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A Comparative Study of Prony Based Method for Identification of Low-Frequency Oscillations in the Power System

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Abstract — *In a modern power system, low-frequency electromechanical oscillations get triggered due to many reasons like a sudden change in load; these oscillations may lead to power system instability if the oscillations are not damped, which may finally lead to the collapse of the system. Hence accurate and precise estimation of the parameters of low-frequency oscillation in a power system is of utmost importance. In this research paper, the performance of two Prony based methods is compared for identifying dominant low-frequency oscillations. The performance is compared in terms of attenuation factor and frequency of oscillation with different noise levels and sampling rates of the Phasor Measurement Unit (PMU) with the synthetic signal generated in MATLAB and real-time data obtained from Western Electricity Coordinating Council (WECC).*

Keywords — *Attenuation factor, Low-frequency oscillations, Phasor Measurement Unit, Power System, Prony Method.*

I. INTRODUCTION

The modern power system is highly interconnected to share increasing load demand effectively, and it is essential to maintain the stability of the power system. Due to the interconnection of the power system and randomly varying loading conditions, low-frequency electromechanical oscillations get triggered in the power system, and if these oscillations decay with time, it will lead to the stable operation of the system on the other hand if the oscillation grows with increasing amplitude it will lead to unstable system operation [1]–[2]. So, it is essential to identify these low-frequency oscillation parameters to keep the power system within a stable operating region. Local modes of oscillation have frequencies in the range of 0.7 to 2.0 Hz and are due to a single generator or a single plant, whereas the frequency range of inter-area modes of oscillation lies between 0.1 to 0.8 Hz and are mainly due to groups of generators or groups of plants [1]–[4].

In earlier decades, modal analysis was an important method for estimating low-frequency oscillations occurring in the system. As the operating point of the power system changes continuously due to load-generation variation and various contingencies, it is very difficult to establish an accurate model of the power system all the time [5]. To overcome this drawback, the application and fast development of the Phasor Measurement Unit (PMU) have enabled identifying the low-frequency oscillation parameters of the power system. PMU provides time-stamped measurements of current, voltage across the transmission line. The data collected is transmitted through a communication channel to the control center, and the dynamic parameters of low-frequency oscillations (frequency and attenuation factor) are estimated, which further allows the system operator to take any control action if necessary to keep the power system in stable operating conditions[6]–[10].

There are several measurement-based signal processing algorithms that estimate parameters of low-frequency oscillations; the signal processing algorithm must estimate true modes present during oscillations; otherwise, it may lead to improper control action by the operator. The signal processing algorithms include the Prony algorithm, the Eigensystem Realization algorithm, Matrix Pencil, etc. [11]–[17]. Different algorithms have their pros and cons. The objective of this research paper is to compare the performance of two Prony based methods, i.e., the Prony algorithm and the Multi Prony algorithm. The paper is organized as follows. Section II provides an overview of Prony based methods. Section III provides results and performance comparison of both algorithms on the synthetic signal with the varying noise level and PMU reporting rates, the real-time signal obtained from WECC, and Section IV presents the conclusion of this paper.

II. PRONY BASED METHODS

A. Prony Algorithm

The Prony algorithm is a signal processing technique to estimate damping (attenuation factor) and frequency components present in the given signal. In this, a linear prediction model is developed that fits the signal, and then roots of the polynomial and eigenvalues are calculated. The mathematical model related to the Prony algorithm is described in [11], [13].In this research paper, model order estimation is based on singular values obtained from the autocorrelation matrix of the signal described in [15].

B. Multi-Prony Algorithm

In the multi Prony algorithm, the drawback with the standard Prony algorithm is overcome, i.e., it is independent of the model order of the system. It is based on the fact that irrespective of model order, the true modes appear consistently. In the multi Prony algorithm, two different data windows are considered, and the Prony algorithm is applied to each data window with the same sampling rate. As already stated, true modes will appear in both the windows, so true modes will be then extracted through the sorting method. Complete details about multi Prony algorithm are described in [14].In the multi Prony algorithm, true modes are obtained and are very simple to implement.

III. RESULTS AND PERFORMANCE COMPARISON

In this section, comparative performance analysis is done on the synthetic and real-time signal in MATLAB with different levels of noise and varying PMU reporting rates for both Prony and multi Prony algorithms. In this research paper, a low pass filter is used for both Prony and multi Prony algorithm to filter out measurement noise, and then algorithm performance is analyzed on synthetic signal and real-time signal. For all analyses, simulation is run for 50 trials, and the average value obtained is given in the below table.

