

# Probabilistic Design Analysis of Sustainable Unconventional Materials Used In Concrete Mix Aggregates: A Novel Statistical Analysis Based Eco-Friendly Approach

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**Abstract** - The work presented here is to make an investigation of the materials, which are uncommonly used in day to day concrete applications with Fly ash as an exception... The materials that are chosen are Slag, Fly ash and rice husk ash.

A thorough probabilistic analysis of these material properties is made here, as they will be further used to develop a mixed design. Some of the physical properties selected for the analysis are specific gravity, density, grading, voids variation etc. These properties directly affect the properties of concrete, both hardened and fresh state. It is well established by A.M.Neville<sup>2</sup> et al. that the variations in the strength of concrete are a result of random variations in the material properties.

The modelling of material properties is carried out considering the statistical behaviour as a continuous distribution. The common distributions investigated are normal, lognormal, and beta distributions and most of the variables are found to conform to more than one of these distributions mentioned above. The fitting of distributions is done by using the Kolmogorov-Smirnov test, considering the distributions as nonparametric up to 5 % of probability of failure. The mean, standard deviation and variance are used to quantify the material properties

A detailed laboratory analysis is made, and the data obtained is given intense statistical treatment. The variables are found to follow, predominantly normal distribution and are confirmed by the k-s goodness of fit test to detect the distribution model.

**Keywords** - Probabilistic, Kolmogorov-Smirnov (ks) test, statistic (D), Normal.

## I. INTRODUCTION

The Mix Designs for concrete mixes that are currently practised are based on a semi-Probabilistic approach, and they do not take into account the influence of material properties of the fresh and hardened concrete. Various tables

and charts referred to in mix designs are all based on experimental studies made by extensive work by researchers, and they do not consider the variations in material properties while designing the mixes. Several mixed design methods are currently practised and adopted in various countries with their own methods of design. Normally the material properties which have a direct influence on the concrete mix design are specific gravity, bulk density, fineness modulus, moisture content etc. These properties are chosen depending on the functionality and type of material. The strength or any other property of concrete is always an effect of the *unique* combination of all these material properties, which interact and make the concrete possess specific engineering characteristics or behaviour. Hence to develop better concrete mix design procedures, such variations in these material properties are to be established and evaluated using statistical tools.

Some attempts have also been made by adopting mixture design methods to optimize mix proportions for HPC, and researchers have found considerable success in their attempts, as seen in a paper published by Marcia J .Simon<sup>9</sup> et al.

In most routine mix design procedures, all basic material properties are found by calculating the mean of a set of the number of observations in the laboratory. Few basic statistical applications that are currently practised are generally restricted to the determination of mean and variance of cube strengths, which leads to the determination of target mean strength. A statistical approach of optimizing the mix design procedures have been attempted by Shamshad Ahmad<sup>6</sup> et al. using the statistical experiment design methods by applying the statistical tools and procedures. A serious attempt has also been made by Kumar<sup>7</sup>, S.V. et al. to develop standard mix design procedures for concrete, applying particle packing theories and have come out with a particle packing model which can be used as a guide for proportions.



The aim of the current study is to detect the distributions for all those material properties, which will be used as a base for developing a mixed design founded on probabilistic analysis.

Probabilistic models are one of the important tools adopted in the fields of engineering. Under Probabilistic analyses of concrete mix design, the variables are subjected to probabilistic studies, and their variations are expressed as a probabilistic model expressed in terms of statistical parameters viz mean and standard deviation or any such statistical parameters depending on the distribution. The application of probability theory is very appropriate to study the influence of material properties as a random phenomenon and is well described and justified by R.Ranganathan<sup>3</sup> et al. Usually, most of the concrete material properties follow some of the important distributions such as Normal, Lognormal, the beta of the distribution etc. depending on their innate behaviour. The most widely used and common distribution, which is satisfied by many random variables, is normal distribution, as explained by Douglas C.Montgomery<sup>1</sup> et al. It has also been investigated thoroughly by Shrikanth.B.Vanakudre<sup>4</sup> et al., in his research that many basic material properties of concrete compliance to normal and lognormal distributions. In the probabilistic concrete mix design approach, the variables are not expressed by their single average of mean or as a discrete value obtained from laboratory analysis, as done in conventional mix design methods, but rather as a mathematical expression or as a model, with the appropriate fitting distribution. The best-fitting distributions are identified from the goodness of fit methods, and the variables are modelled accordingly.

**II. MATERIALS USED UNDER THE STUDY**

- Steel slag ( 10 mm and downsize up to 150 microns)
- Fly ash
- Rice husk ash

**A. Steel Slag**

The slag is collected from Jindal steelworks (JSW) situated at Toranagal 30 KM away from Bellary city. Steel slag is a dark grey granular material obtained as a byproduct in the manufacturing of steel from steel SMS plants.

This material is also called Basic oxygen furnace slag or LD Converter slag. This slag is formed when molten floating slag in the steel manufacturing process, quenched in water. Due to sudden cooling effect, the granulation occurs and is very hard and tough. It has a specific gravity of 3.12 and is very dense and compact and strong. It is highly angular and has very low water absorption, high crushing strength and volume stability. This material can be considered as a replacer for the natural sand in mortar or concrete.

The chemical analysis of the slag is given in the Table 1.

**TABLE 1: CHEMICAL ANALYSIS OF SLAG\*.**

Oxides	Sampl e 1 Perce nt	Sampl e 2 Perce nt	Sampl e 3 Perce nt	Sampl e 4 Perce nt	Sampl e 5 Perce nt
Gain On Ignition	2.13	2.81	3.20	2.90	2.88
SiO <sub>2</sub>	10.92	12.01	11.84	10.24	10.08
Fe <sub>2</sub> O <sub>3</sub>	32.99	36.21	40.46	32.52	34.83
Al <sub>2</sub> O <sub>3</sub>	4.44	3.23	4.85	3.14	3.53
CaO	37.48	35.26	30.05	38.83	37.63
MgO	5.43	4.87	4.65	5.58	4.29
Na <sub>2</sub> O	0.12	0.12	0.06	0.38	0.45
K <sub>2</sub> O	-	-	-	0.12	0.13
Mn <sub>2</sub> O <sub>3</sub>	7.29	7.06	6.51	7.71	7.70
CL <sup>-</sup>	0.016	0.043	0.036	0.028	0.023
Sulfide sulphur	0.65	0.66	0.68	0.67	0.62

\*Source: M/s Jindal Steel Works

From table 1, it is observed that the percentage of iron and calcium oxides are high, but as the material is formed in a highly fused condition, the individual oxides are non-reactive. Silica is present with an average value of 11%. The average value of iron present in slag is 35 % and is also non-reactive as it exists in infused and combined form with other oxides.

**B. Fly Ash**

Fly ash is collected from the thermal power plant situated at Kudithini, 20 K.M. away from Bellary. It is the material accumulated from ESP at the third stage of collection, and hence it is a fine powder. As the percentage of silicon dioxide is high, the fly ash belongs to Class F. The composition of the fly ash is given in Table 2.

**TABLE 2. ELEMENTAL FLY ASH ANALYSIS\*.**

Elemental Oxides	Sample 1 Percent	Sample 2 Percent	Sample 3 Percent
Silicon Di-Oxide	62.89	63.35	63.90
Aluminum Oxide	28.66	27.13	28.24
Calcium Oxide	2.23	2.40	2.21
Magnesium Oxide	0.12	0.02	0.11
Manganese Oxide	0.01	0.01	0.01
Titanium Di-Oxide	0.60	0.88	0.56
Sodium Oxide	0.07	0.10	0.10
Potassium Oxide	0.92	0.88	0.91
Iron Oxide	4.14	4.82	3.59
Phosphorous Oxide	0.25	0.27	0.25
Sulphur Tri-oxide	0.11	0.14	0.12

\*Source: Karnataka Power Corporation, KUDITHINI

From Table 2, it is observed that the silica content is high and calcium oxide is low, which implies, the fly ash belongs to class F. The other compound with a higher percentage is aluminium oxide, having an average value of 27 %. There are various other oxides present in minor percentages. This fly ash source is a useful pouzoulona, as it contains a high percentage of reactive silica.

**C. Rice Husk Ash**

The rice husk ash has high reactive pouzoulzna. Rice husk ash is obtained from KC-Contech, Chennai, sold under the market name Hyper 2000. The material has a deep grey colour and is very fine. The composition is given in Table 3.

**TABLE 3. ELEMENTAL RICE HUSK ANALYSIS\***

Elements	Sample 1 (Percent)	Sample 2 (Percent)	Sample 3 (Percent)
Loss on Ignition	8.71	7.00	3.07
Silica(as SiO <sub>2</sub> )	83.60	84.00	90.47
Aluminum(as Al <sub>2</sub> O <sub>3</sub> )	3.05	3.84	3.13
Iron(as Fe <sub>e</sub> O <sub>3</sub> )	1.10	0.60	0.32
Titanium(as TiO <sub>2</sub> )	NIL	NIL	NIL
Calcium(as CaO)	1.80	2.85	1.96
Magnesium(as MgO)	1.28	1.35	0.35
Sodium(as Na <sub>2</sub> O)	0.17	0.13	0.22
Potassium(as K <sub>2</sub> O)	0.29	0.23	0.48

\*Source: KC-Contech, Chennai

The rice husk ash has a good silica percentage of about 85 %, which is very fine. Hence it is also very reactive. Loss on Ignition at an average value of 6 % indicates the presence of unburnt carbon or organic material.

