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Original Article

Designing an Effective Hybrid Control Strategy to Balance a Practical Inverted Pendulum System

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Abstract *- Controlling an inverted pendulum system to be successfully balanced is one of the most conventional and significant control problems. An inverted pendulum system typically consists of a cart and a free-rotating rod. The control goal is to maintain a vertical position of the rod while the cart must be regulated to follow a given desired trajectory satisfying an acceptable tolerance. To obtain such a control objective, the system should be separated into two simultaneous control phases: rotational angle control for the rod and position one for the cart. This work mainly focuses on designing an effective control scheme for both a mathematical model and a practical prototype of the inverted pendulum system. The control methodology is a reasonable integration of two conventional PID controllers and a proper metaheuristic optimization technique, e.g., PSO (particle swarm optimization). Simulation results on the Simulink model and experiment results on the practical one demonstrate the feasibility and effectiveness of the control strategy proposed in this study.*

Keywords *- Inverted pendulum, PID, PSO, Scaling factors, Balancing control.*

I. INTRODUCTION

It is a fact that there are a huge number of nonlinear control problems concerning an original control of an inverted pendulum [1-5]. A typical model of an inverted pendulum consists of a free-rotating pendulum rod attached to a small cart which usually moves on a directional bar. Figure 1 shows the practical model of an inverted pendulum system designed in this work. The pendulum rod can rotate freely up to 360 degrees while the small cart moves along a one-meter-directional guide bar. A 24VDC motor drives the motion of the cart. It is noted that the velocity of the cart was kept at a constant value for experiments implemented in this study.

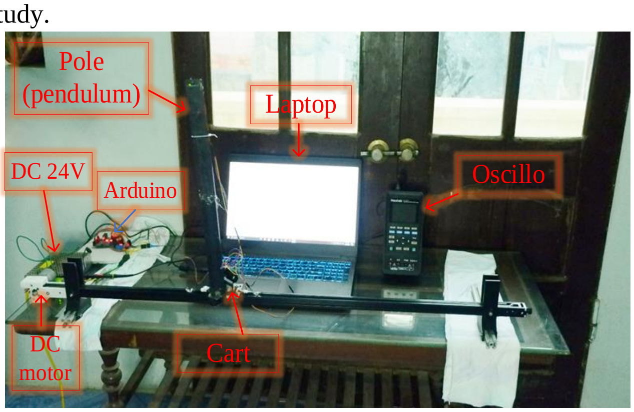

Fig. 1 The practical inverted pendulum system designed in this study

The control problem in this model has been solved in different ways. However, a classical solution should be mainly to control the dynamic force on the cart so that the pendulum is balanced at a certain position, e.g., a vertical position with a 90-degree angle [3]. This can be considered a control problem with only one input variable but many output variables (single input - multiple outputs). One input variable represents the force on the cart. Two outputs are the position of the cart and the deviated rotational angle between the pendulum rod and a standard direction (usually a vertical one). These output variables need to be successfully controlled to obtain the balance of the pendulum, bringing the system to a stable state.

Traditional control techniques may not be able to solve the inverted pendulum control problem efficiently. Some previous studies already proposed a number of control solutions such as PID, fuzzy logic, and neural networks for this nonlinear control problem [6]. A fact is that the conventional PID regulators can be improved much better [7]. This paper focuses on designing a new inverted pendulum model using conventional PID controllers based on the particle swarm optimization (PSO) algorithm [8-10]. The control methodology cooperates between a PID – based deviation angle controller and a PID-based position one. This hybrid integration can ensure the balance of the rod and control of the cart as desired.

Remember that each PID controller used for the inverted pendulum system has three parameters to be determined, i.e., gain, integral, and derivative factors. In this study, six parameters, twice the three mentioned above, should be determined using an appropriate optimization technique. In this study, the PSO algorithm [11-15], one of the most effective metaheuristic optimization techniques, will be applied to optimize these six parameters. The results of simulation on MATLAB/Simulink software and a real experimental model have shown the outstanding effectiveness of the proposed control strategy.

The structure of this paper is divided into five sections. After the Introduction section, section 2 presents a mathematical model of the cart-inverted pendulum system. Then, section 3 proposes the PSO algorithm applied to the design of a PID controller. Next, section 4 focuses on representing numerical results of stimulation on MATLAB/Simulink software and experiments on the practical inverted pendulum model shown in Fig. 1. The last section consists of the conclusion and future work concerning this study.