A. Synthetic Signal

Consider the synthetic signal as given below:

$$x_1(t) = \cos(2\pi * 0.4t)\, e^{-0.3t} \quad (1)$$

The above signal is a single-mode signal with a frequency component of 0.4 Hz and an attenuation factor of 0.3. The analysis is performed for both the algorithms with 60 samples per second as the sampling rate and different levels of noise.

TABLE I

ANALYSIS OF SIGNAL X_1(T) BY PRONY ALGORITHM

Model Order	Noise	Estimated Frequency in Hz	Attenuation Factor
1	No noise	0.4	0.3
1	50dB	0.39	0.31
1	35dB	0.38	0.29
1	20dB	0.37	0.28

TABLE II

ANALYSIS OF SIGNAL X_1(T) BY MULTI PRONY ALGORITHM

Noise	Estimated Frequency in Hz	Attenuation Factor
No noise	0.4	0.3
50dB	0.4	0.3
35dB	0.39	0.3
20dB	0.38	0.31

From the above analysis for a single-mode signal, it is very clear that both algorithms can estimate true modes with good accuracy.

Consider the second synthetic signal as given below:

$$x_2(t) = \cos(2\pi * 0.4t)\, e^{-0.3t} + \cos(2\pi * 0.2t)\, e^{-0.6t} \quad (2)$$

The signal $x_2(t)$ has two modes with a frequency component of 0.4 Hz and 0.2 Hz, respectively, and an attenuation factor of 0.3 and 0.6, respectively. The analysis is performed for both the algorithms with 60 samples per second as the sampling rate and with different levels of noise. In this signal, attenuation factor and frequency components are spaced apart from each other. The plot of real and estimated signal $x_2(t)$ with SNR of 20dB is shown in Fig.1

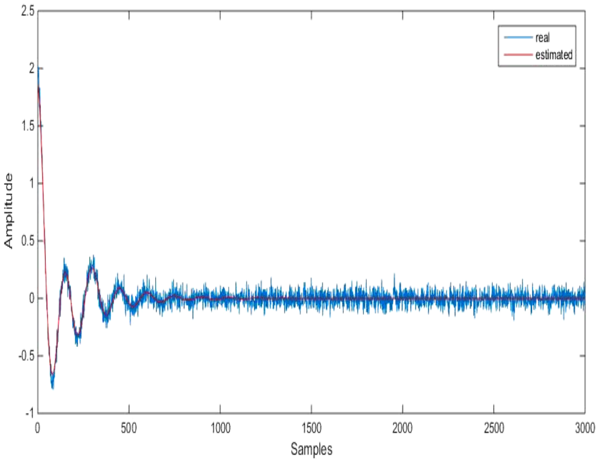

Fig. 1 Real and estimated signal $x_2(t)$ by multi Prony algorithm for 20dB noise

TABLE III

ANALYSIS OF SIGNAL X2(T) BY PRONY ALGORITHM

Model Order	Noise	Estimated Frequency in Hz	Attenuation Factor
2	No noise	0.4	0.3
		0.2	0.6
2	50dB	0.4	0.29
		0.19	0.61
2	35dB	0.39	0.31
		0.19	0.59
3	20dB	0.38	0.28
		0.17	0.58
		0.8	0.7

TABLE IV

ANALYSIS OF SIGNAL X2(T) BY MULTI PRONY ALGORITHM

Noise	Estimated Frequency in Hz	Attenuation Factor
No noise	0.4	0.3
	0.2	0.6
50dB	0.4	0.3
	0.2	0.6
35dB	0.4	0.3
	0.19	0.59
20dB	0.39	0.29
	0.2	0.6

From the above analysis for two-mode signal $x_2(t)$, it can be concluded that the accuracy for estimating the frequency and attenuation factor of the multi Prony algorithm is better than the Prony algorithm.

The main shortcoming of the Prony algorithm is that as the noise level in the signal increases, then model order estimation isn't accurate, and hence fictitious modes are also estimated, which isn't the part of the signal. Thereby, exact true modes are not estimated by the Prony algorithm if the noise increases, whereas for multi Prony algorithms, true modes are identified with better accuracy.