**III. MATERIAL PROPERTIES**

The following material properties are considered, and their statistical distributions are worked out:

The number of samples is 30 for each variable

- i) Specific gravity
- ii) Density (vibration compacted)
- iii) Voids
- iv) Fineness Modulus

**TABLE 4. STATISTICAL DISTRIBUTION OF RANDOM VARIABLES**

SI NO	Random variables	Mean ( $\mu$ )	Standard Deviation ( $\sigma$ )	Variance ( $\sigma^2$ )	CO V	Distribution
<b>SLAG</b>						
1	Specific gravity	3.173	0.1093	0.01195	3.4450	N*
2	Density, gm/cc	1.990*	0.0287	0.00082	1.438	N
3	Percent voids	36.25	2.226	4.9538	6.14	N
4	Fineness modulus	4.3104	0.265	0.0702	6.146	N
<b>FLY ASH</b>						
1	Specific gravity	2.5283	0.06566	0.00431	2.597	N
2	Density, gm/cc	1.37131	0.0073	5.33057e-005	0.5324	N
3	Percent voids	45.7427	0.2888	0.0834	0.6313	N
<b>RICE HUSK ASH</b>						
1	Specific gravity	2.056	0.03112	0.00097	1.514	N
2	Density, gm/cc	0.759	0.0187	0.00035	2.465	N
3	Percent voids	63.394	0.74933	0.5615	1.182	N

N - NORMAL DISTRIBUTION

**A. Slag**

**a) Specific gravity**

The Normal distribution plot for the specific gravity data from figure 1 indicates that the data is concentrated at the centre of the curve and not much scattered. The curve has a good peak and is not much skewed. Table 1 shows the standard deviation for slag, which is very small with a very low coefficient of variation. The mean specific gravity value is 3.17306. The data has a maximum value of 3.3875 and a minimum value of 3.0098, showing a range of 0.3777.

The plot for the goodness of fit from figure 2 between empirical and the standard normal distribution shows the normal distribution fits well for the data. The goodness of fit is done using the KS test, and the maximum test statistic permissible from tables is 0.24 for a sample strength of thirty. From the graph, we observe that the maximum D measured is 0.2333; hence it is concluded the data fit the normal distribution.

**b) Density**

The normal distribution curve drawn in figure 3 shows a few peaks, data not much scattered and concentrated. It is observed from Table 1 the density of slag is very fairly constant, with minimum and maximum values are 1.938 and 2.1495 gm/cc. The range of the density values is 0.2115; being very low confirms good homogeneity of the material. The low standard deviation shows less scattering of data with good convergence.

The K-S Goodness of fit from figure 4 shows the data for the close packing density condition, and the normal distribution fits well. The maximum test statistic D measured is 0.2 against the permissible value of 0.24.

**c) Percentage Voids**

The mean of percentage voids is 36.9, which indicates a good packing property of the material and also due to the good gradation of the slag size varying from 10 mm to 150 microns. It means that the effect of compaction properties of slag has a less pronounced influence on the properties of concrete.

However, the percentage voids have shown a tendency to have higher deviation, variance and coefficient of variation. Data fits well for percentage voids for the normal distribution.

The ks test confirms normal distribution, with Dmax as 0.1918 from figure 5.

**d) Fineness modulus**

The histogram & normal curve from figure 6 shows that the gradation and spread of data are unusual. The data is concentrated at the ends. The maximum, minimum and mean values are recorded as 4.84, 4.017 and 4.447, respectively. The range of the data is 0.823

However, the standard deviation value shows that fluctuation fairly exists. The coefficient of variation at 7.017

confirms the variation in the fineness modulus. It also indicates that it can be considered probabilistic.

Still, as a whole, the data fits well for the normal distribution and the Goodness of fit of the same is verified by the Kolmogorov-Smirnov test. It can be observed from the figure that as the test statistic 0.1918 is well below the permissible value for 30 samples at a 5 % significance level, the fitted distribution is considered normal.

### **B. Fly Ash**

#### **a) Specific gravity**

From the normal distribution curve shown in figure 8, it is observed that the data spread is less and do not possess high or low values of specific gravity. It shows that experimental errors are less pronounced and results are more reliable. Further, the distribution is not much skewed and has a good peak.

Fly ash has a mean specific gravity value of 2.528, with high and low values are 2.62 and 2.423. The data has a range of 0.197, which shows the results are consistent and errors are minimized. The Specific gravity values from above table 1 indicate that the standard deviation & variance are very small. The normal curve plot is shown in figure 9.

The Goodness of fit is done using the KS test, and the maximum test statistic permissible from tables is 0.24 for thirty samples. The maximum D measured is 0.2232; hence the fitted distribution is normal.

#### **b) Density**

The normal distribution curve shown in figure 11 shows a good peak, indicating the concentration of data at the centre, and from table 1, the mean is 1.371. The maximum and minimum values from the data are 1.385 and 1.357gm/cc, with a range of 0.028. Hence it shows good homogeneity of material and consistent results.

The standard deviation from table 1 shows a very low value of 0.0073 and has a very small coefficient of variation of 0.5324. The goodness of fit test is made with the ks test, and figure 12 gives the details. The empirical and standard distributions match each other, with D maximum measured from the graph at 0.098, which confirms the normal distribution. The 90 %, 50 % and 5 % density values are shown in figure.

#### **c) Percentage voids**

The density function for voids is shown in figure 13, which shows a good peak and less scattering of data. The mean of the data is 45.7427, having low and high values observed as 41.218 and 46.325. From table 1, we observe standard deviation is 0.2888, indicating the variations in observed data is low. The data fit normal distribution as shown in figure 14 with maximum D measured as 0.2031, by ks test.

### **C. Rice Husk Ash**

#### **a) Specific gravity**

The density function plot is drawn as shown in figure 15. The data has a mean value of 2.0559 and has high and low values observed as 2.00 and 2.13. The data has a range of 0.13, which shows consistency in the data. The normal curve has a good peak, indicating less spread and has a 90% value of 2.091. The histogram exhibits no skewness.

The statistical results from table 1 indicate the data has a very low standard deviation and coefficient of variation. The goodness of fit by the Kolmogorov test indicates the data very well fit the normal distribution. The test statistic Dmax is measured from the plot and shown in figure 16 is 0.163, which is well below the maximum permissible value of 0.24 for thirty samples.

#### **b) Density**

The density function plot is shown in figure 17 shows a good peak and less spread. The mean value of 0.759 gm/cc shows the material is light and has low bulk density. The standard deviation is computed as 0.0187 and the coefficient of variation as 2.465. The goodness of fit test from figure 18 shows that the data conform to normal distribution.

#### **c) Percentage voids**

The histogram is plotted in figure 19 is balanced and displays a good peak without skew. However, the voids determined high, at a mean value of 63.4 %, can be due to low specific gravity and the lightness of the material. The low standard deviation of 0.749 and low coefficient of variation indicates good consistency of the data. The goodness of fit test is shown in figure 20, and the test statistic D max, measured as 0.1118, confirms the normal distribution.

## **IV. CONCLUSION**

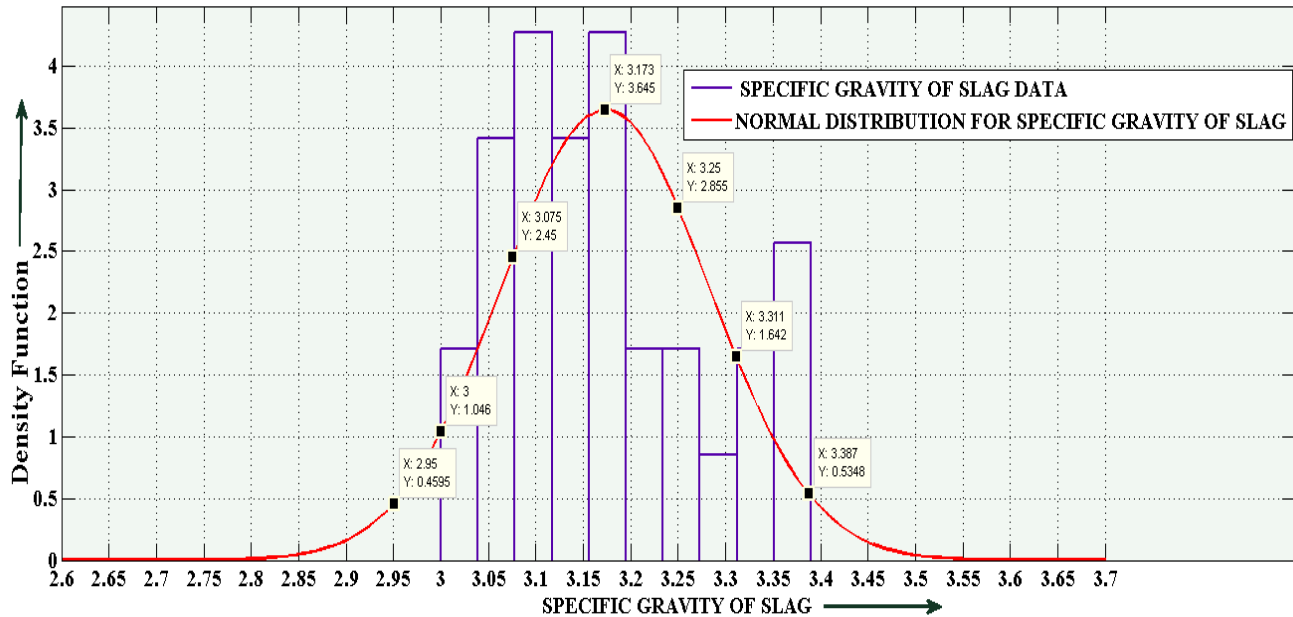
- As observed from table No 1, all the variables follow the normal distribution.
- The coefficient of variation for specific gravity of slag is around 3 %, which shows that the data is fairly consistent.
- The packing properties of the slag are good, with the mean standing at 36% voids (64% solids), and it can be reasoned for the good grading properties of the material. This can be substantiated by the fact that the average F.M. value is 4.3 from Table 1, and the fineness modulus value for 10 mm aggregate according to the *fuller Thomson formula* is approximately 3.9-4.0. Hence the adopted value is much closer to the theoretical value, which is the straight line gradation for the least voids packing property of the material. But the variation in the packing properties is quite high, which may be probably due to the time of vibration and grade of the material.