2. A Mathematical Model of an Inverted Pendulum

Modeling a control plant is an initial step to designing a proper control strategy. The more exact the mathematical model, the more feasible the control methodology can be. To build the mathematical model of an inverted pendulum system, it is necessary to use Newton's laws to analyze the dynamic responses of the system. Considering the assumptions indicated in Table 1, the mathematical model representing the inverted pendulum system is expressed in (1).

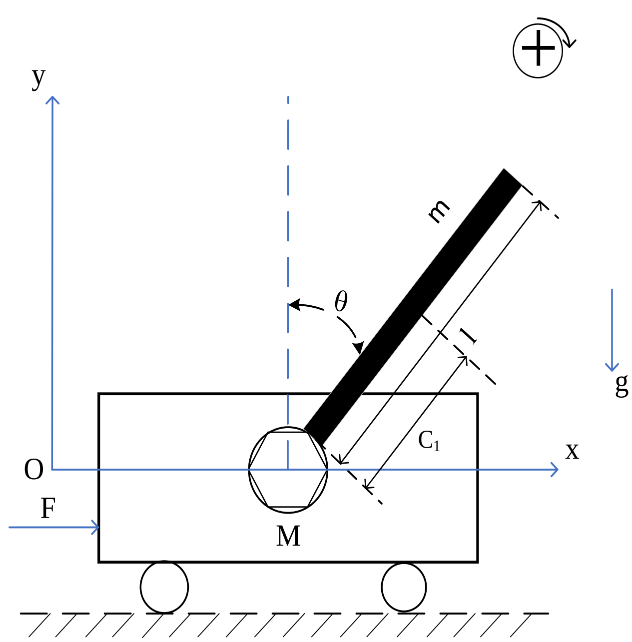

Fig. 2 Free body diagram of inverted pendulum

Table 1. Assumption for the inverted pendulum system

Symbol	*Parameter*	*Value*
M	Mass of the Cart	1.15 kg
m	Mass of the pendulum	0.35 kg
l	Length from center of the cart to pendulum	0.4 m
θ	Pendulum angle from vertical	Rad
F	Force applied to the cart	N
C_1	Length from the center of the cart to the center of the pendulum	0.28 m
J_1	Moment of inertia	0.002 kg.m^2
q	State variable, the inverted pendulum system on the cart has two state variables, x (position of the cart) and θ (the deviated angle of the pendulum bar)	
g	Gravity	9.81 m/s^2

$$\begin{cases}(m+M)\ddot{x}+mC_1\ddot{\theta}\cos\theta-mC_1\dot{\theta}^2\sin\theta=F\\ mC_1\ddot{x}\cos\theta+(J_1+mC_1^2)\ddot{\theta}-mC_1g\sin\theta=0\end{cases} \tag{1}$$

The model indicated in (1) can be written in the following matrix type:

$$M(q)\ddot{q}+V_m(q,\dot{q})+G(q)=\begin{bmatrix}F\\0\end{bmatrix} \tag{2}$$

Where q is the state variable that representatively denotes two considerable quantities: the position of the cart x and the deviated angle of the pendulum θ. Matrices $M(q)$ $V_m(q,\dot{q})$ and $G(q)$ are given as follows:

$$\begin{cases}M(q)=\begin{pmatrix}m+M & mC_1\cos\theta\\ mC_1\cos\theta & J_1+mC_1^2\end{pmatrix}\\ V_m(q,\dot{q})=\begin{pmatrix}0 & -mC_1\dot{\theta}^2\sin\theta\\ 0 & 0\end{pmatrix}\\ G(q)=\begin{bmatrix}0\\ -mC_1^2g\sin\theta\end{bmatrix}\end{cases} \tag{3}$$

The mathematical model of an inverted pendulum system presented in (1) or (2) will be applied to design a balancing control strategy for the inverted pendulum, studied below.