Consider the third synthetic signal as given below:

$$x_3(t) = \cos(2\pi * 0.3t)\, e^{-0.4t} + \cos(2\pi * 0.35t)\, e^{-0.8t} \qquad (3)$$

The signal $x_3(t)$ has two modes with a frequency component of 0.3 Hz and 0.35 Hz, respectively, and an attenuation factor of 0.4 and 0.8, respectively. The analysis is performed for both the algorithms with 60 samples per second as the sampling rate and with different levels of noise. In this signal, $x_3(t)$ frequency components are closely spaced to each other.

TABLE V

ANALYSIS OF SIGNAL X3(T) BY PRONY ALGORITHM

Model Order	Noise	Estimated Frequency in Hz	Attenuation Factor
2	No noise	0.3	0.4
		0.35	0.8
2	50dB	0.3	0.4
		0.34	0.79
2	35dB	0.31	0.39
		0.33	0.75
3	20dB	0.32	0.35
		0.32	0.70
		0.7	0.5

TABLE VI

ANALYSIS OF SIGNAL X3(T) BY MULTI PRONY ALGORITHM

Noise	Estimated Frequency in Hz	Attenuation Factor
No noise	0.3	0.4
	0.35	0.8
50dB	0.3	0.4
	0.35	0.79
35dB	0.3	0.39
	0.34	0.79
20dB	0.31	0.41
	0.35	0.81

For two-mode signal $x_3(t)$, from the analysis, it is clear that accuracy for estimating frequency and attenuation factor of the multi Prony algorithm is better than the Prony algorithm. For signal $x_3(t)$, the frequency components are very closely spaced to each other, and the Prony algorithm isn't able to estimate true modes present in the signal at higher noise level and is not able to discriminate the modes, and a fictitious mode is estimated by Prony algorithm at 20dB noise level. On the other hand, the multi Prony algorithm can estimate true modes present in the signal with higher accuracy.

Consider the fourth synthetic signal as given below:

$$x_4(t) = \cos(2\pi * 0.2t)\, e^{-0.3t} + \cos(2\pi * 0.5t)\, e^{-0.6t} + \cos(2\pi * 0.8t)\, e^{-0.9t} \quad (4)$$

The signal $x_4(t)$ has three modes with a frequency component of 0.2 Hz, 0.5Hz, and 0.8 Hz, respectively, and an attenuation factor of 0.3, 0.6, and 0.9, respectively. The analysis is performed for both the algorithms with 60 samples per second as the sampling rate and with different levels of noise.

TABLE VII
ANALYSIS OF SIGNAL X_4(T) BY PRONY ALGORITHM

Model Order	Noise	Estimated Frequency in Hz	Attenuation Factor
3	No noise	0.2	0.3
		0.5	0.6
		0.8	0.9
3	50dB	0.2	0.3
		0.51	0.61
		0.79	0.89
4	35dB	0.15	0.24
		0.47	0.57
		0.77	0.86
		0.3	0.2
4	20dB	0.14	0.25
		0.55	0.57
		0.75	0.81
		0.4	0.21

TABLE VIII
ANALYSIS OF SIGNAL X_4(T) BY MULTI PRONY ALGORITHM

Noise	Estimated Frequency in Hz	Attenuation Factor
No noise	0.2	0.3
	0.5	0.6
	0.8	0.9
50dB	0.2	0.3
	0.5	0.6
	0.8	0.89
35dB	0.19	0.31
	0.49	0.6
	0.8	0.89
20dB	0.18	0.29
	0.48	0.61
	0.81	0.91

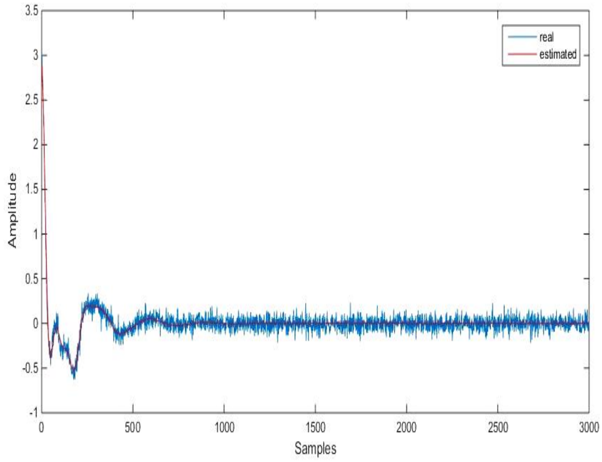

Fig. 2 Real and estimated signal x_4(t) by multi Prony algorithm for 20dB noise

For three-mode signal x_4(t), from the analysis, it is clear that the multi Prony algorithm has good accuracy for estimating true modes present in the signal having three components, whereas the Prony algorithm suffers the problem of true model order identification and thereby true modes present in the signal isn't estimated in higher noise level.

