,,$$
$$P_i(\mu)=\textstyle\sum_n P_i[\mu]\mu^n\,,\quad W_i(\mu)=\sum_n W_i[\mu]\mu^n \qquad (30)$$

Consequently, the steps in unified-HEPM for hybrid ac-dc systems are enumerated as :

1. Formulate the hybrid algebraic PFE using (15) and (16).
2. Embed the hybrid PFE with embedding parameter μ using (22) - (27) for the hybrid ac-dc systems
3. Considering (30) compose power series expansion of all unknown variables.
4. Separate the real and imaginary part of unknown variables and admittance in recursive relation.
5. Determine the recursive relation by equating the power series coefficient of μ on both sides of equation.
6. Find the germ solution at $\mu = 0$ in embedded set of equations (22) - (27).
7. Solve the resulting linear non homogeneous algebraic equation to calculate the power series coefficient of unknown quantities.
8. Extend the region of convergence of power series using the Pade` approximant and evaluate the solution of unknown variables at $\mu = 1$.

Thus, the solution evaluated by following unified-HEPM algorithm provides the unique stable high voltage solution because of extending the region of convergence domain analytically and using Stahl's theorem [26]. Moreover, it also signals the non-existence of solution unequivocally.

IV. CASE STUDY OF UNIFIED-HEPM

In order to illustrate the developed unified-HEPM, a modified four bus system is considered as shown in Fig. 5, where the bus 3 is a dc load bus. Since, the system is composed of ac and dc generators and loads two voltage source converters are considered in case study connected between bus 1-4 and bus 3-4. As the dc load requires only the active power both converters act as ac-dc rectifier. With intention to mainly focus on holomorphic embedding formulation of hybrid ac-dc systems, the efficiency of converter is considered to be 100%. If it is other than 100% then it results only in the scaling of solution and does not affect the embedding of PFEs.

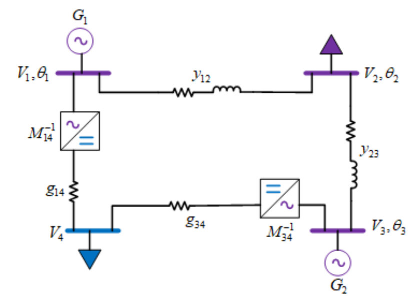

Fig. 5:Modified 4 bus Hybrid ac-dc system

Algorithm 1: Unified-HEPM for hybrid ac-dc modified four bus system

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Results: Solution of the PFE for unknown parameters to be determined according to Table I.

Initializations:

- Calculate the modified Y-bus matrix from given admittance along with shunt elements at all buses.
- In addition find out germ solution which will correspond to no generation and no loss case as initial step in recursive formulation.
- Set the step count (n) to 1 initially

whiledifference in current and previous voltages at every bus in p.u. < 1e-04

do

1. Using germ solution, calculate reactive power at PV buses, active power at V node, and the real and imaginary parts of voltages coefficients at all buses.
2. Formulate power series using calculated coefficients.
3.Determine Pade' approximant of all the unknown parameters
4.Using $\mu = 1$ in Pade' approximated series of unknowns obtain the voltages in rectangular form.
5. Store the unknown parameters.
6. n++.

end

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & C & 0 & 0 & 0 & 0 \\ G_{21} & -B_{21} & G_{22} & -B_{22} & G_{23} & 0 & 0 & 0 \\ B_{21} & G_{21} & B_{22} & G_{22} & B_{23} & 0 & 0 & 0 \\ 0 & 0 & G_{32} & -B_{32} & (G_{32}+G_{34}^{dc}) & 0 & -G_{34}^{dc} & 0 \\ 0 & 0 & B_{32} & G_{32} & B_{32} & 1 & 0 & -G_{34}^{dc} \\ -G_{41}^{dc} & 0 & 0 & 0 & -G_{43}^{dc} & 0 & (G_{43}^{dc}+G_{41}^{dc}) & 0 \\ 0 & -G_{41}^{dc} & 0 & 0 & 0 & 0 & 0 & (G_{43}^{dc}+G_{41}^{dc}) \end{pmatrix} \begin{pmatrix} V_{1re}[n] \\ V_{1im}[n] \\ V_{2re}[n] \\ V_{2im}[n] \\ Q_{3im}[n] \\ V_{3im}[n] \\ V_{4re}[n] \\ V_{4im}[n] \end{pmatrix} = \begin{pmatrix} \gamma_{n0} \\ \gamma_{n1} \\ Re\{S_2^* W_2^*[n-1]\} \\ Im\{S_2^* W_2^*[n-1]\} \\ Re\{P_3 W_3^*[n-1] - j\sum_{k=1}^{n-1} Q_3[k] W_3^*[n-k]\} \\ Im\{P_3 W_3^*[n-1] - j\sum_{k=1}^{n-1} Q_3[k] W_3^*[n-k]\} \\ P_4 W_4^*[n-1] \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ -B_{23} \\ G_{23} \\ -B_{23} \\ (G_{32}+G_{41}^{dc}) \\ 0 \\ -G_{43}^{dc} \end{pmatrix} V_{3re}[n] \quad (28)$$