3. Propose A Hybrid Control Strategy For The Inverted Pendulum System

3.1. Hybrid Control Strategy

It is known that PID (Proportional – Integral – Derivative) controllers have become popular and widely used in theory and practical industry [10]. These conventional regulators have appeared in almost fields of industrial factories because of their effective controllability, low cost, and simple design. A typical method for tuning PID controller is Ziegler – Nichols method. However, turning a PID controller is always difficult because of the time-consuming experimental process, noise, external forces, and errors from the measuring devices or sensors working with the PID controller. To solve this problem, it is essential to find out the best mathematical model of the plant.

Therefore, it can be seen that tuning PID's parameters are unlikely to achieve the most optimal value because it is usually based on the experience of long–term engineers. In this paper, the authors propose research on tuning PID methods combined with a particle swarm optimization algorithm as a solution that finds a set of parameters automatically following the direction of biomimetic optimization.

3.2. Particle Swarm Optimization Algorithm (PSO)

The PSO algorithm is a popular metaheuristic–based optimization technique, effectively solving many optimal problems [11-19]. This optimization method is inspired by biology theory to search for optimal points by minimizing an objective function which can also be denoted as the fitness function. This algorithm is inspired by the activity of schools in nature, such as schools of fish and flocks of birds in the process of searching for food. An overview of a typical PSO algorithm can be found in [8, 15]. Figure 3 and Fig. 4 present a search process for optimal points and a typical flowchart of the PSO algorithm, respectively. It is noted that the PSO mechanism is usually employed for determining optimal parameters in control problems, i.e., finding appropriate sets of PID factors which will be described in the hybrid control strategy below.