The PMU reporting rates in the practical power system vary from 10Hz to 100Hz. Now both the algorithms are compared for the performance with varying PMU reporting rates and a noise level of 35dB for signal x_2(t).

TABLE IX
ANALYSIS OF SIGNAL X_2(T) WITH VARYING PMU RATES BY PRONY ALGORITHM

Model Order	PMU Reporting Rate (Hz)	Estimated Frequency in Hz	Attenuation Factor
2	70	0.4	0.3
		0.2	0.6
2	50	0.39	0.31
		0.21	0.59
3	30	0.37	0.28
		0.22	0.57
		0.5	0.7

TABLE X
ANALYSIS OF SIGNAL X_2(T) WITH VARYING PMU RATES BY MULTI PRONY ALGORITHM

PMU Reporting Rate (Hz)	Estimated Frequency in Hz	Attenuation Factor
70	0.4	0.3
	0.2	0.6
50	0.39	0.31
	0.19	0.59
30	0.38	0.31
	0.21	0.61

For two-mode signal x_2(t) with varying PMU rates and with the noise level of 35dB, from the above analysis, it is clear that the multi Prony algorithm can track true modes present in the synthetic signal for different PMU rates, whereas for lower PMU rate Prony algorithm isn't able to identify true model order of the signal and true modes present in the signal due to which it provides fictitious modes in the result.

B. Real-Time System-WECC

The probe data obtained from the real-time WECC system on 14th September 2005 is used to compare the performance of both Prony and multi Prony algorithms. There are two main windows taken into consideration for analysis purposes. In main window 1, due to probing of ±125 MW, a single frequency component with a frequency of 0.318 Hz and 8.3% damping is observed as reported in [7], [9]. In the same way for main window 2, after probing, a single frequency component with a frequency of 0.315 Hz and 7.88% damping is observed.

For analysis purposes, white Gaussian noise with SNR 30 dB is added to the signal, and analysis is shown in the table below. The plot of probing data is shown in Fig.3

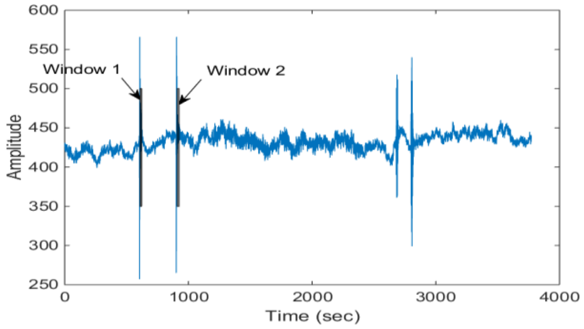

Fig. 3 Probing data of WECC system for real power flow [7]

TABLE XI
ANALYSIS OF WECC SYSTEM BY PRONY ALGORITHM

Model Order	Main Window	Estimated Frequency in Hz	Damping (%)
1	1	0.312	8
2	2	0.318	7.4
		0.4	3

TABLE XII
ANALYSIS OF WECC SYSTEM BY MULTI PRONY ALGORITHM

Main Window	Estimated Frequency in Hz	Damping (%)
1	0.316	8.15
2	0.313	7.58

From the analysis of real-time signals with the Prony algorithm and multi Prony algorithm, it can be concluded that the Prony algorithm isn't reliable and accurate for identifying true modes present in real-time signal whereas multi Prony algorithm has higher accuracy to estimate the true modes present in the signal. The Prony algorithm identifies one fictitious mode in the second window, which isn't present in the signal, whereas the multi Prony algorithm estimates true modes for both the windows.

IV. CONCLUSION

In this research paper, two Prony based methods, i.e., Prony algorithm and multi Prony algorithm, are compared for estimation of the parameters of low-frequency oscillation in the power system. Based on the analysis performed on different synthetic signals from single-mode to three modes, it is very clear that the Prony algorithm is very sensitive to noise and provides fictitious modes in the presence of noise; model order estimation is also a drawback for the Prony algorithm. On the other hand, the multi Prony algorithm is simple, robust and provides results with better accuracy, and is more reliable for identifying true modes present in both synthetic and real-time signals even in the presence of noise.

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