- The fineness modulus of slag is found with a mean of 4.3, and the value indicates it is well distributed with regard to coarse and fines and is well balanced. But as the material has a variance of 6.14 % and with high and low values at 4.84 and 4.02, which indicates that the material is not homogeneous, means all size fractions are not evenly distributed.
- The Specific gravity of fly ash shows a mean of 2.528, with a variance of 2.6 %, which implies the fly ash has consistent material quality.
- The compacted density of fly ash is 1.371 gm/cc and has a very low standard deviation and coefficient of variation.
- The specific gravity of rice husk ash is found with a mean of 2.0559, which indicates the material has significantly less weight, which depends on its composition.
- The density of rice husk ash is low at 0.759 gm/cc, indicating it is in a very fine state.
- Percentage of voids for slag (36%), fly ash (46%), and rice husk ash (63%), data shows that, except slag, as the material is lighter, they can't be compacted to higher densities. Also, as RHA and FA tend to be less well graded, the voids are relatively higher. Hence they are difficult to compact under gravity.

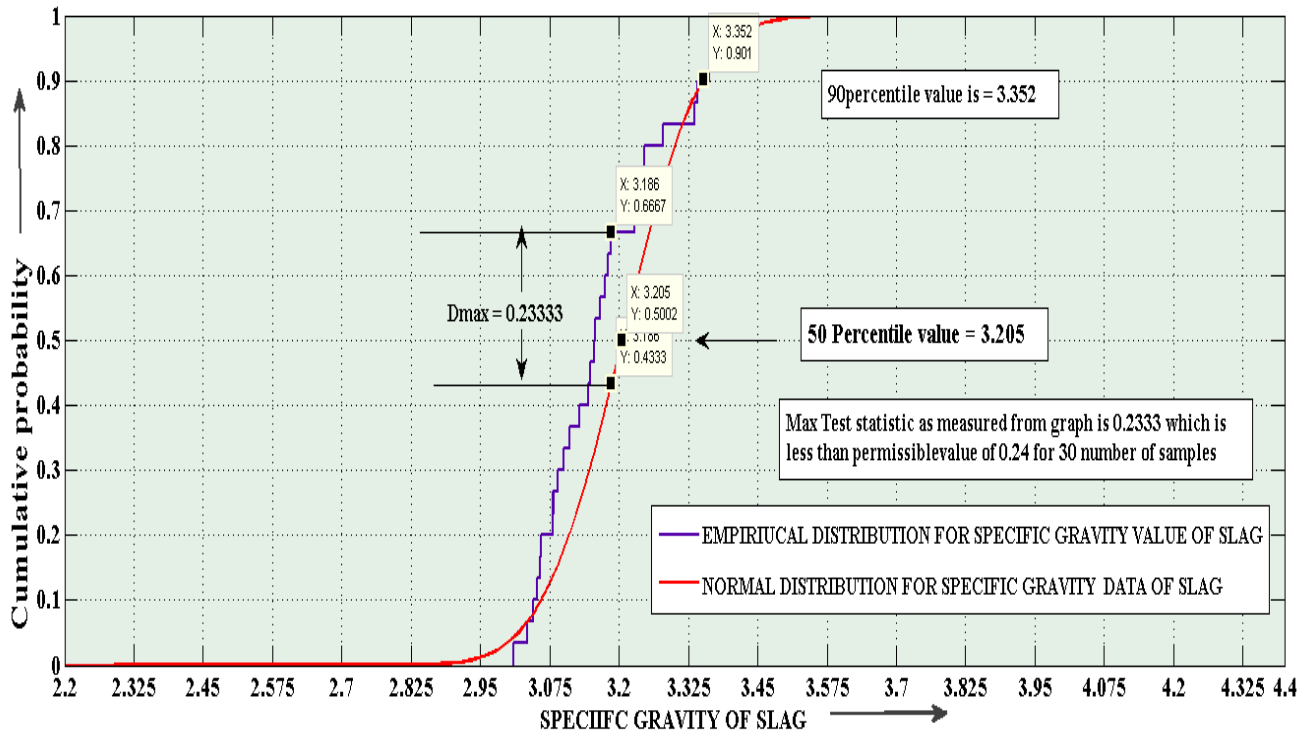
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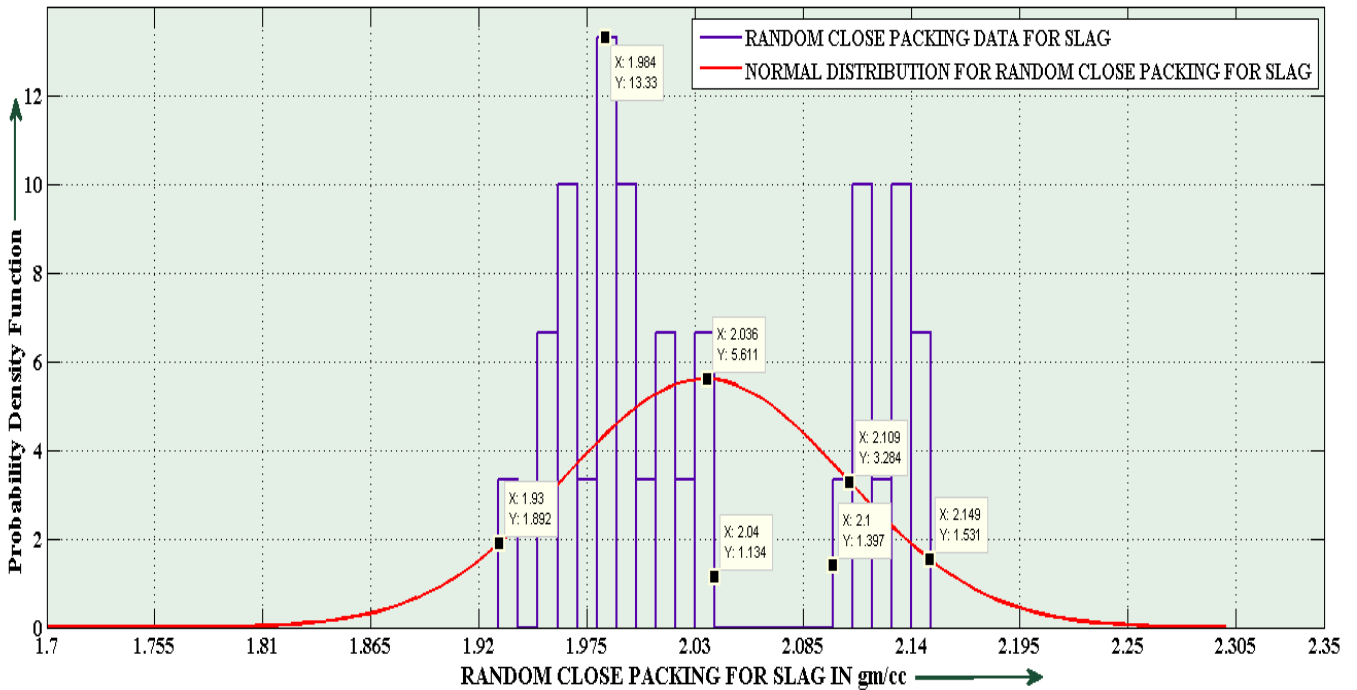
**DISTRIBUTIONS**