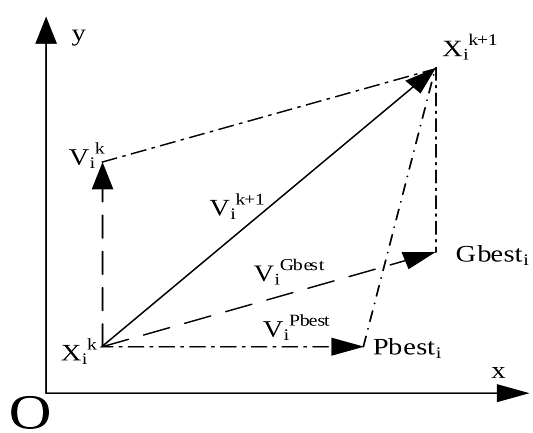

Fig. 3 The definition of changing search points in a typical PSO algorithm

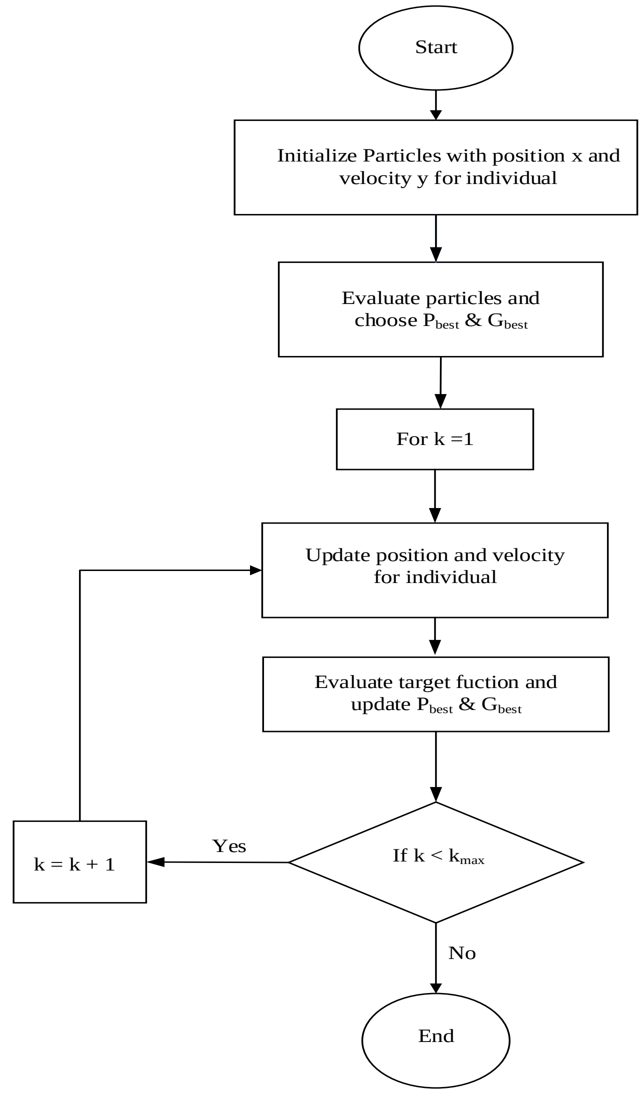

Fig. 4 A typical PSO algorithm flowchart

3.3. Application of particle swarm optimization algorithm to design a hybrid control strategy

The hybrid control strategy using a traditional PID regulator and the PSO algorithm as a proper integration is illustrated in Fig. 5. The objective function is a formula that an algorithm feeds data to calculate predictions. Since the desired requirement is to minimize error $e(t)$ of the output, the objective function can be described as follows:

$$J = \int_0^T e_\theta(t).tdt = \int_0^T \left(\theta^* - \theta(t)\right).tdt \quad (4)$$

Where $\theta^*(t)$ and $\theta(t)$ denote the reference angle, the real angle deviated from the vertical axis of the pendulum.

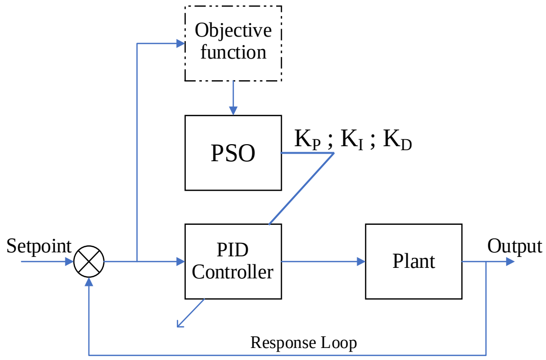

Fig. 5 Structure of a hybrid PSO~PID controller

In the PSO algorithm applied to the PID controller, each particle contains three parameters, K_P, K_i, and K_d, which means the search space of these parameters. We have the algorithm procedure of the PSO~PID control system as follows:

Step 1: Initialize for each particle in a population

1.1: Evaluate each particle's position (X_{ik}) randomly.

1.2: Evaluate each particle's velocity V_{ik}.

Step 2: Run the model

2.1: Run a control model with established parameters in the previous time.

2.2: Find parameters: K_p, K_i, and K_d of the PID controller.

2.3: Find the corresponding objective function.

2.4: Evaluate position function Xik following the value of an objective, so-called fitness function.

Step 3: Update the value of position and velocity for each particle

3.1: Update the value of position and velocity.

3.2: Evaluate objective function (fitness).

3.3: If fitness < Pbest_fitness then Pbest = Xik, Pbest fitness = fitness.

3.4: Update the value Gbest for each respective particle with the current minimum position of the objective function in the population.

Step 4: Find the value of the new particle.

If a particle's present value is better than its previous best value, update it.

Step 5: Repeat step 2 until the stopping one of the criteria is satisfied.

Targets of tuning PID parameters by PSO algorithm are:

- Minimizing the objective function.
- Find suitable results for the system and reduce the error. Repeat steps until the stopping criteria are satisfied.

3.4. The working principle of the system is when the PID controller is combined with the PSO algorithm on the Arduino platform.

Assuming that there are two major phases given below:

- Initializing phase:

Establish a random PID controller parameters model based on the PSO algorithm.

- Processing phase and ending phase:

After creating a PID controller parameter, data is uploaded to the Arduino mainboard via COM. Then, Arduino will write PID controller parameters in the PID program on the mainboard. When Arduino initializes the inverted pendulum system, it also calculates the objective function with PID's parameters in the previous time. After ending the program, Arduino will give feedback on the present value to the PSO program on the PC platform to tune for the next loop base on response data. After that, the PSO program will check whether the stopping criteria are satisfied or not, if not, then go back to the loop until the satisfying stopping criteria then give the most suitable PID parameters in repeating processing. Figure 6 also describes this control methodology in a block diagram form.