where,

$$V_{3re}[n] = \gamma_{n0} + \gamma_{n1} \frac{V_3^{sp^2} - 1}{2} - \frac{1}{2}\left(\sum_{k=1}^{n-1} V_3[k] V_3^*[n-k]\right) \quad (29)$$

A. Unified-HEPM algorithm for hybrid ac-dc systems

Based on the procedural steps presented in Section III, an algorithm for modified four bus case study system using unified-HEPM is constructed. At the same time, mathematical formulation following this algorithm is given by (28). All the unknown complex quantities are separated in real and imaginary part to get the consistent set of simultaneous equations that are solved using recursive relation. This recursive relation is valid $n > 1$. Germ solution corresponding to $\mu = 0$ results in the initial start of recursive algorithm (28) and is equivalent to no generation no loss case physically. The calculated germ for four bus system is $1 + j0$ at all the buses including dc and slack bus. In order to get consistent set of equations dc bus voltage and slack bus voltage is separated in real and imaginary parts. Following the Algorithm 1, the power series coefficients obtained for bus 4 are such that its imaginary part consequently results in zero.

B. Modified four bus hybrid ac-dc system Results and Discussion

The various parameter considered for simulation and validation of proposed unified-HEPM for a four bus system are listedin Table II and Table III.

Table II:Transmission line data for modified four bus hybrid system

From Bus	To Bus	Resistance	Reactance	Shunt admittance
1	2	0.01008	0.05040	0.05125
1	4	0.00744	-	0.03875
2	3	0.00744	0.03720	0.03875
3	4	0.01272	-	0.06375

Table III: Bus data for Modified four bus hybrid system

Bus No	Bus Type	Generator		Load		Sepcified Voltage
		P_{gi}	Q_{gi}	P_l	Q_l	
1	Slack bus	-	-	0.5	0.3099	1.00
2	ac-PQ bus	0	0	1.7	1.0535	-
3	ac-PV bus	0	0	2.0	1.2394	-
4	dc-P node	318	-		-	1.02

The transmission line data is given in Table II together with bus power data in Table III. All the quantities considered in four bus study system are in p.u.

Table IV: Converged Power Flow Solution

Unknown quantities	V1	V2	V3	Q3	V4
Power flow solution	1	0.999	1.02	1.185	0.9994

Starting with a germ solution power series coefficients are calculated leading to the converged power flow solution as shown in Table IV.

The output from Matpower shown below, for same modified case study, depicts that it does not converge in 10 steps for given initial choice of 1 p.u. for all voltages.

```
>>runpf(case4gs_hybrid)

MATPOWER Version 7.0b1, 31-Oct-2018 -- AC Power
Flow (Newton)

Newton's method power flow did not converge in 10
iterations.

>>>>>Did NOT converge (0.01 seconds)   <<<<<
```

The conventional load flow methods for hybrid ac-dc systems, sequential or unified, are based on the iterative methods as NR methods, GS method which are dependent on the initial condition because of which fails to converge at a unique physical solution. As the proposed unified-HEPM incorporating converter characteristics converge to unique solution in less recursion steps. The results are validate using Matpower which based on NR method solves the given four bus system and due to dependence on initial condition fails to converge.

V. CONCLUSIONS

This paper investigates the novel methodology termed unified-HEPM to calculate the power flow solution of a hybrid ac-dc system reliably and accurately. In the first place, a bus power injection model including converter parameters is developed which provides multivalued solution.Therefore, the holomorphic embedding is employed that evaluates the high voltage power flow solution existing in the physical system and unequivocally signals under its non-existence. The remarkable results of the unified-HEPM developed algorithm applied to four bus hybrid ac-dc system indicate it is independent of initial value choice required in an iterative method, it also solves the power flow equations in the unified way involving ac and dc components. The successful development of unified-HEPM for distribution system is work in process.

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