**FIG 1. FREQUENCY DISTRIBUTION OF SPECIFIC GRAVITY OF SLAG**



**FIG 2. K-S GOF TEST FOR SPECIFIC GRAVITY OF SLAG**



**FIG 3. FREQUENCY DISTRIBUTION OF SLAG**

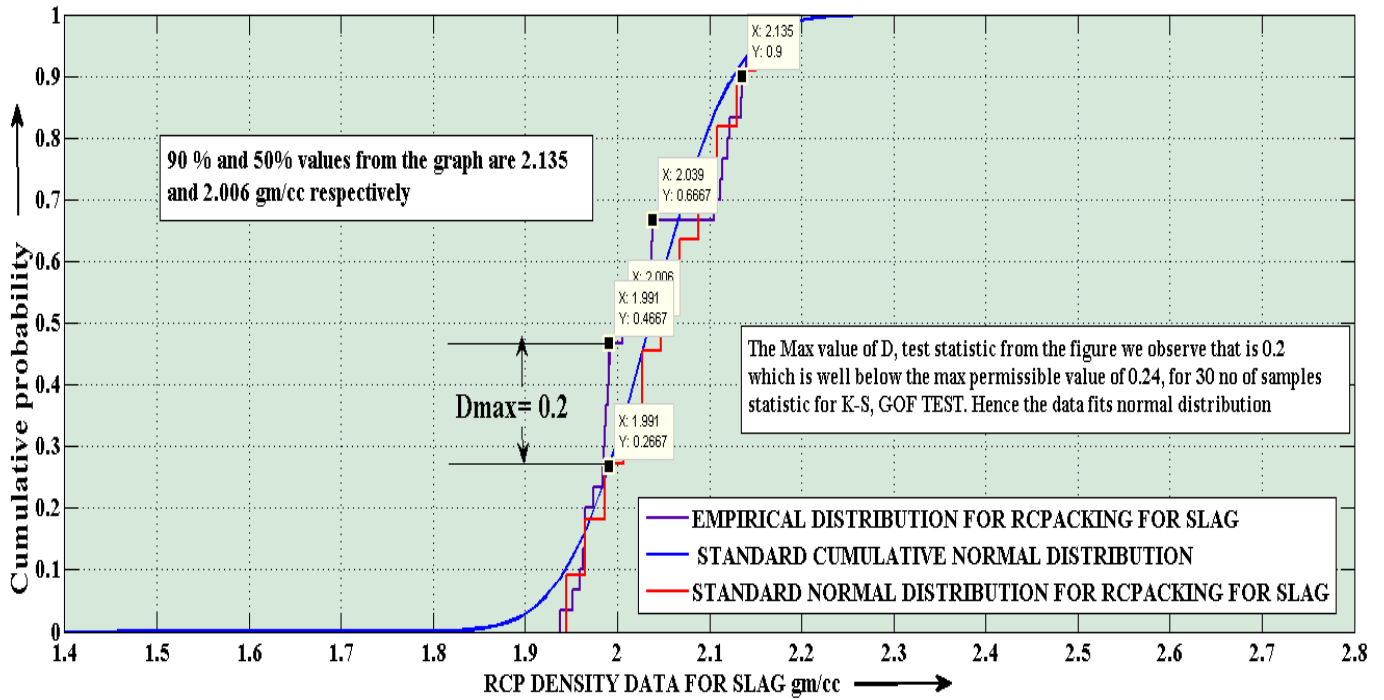


FIG 4. K-S GOF TEST FOR DENSITY OF SLAG

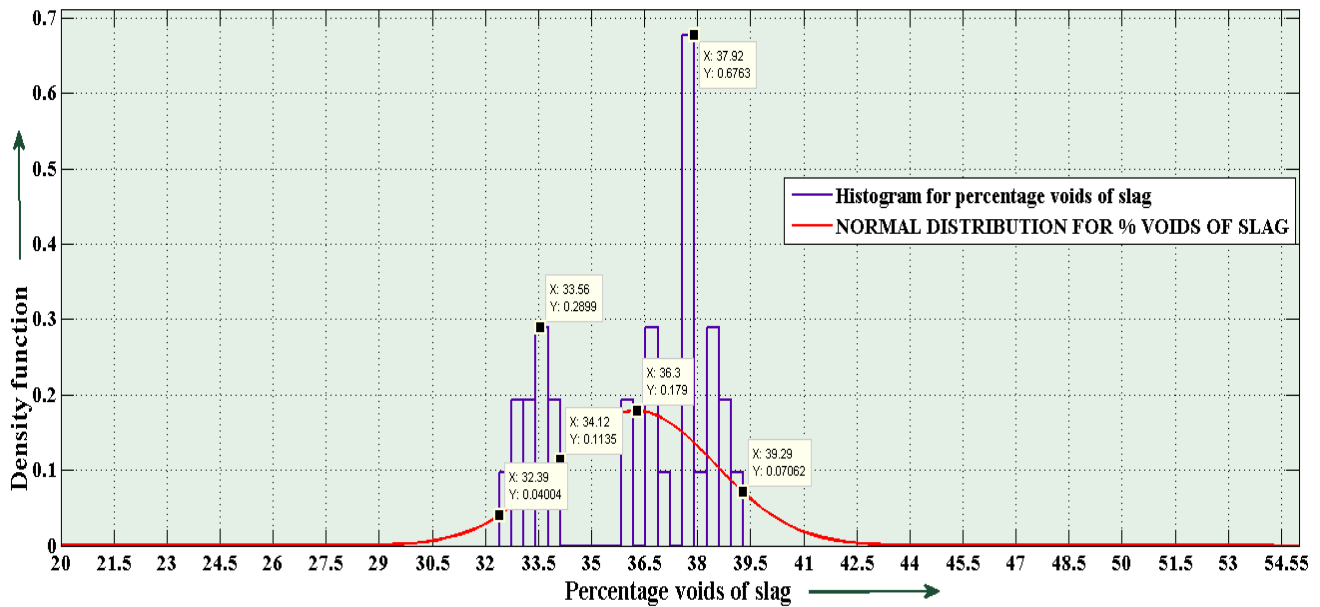
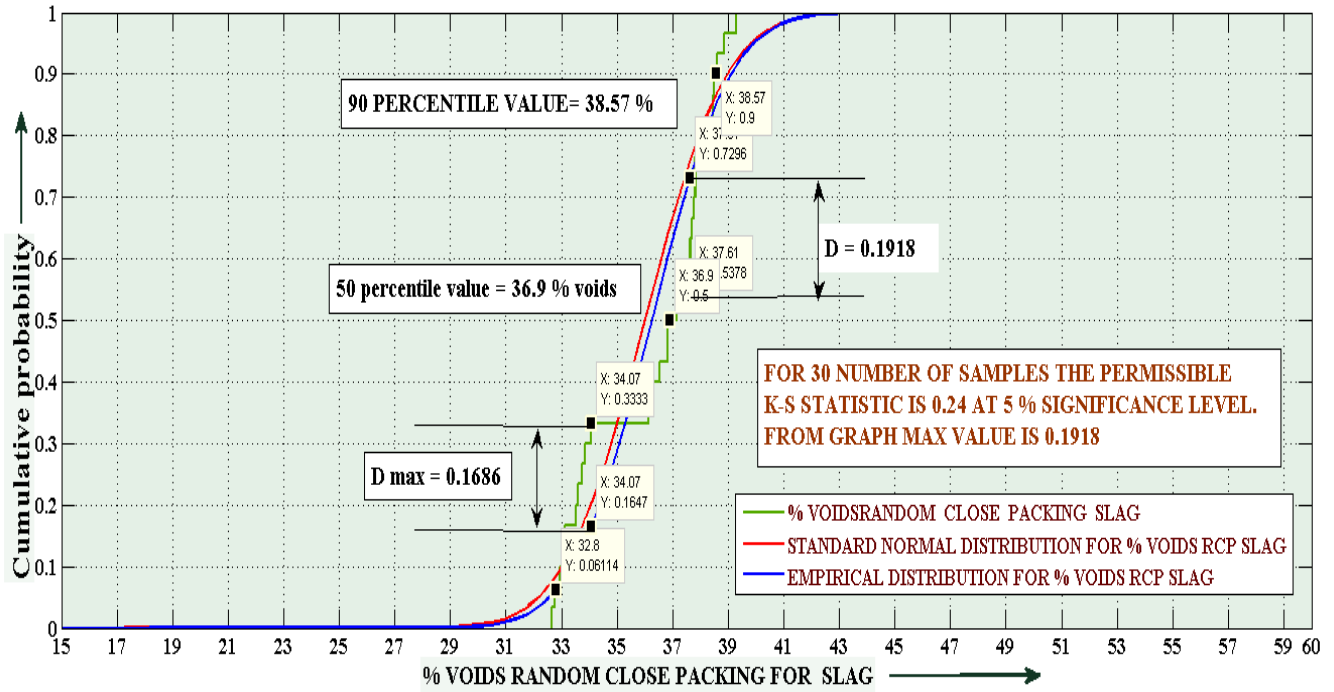
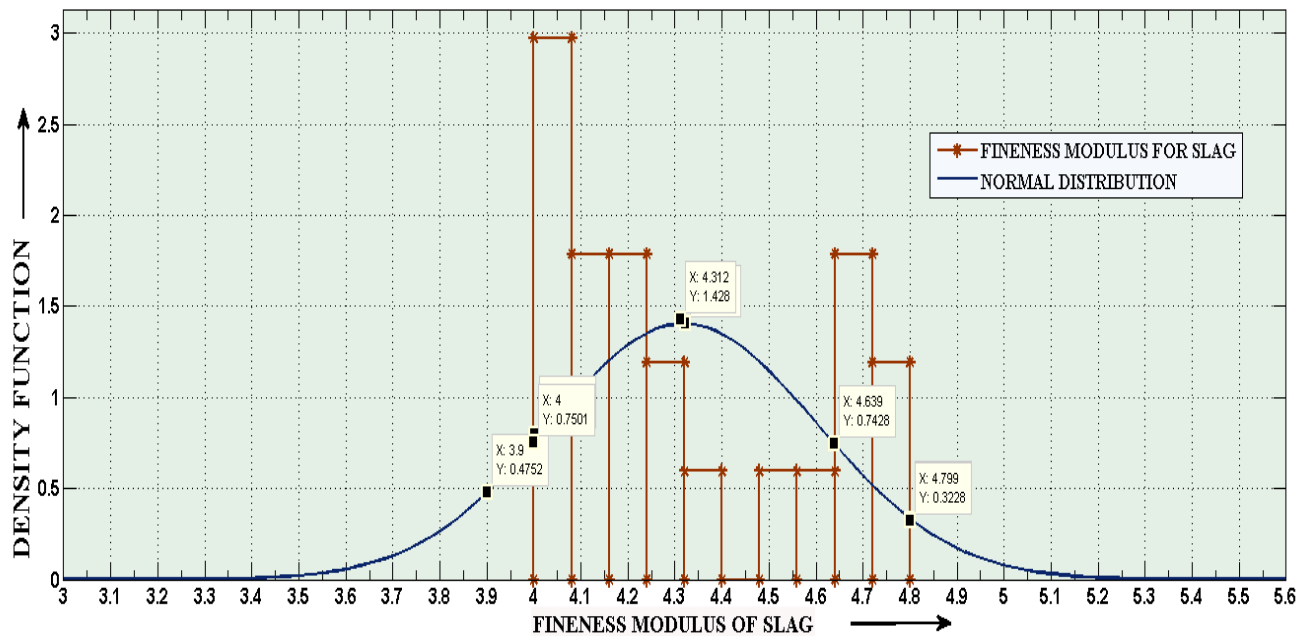