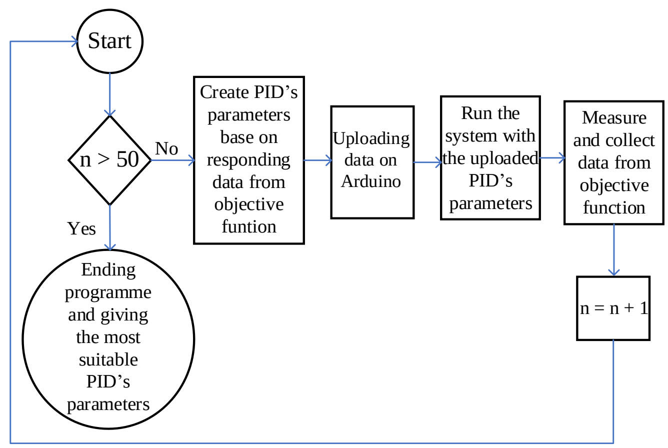

Fig. 6 A flow chart regarding an executed process of Arduino used for the practically inverted pendulum

4. Simulation and Experiment Results

4.1. Simulation via MATLAB/Simulink

To verify the research results presented in the previous parts, simulations and experiments on the actual inverted pendulum model will be conducted in this section. Remember that the numerical simulation results are normally employed to verify the studying theory. In the meantime, the experiment achievements will testify to the applicability of the proposed theory, which can be applied to the manufacturing reality.

The block diagram for the simulation process performed on MATLAB/Simulink is shown in Fig. 7.

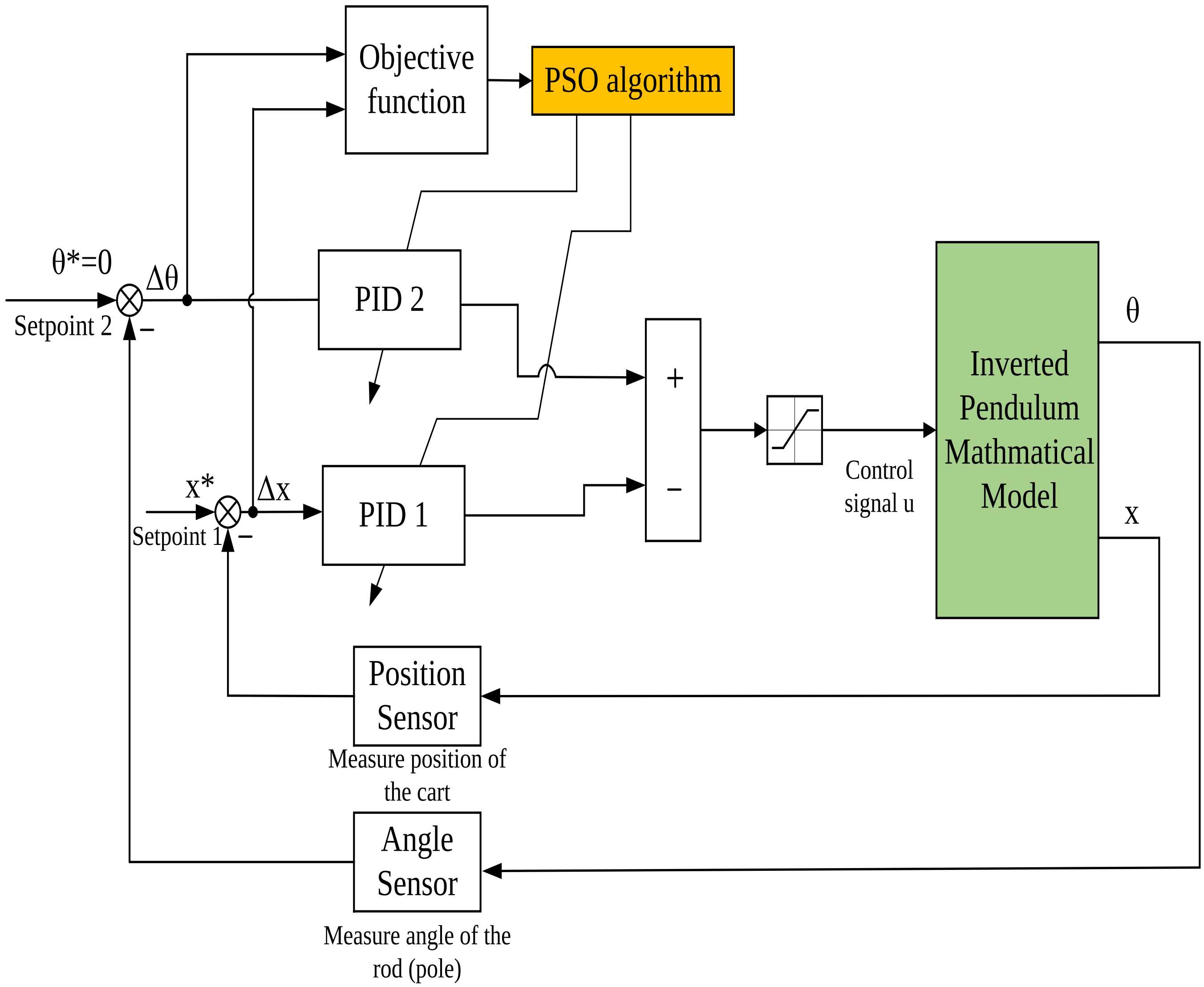

Fig. 7 The PSO – based PID control strategy proposed for the inverted pendulum model

Table 2. Optimal PID's parameters obtained from PSO

No.	Position PID controller	Angle PID controller
1.Gain factor	Kp1 = 65.8000	Kp2 = 84
2.Integral factor	Ki1 = 25.7000	Ki2 = 59.2000
3.Derivative factor	Kd1 = 36.1000	Kd2 = 12.5000

To evaluate the control quality of the proposed control strategy, the following two simulation cases are taken into consideration:

Case 1: It is assumed to set the initial position of the vehicle to be 0.2 m, but the desired position is set to be zero together with the deviated rotational angle. Figure 8 below shows the position deviation of the vehicle and the angle of deviation from the vertical of the rotating rod. This figure clearly shows that both the error of the position and the deviated angle is forced to be zeros at acceptable settling times of less than five seconds. In this perspective, the angle deviation, θ, is controlled to be a desirable value in a very short time, less than two seconds, demonstrating the effectiveness of the proposed control scheme.

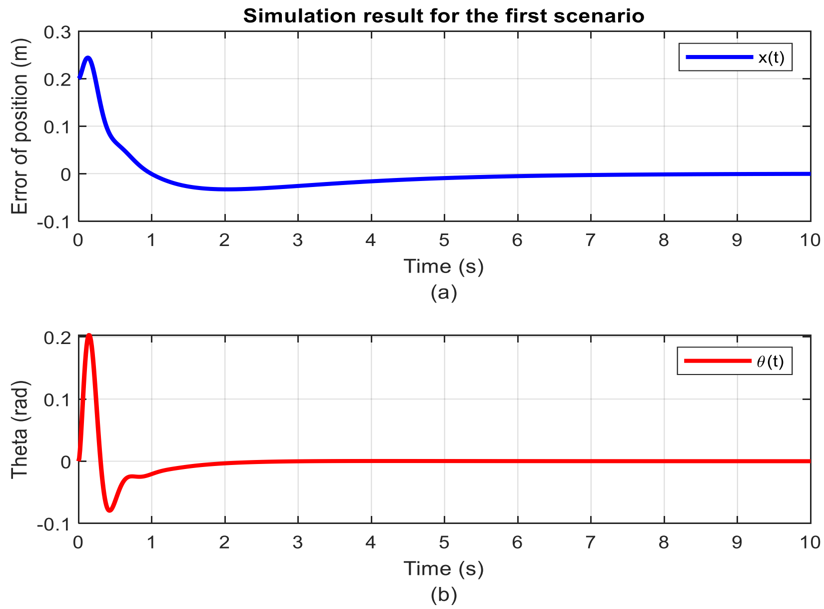

Fig. 8 Simulation results for the first case considering two main parameters, *x*(*t*) and *θ*(*t*)

Case 2: The desired position of the vehicle $r(t)$ is set to change periodically between 0 and 0.2m. Figure 9 below illustrates the desired position, the actual position, the position deviation of the vehicle, and the angle of deviation from the vertical of the rotating rod. It was found from this scenario the proposed hybrid control strategy achieves good control quality, leading to the fast balancing of the rod despite the periodical change of the cart's position. Not only the real position tracks well with this periodical reference, but the rotational angle deviation is completely damped in enough perfectly short settling times.