FIG 5. FREQUENCY DISTRIBUTION OF PERCENTAGE VOIDS OF SLAG





**FIG 6. K-S GOF TEST FOR PERCENTAGE VOIDS OF SLAG**



**FIG 7. FREQUENCY DISTRIBUTION OF FINENESS MODULUS OF SLAG**

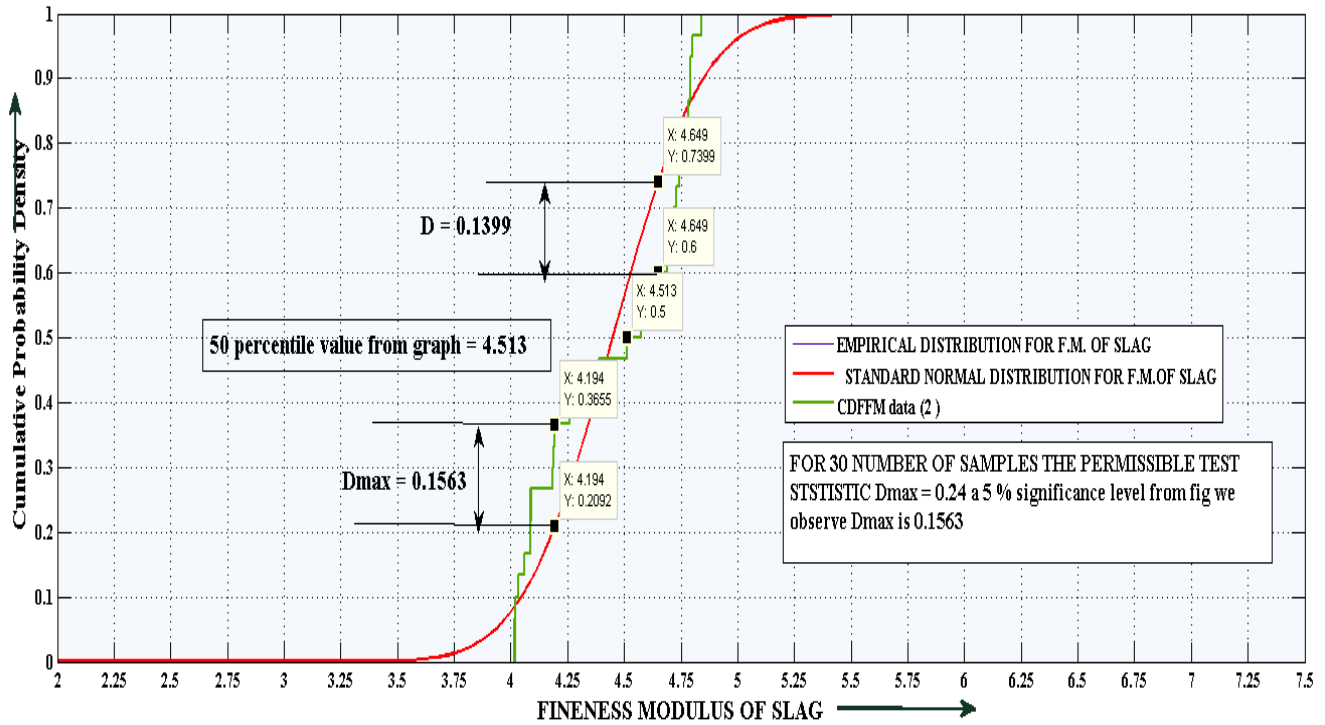


FIG 8. K-S GOF TEST FOR FINENESS MODULUS OF SLAG

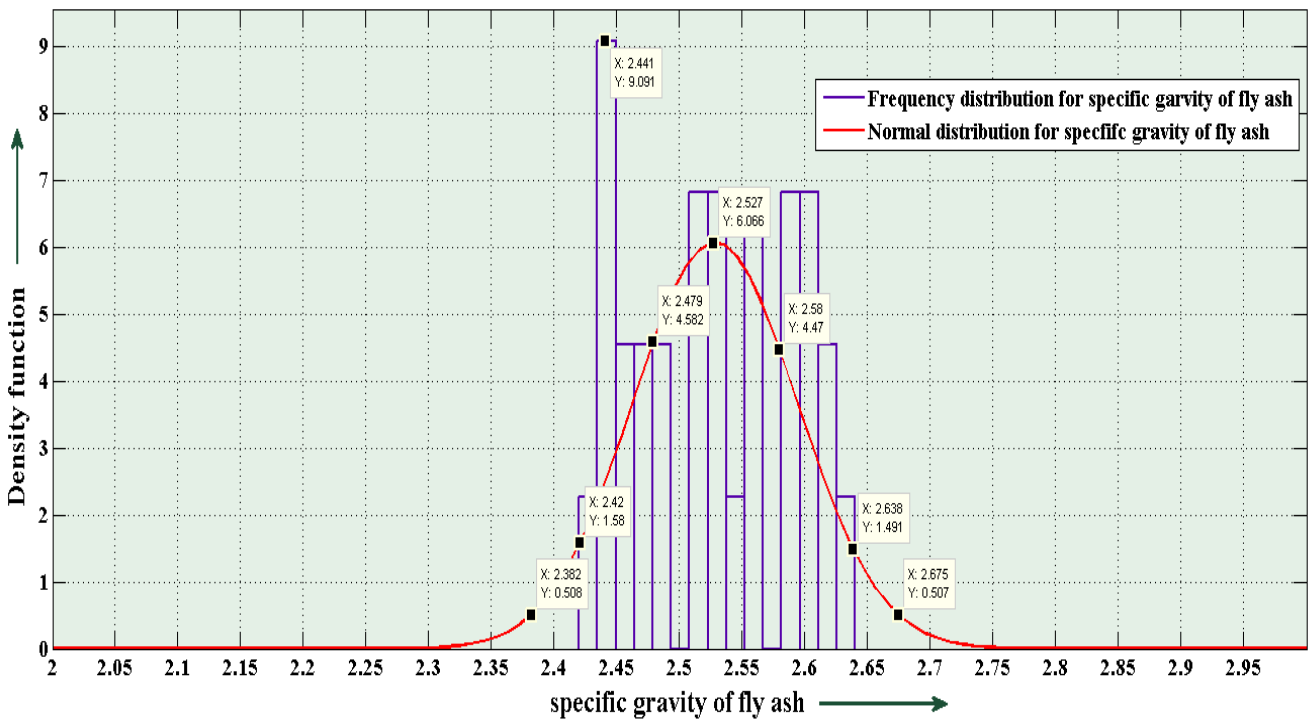


FIG 9. FREQUENCY DISTRIBUTION OF SPECIFIC GRAVITY OF PULVERISED FLY ASH

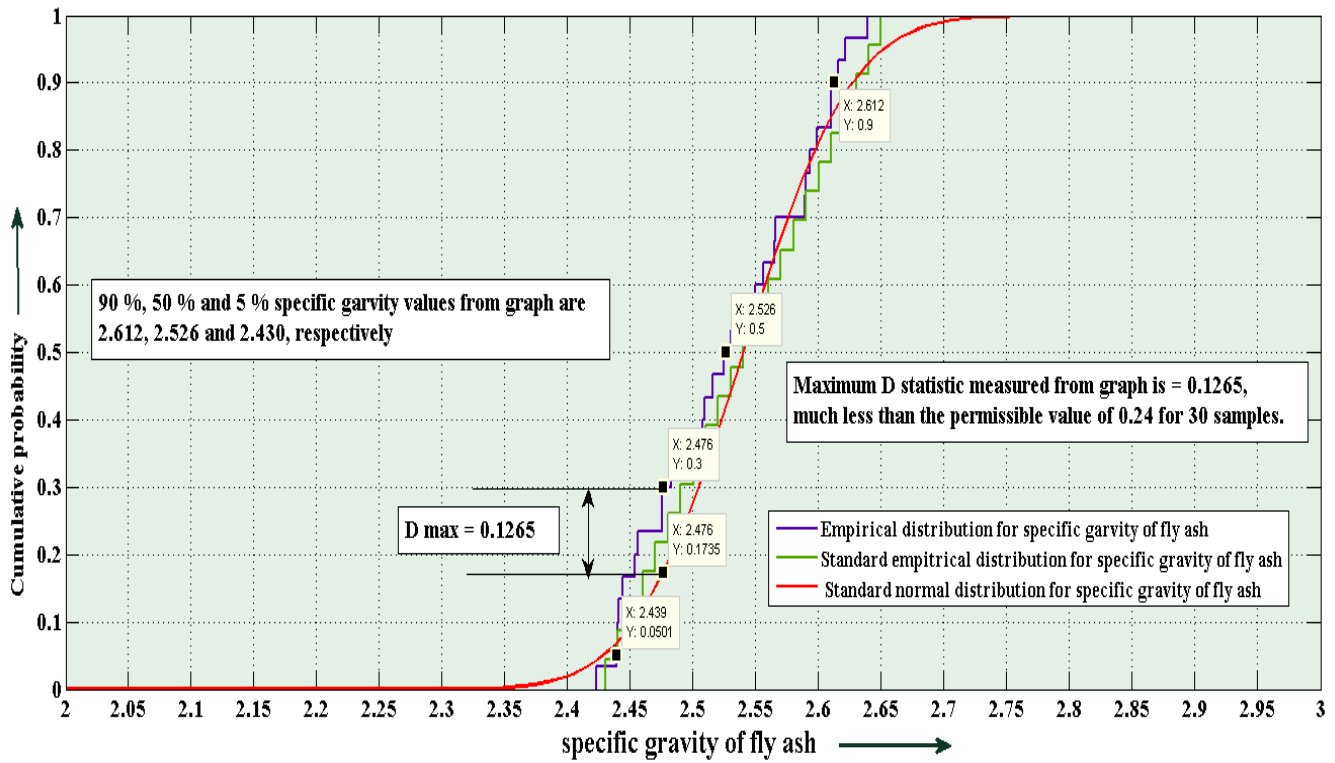


FIG 10. K-S GOF TEST FOR SPECIFIC GRAVITY OF FLY ASH

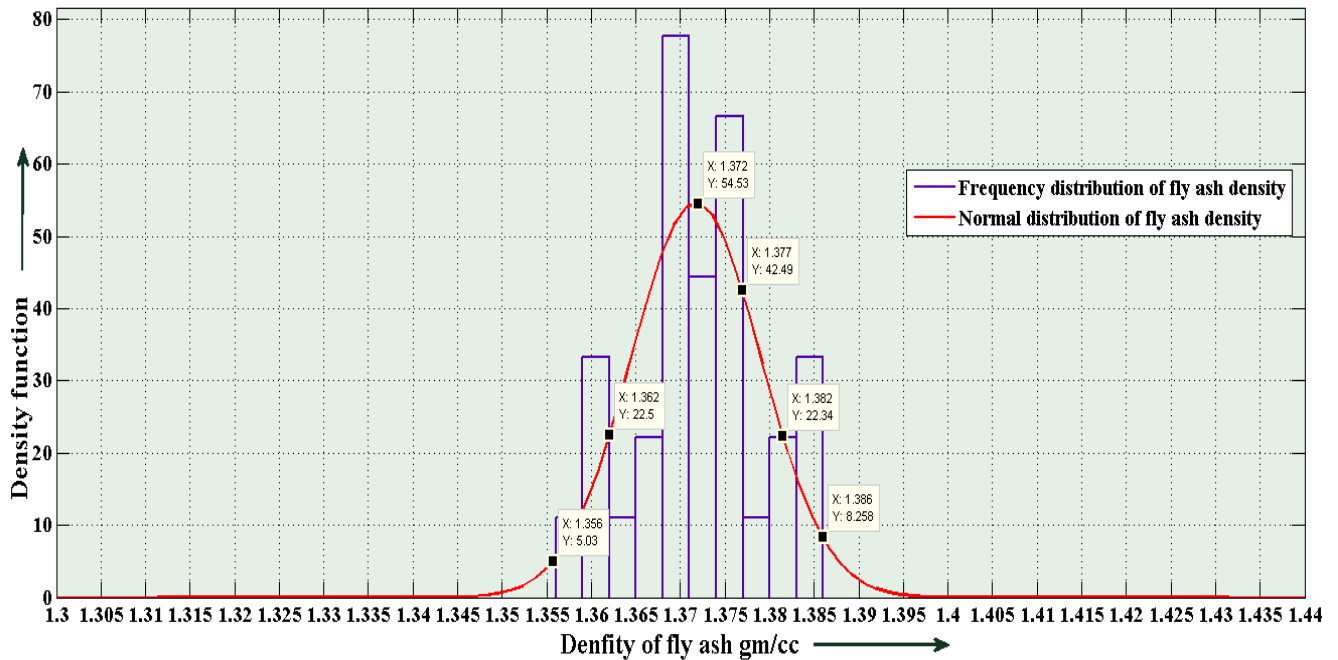


FIG 11. FREQUENCY DISTRIBUTION OF DENSITY OF FLY ASH

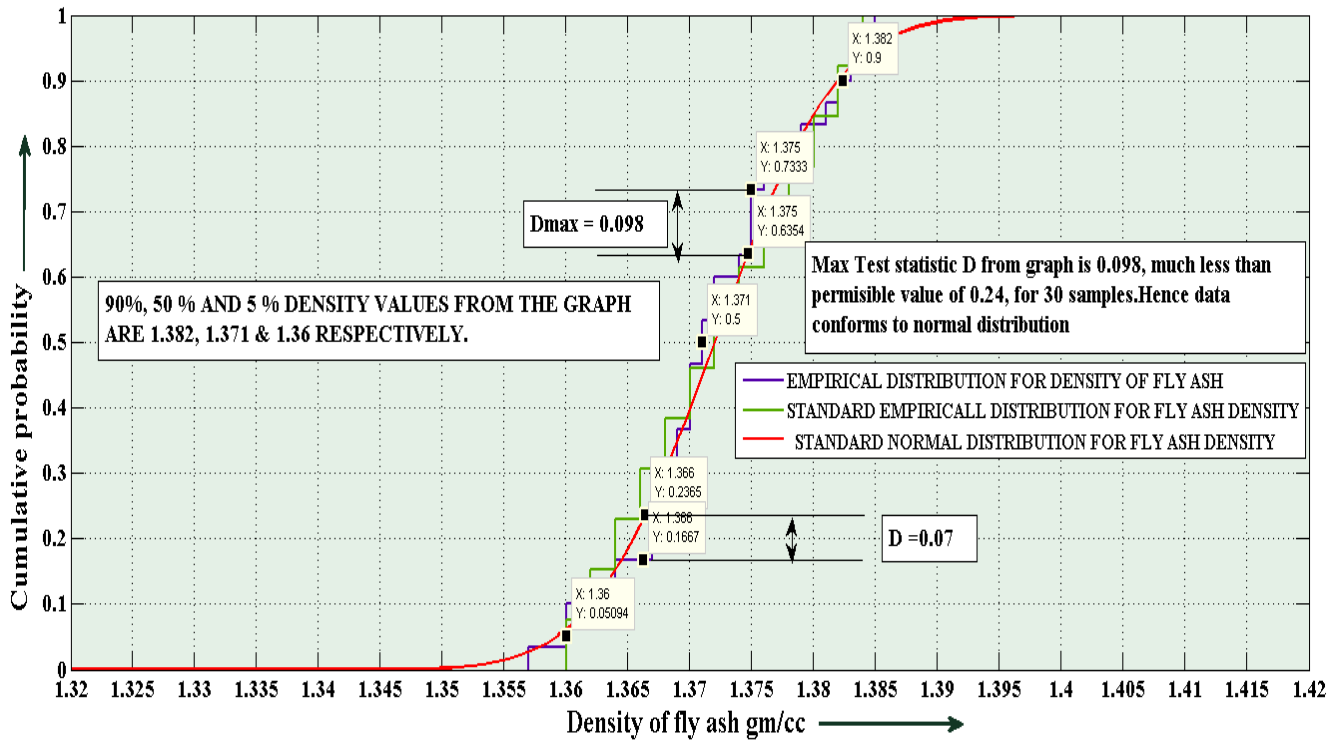


FIG 12. K-S GOF TEST FOR DENSITY OF FLY ASH

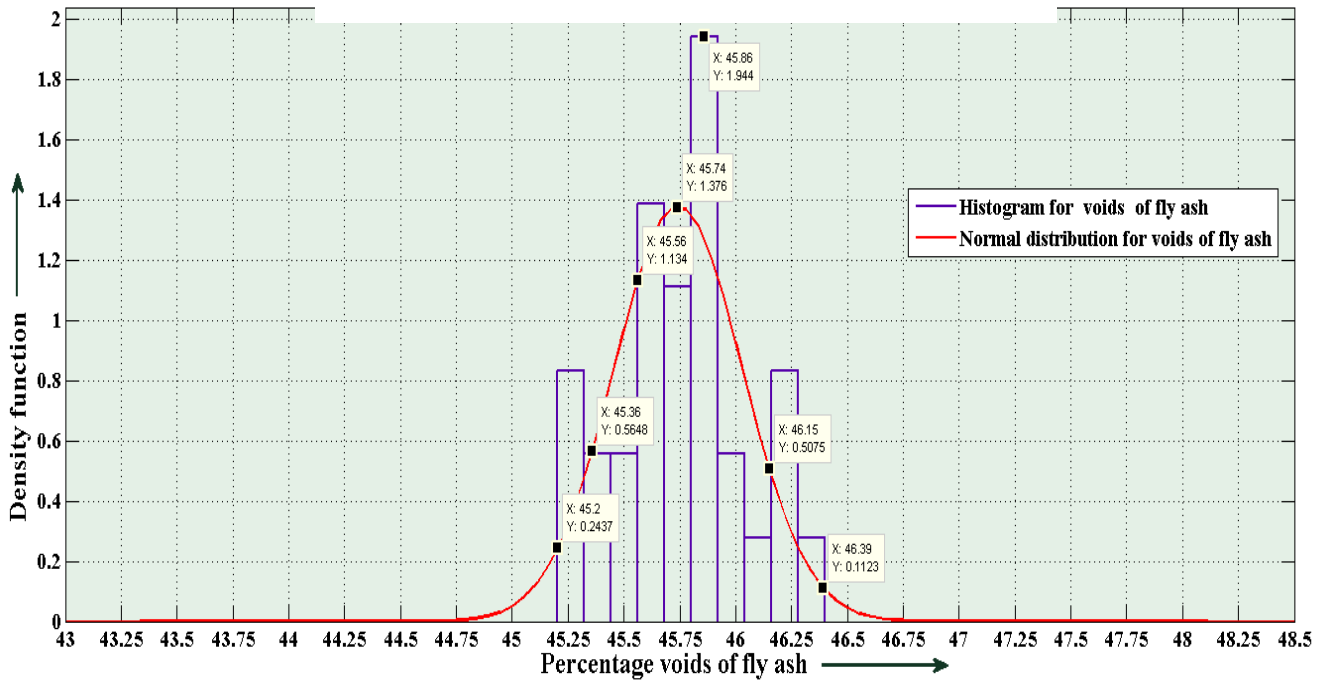


FIG 13. FREQUENCY DISTRIBUTION OF PERCENTAGE VOIDS OF FLY ASH

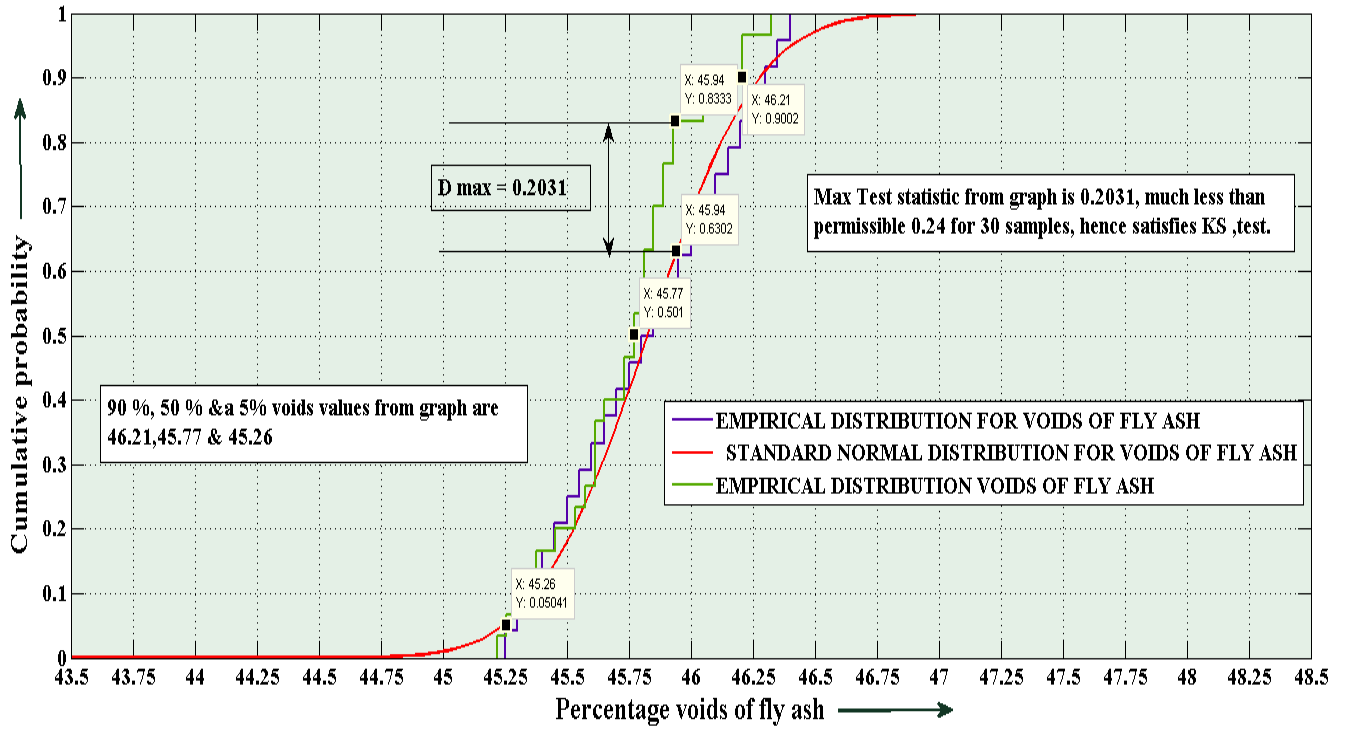


FIG 14. KS GOF TEST FOR PERCENTAGE VOIDS OF FLY ASH

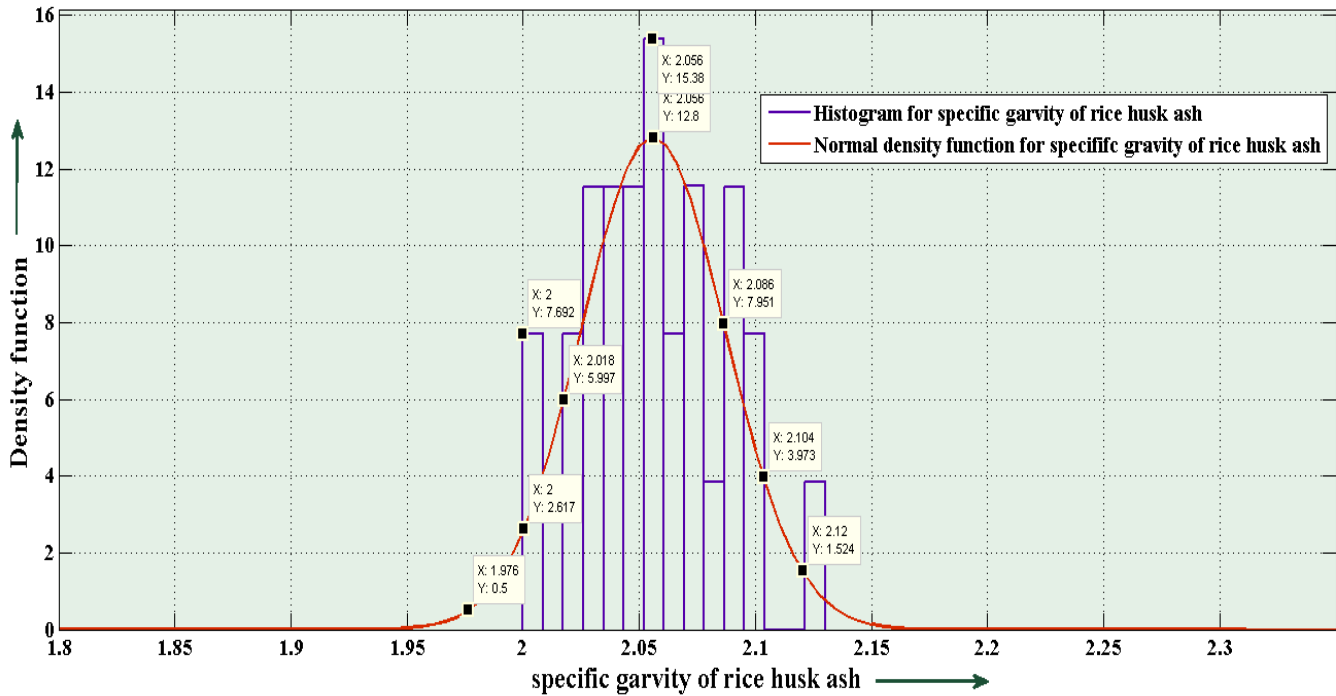


FIG 15. FREQUENCY DISTRIBUTION OF SPECIFIC GRAVITY OF RICE HUSK ASH

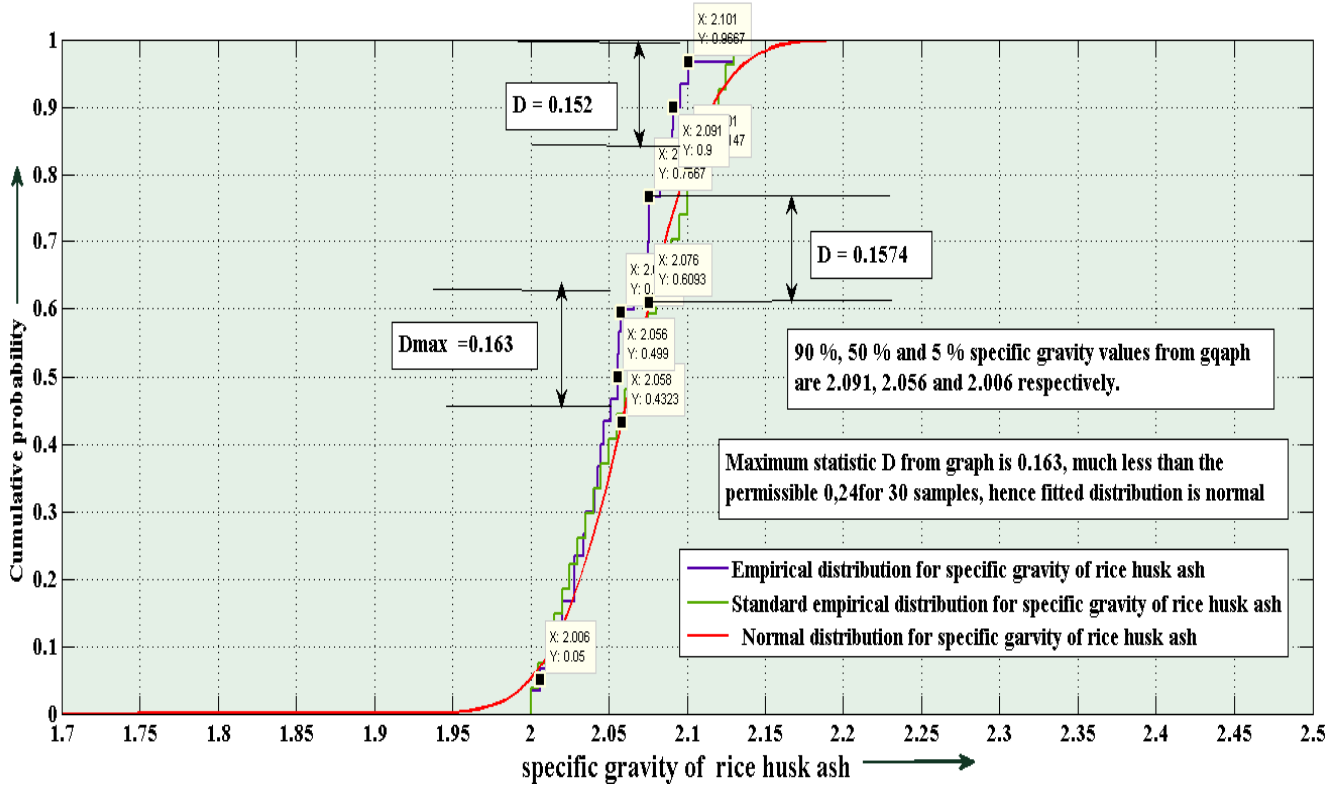


FIG 16. K-S GOF TEST FOR SPECIFIC GRAVITY OF RICE HUSK ASH

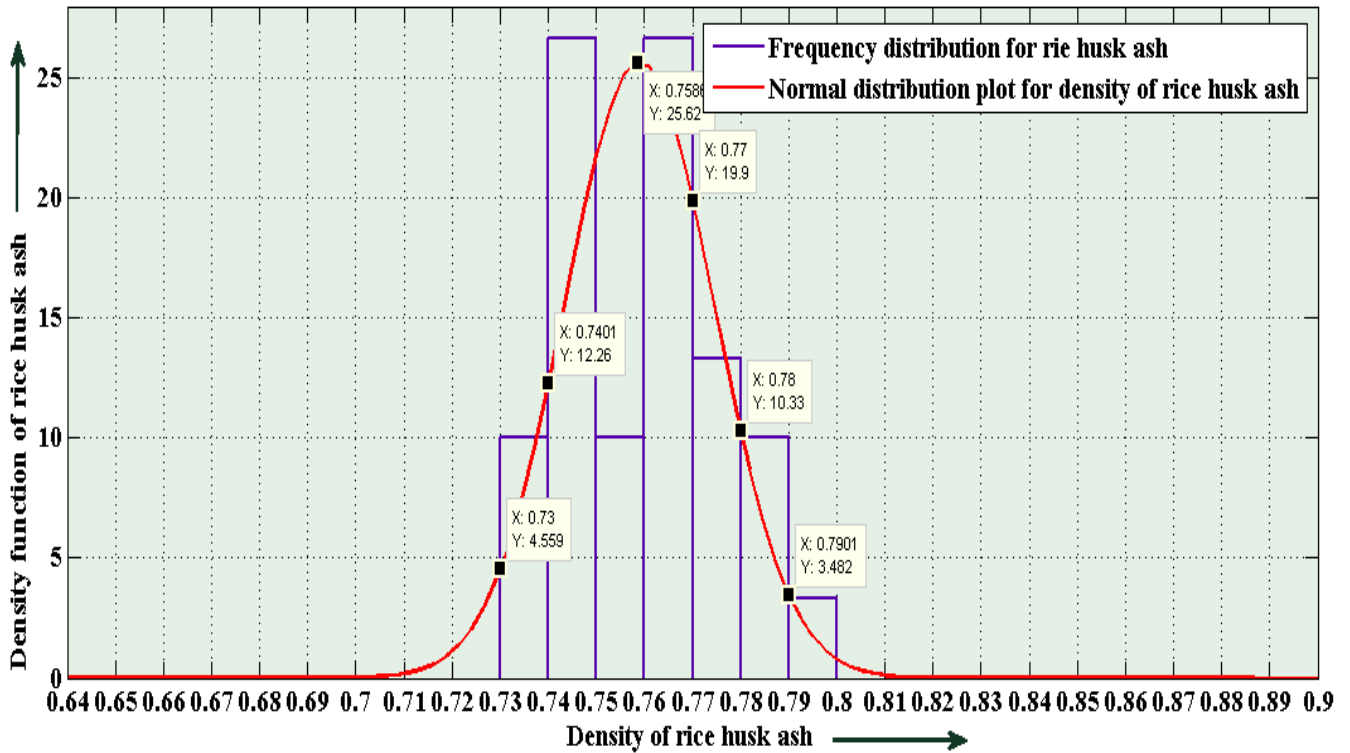


FIG 17. FREQUENCY DISTRIBUTION OD DENSITY OF RICE HUSK ASH

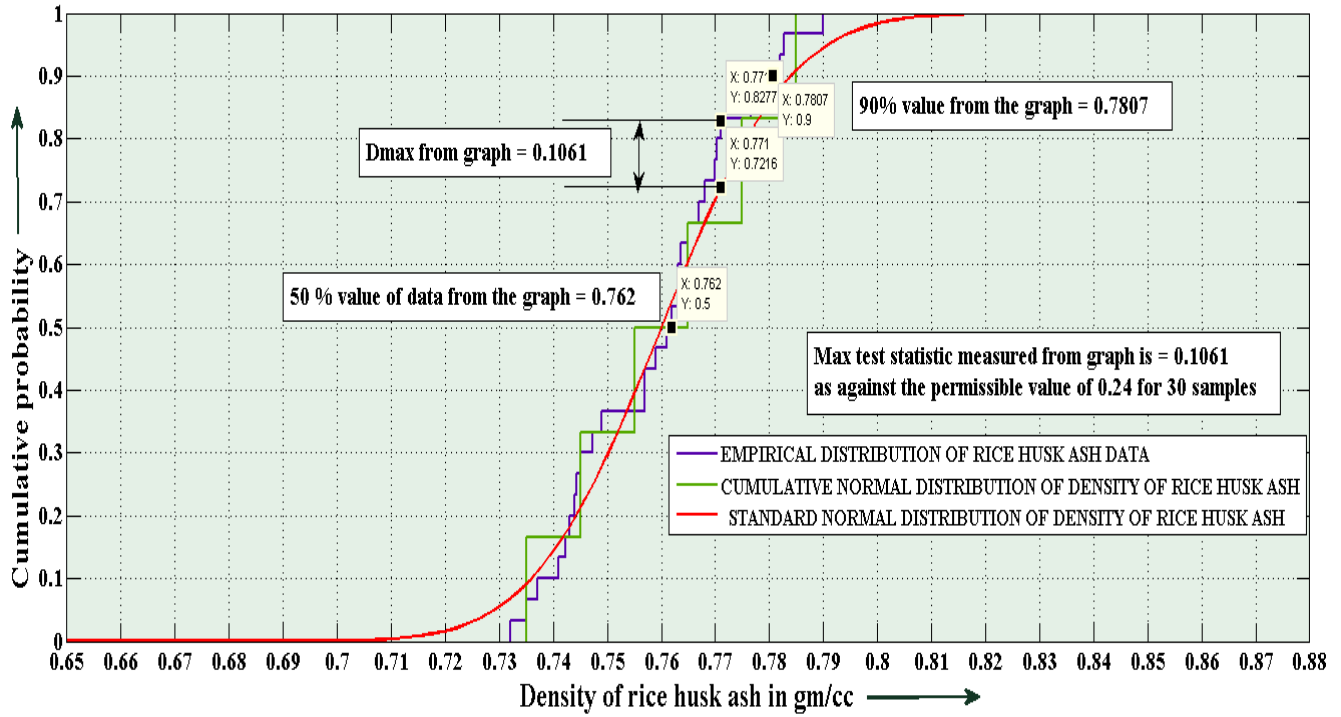


FIG 18. K-S GOF TEST FOR DENSITY OF RICE HUSK ASH

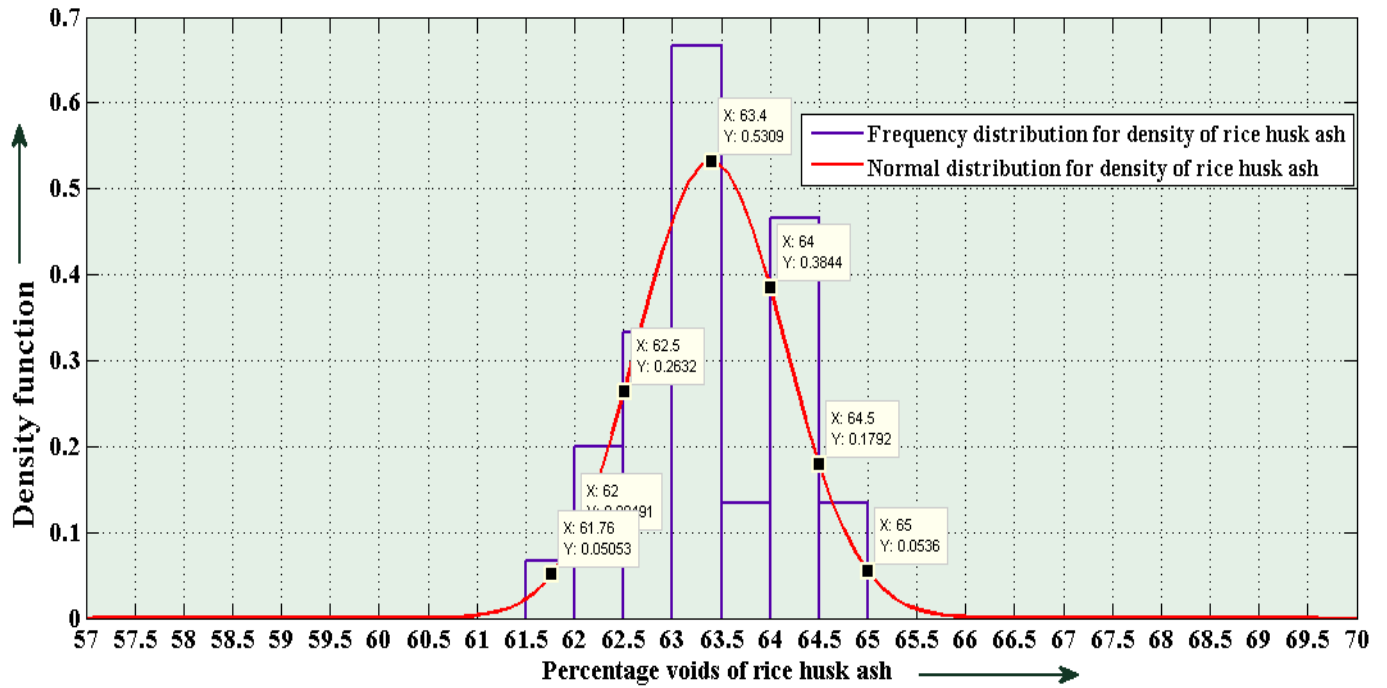
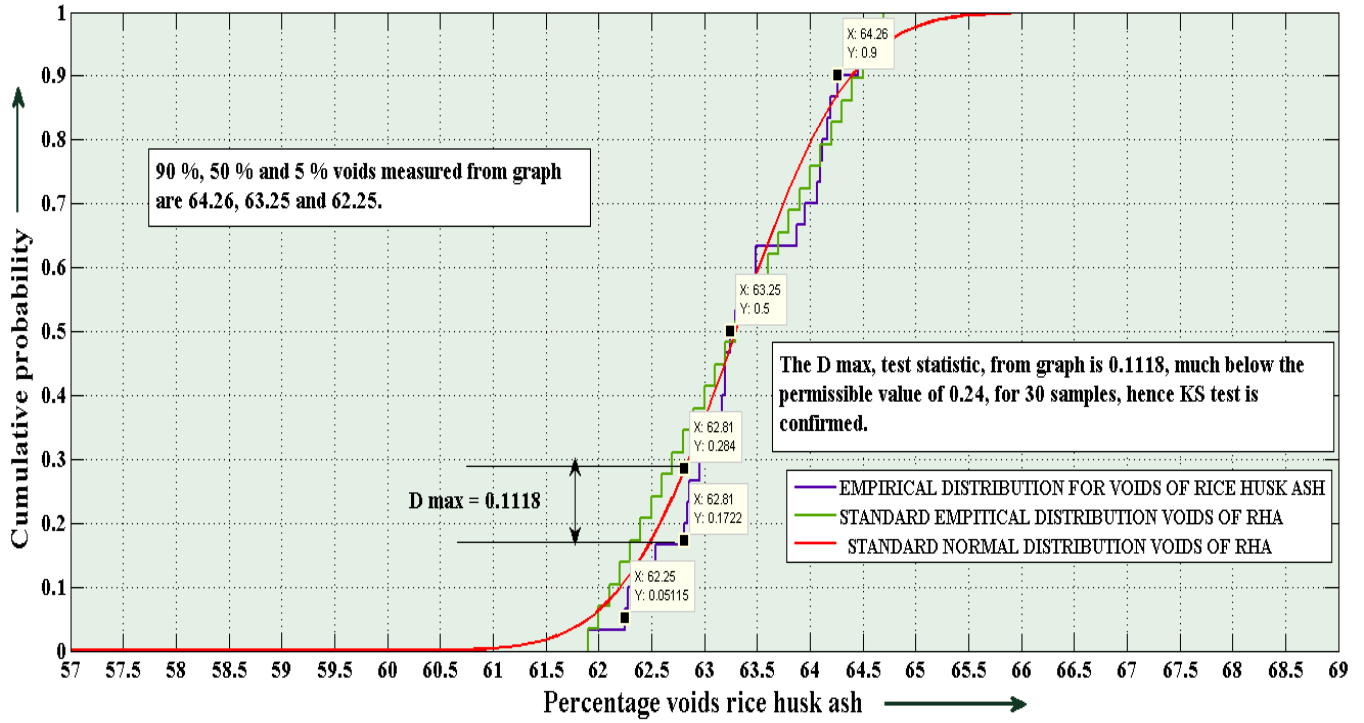


FIG 19. FREQUENCY DISTRIBUTION OF PERCENTAGE VOIDS OF RICE HUSK ASH



**FIG 20. K-S GOF TEST FOR PERCENTAGE VOIDS OF RICE HUSK ASH**