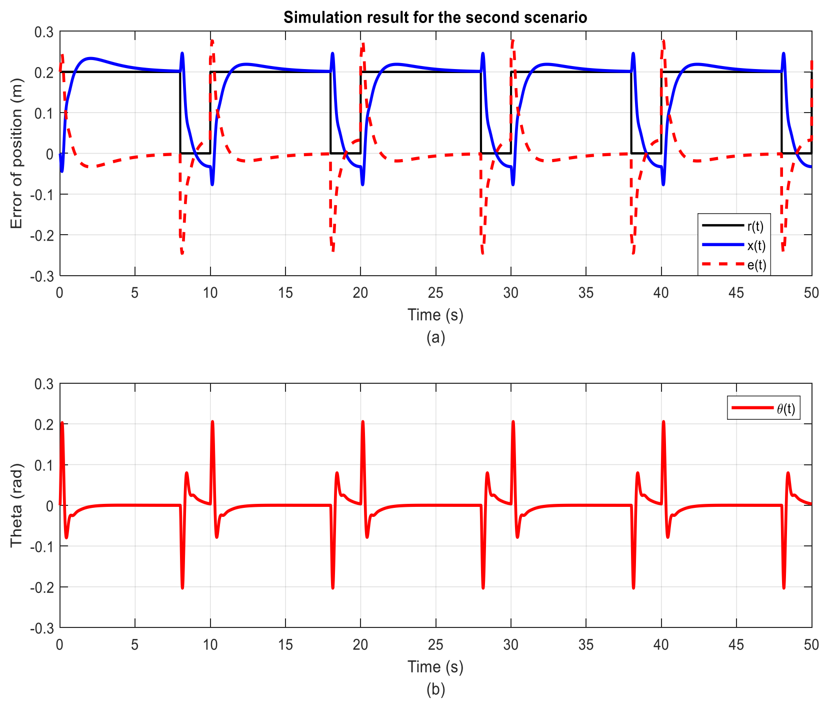

Fig. 9 Simulation results for the second case of major parameters

4.2. Experiments on the practical inverted pendulum model

The practical model built in this study is presented in Fig. 1. To experiment, using MATLAB/Simulink software in combination with the hardware of the inverted pendulum system, the following steps need to be executed:

***Step* 1**: Turn on the PSO program in MATLAB/Simulink.

***Step* 2**: Run the program in MATLAB.

***Step* 3**: Wait until Arduino's 13th light flashes twice (PID's parameters were uploaded successfully from the PSO program to Arduino).

***Step* 4**: Raise the pendulum to the 90-degree position until the 13*th* light goes out, then release the hand to let the system run independently until the 13*th* light comes back on. This is to complete one cycle of the PSO program.

Step* 5**: The process repeats from ***Step 3 to ***Step 4*** until the PSO program reports a set of PID parameters.

Figure 10 illustrates the real control signal measured by an oscilloscope corresponding to the 90-degree vertical position of the practical pendulum model shown in Fig. 1. All signals obtained seem to be stable when the rod of the pendulum is maintained at the vertical position. Experiment results on the inverted pendulum system built in this work verified the practical feasibility of the control strategy proposed. It should be completely obvious the applicability of the hybrid control solution has been proven through both theoretical and practical aspects.

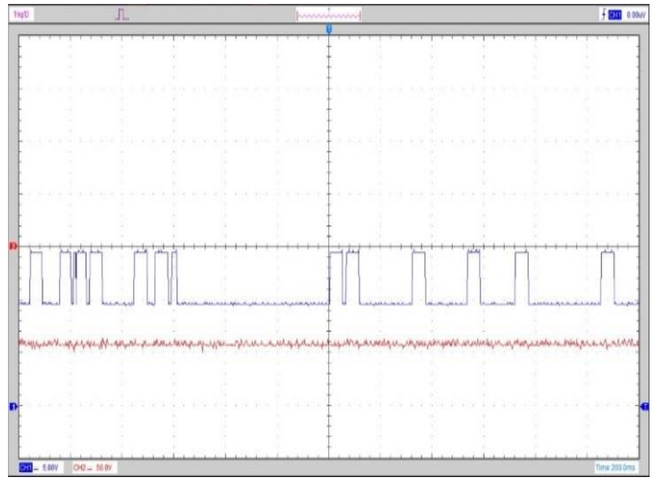

Fig. 10 Control signal measurement in the practical inverted pendulum model

5. Conclusion and Future Work

This paper contributed a comprehensive work from theoretical research, proposing a hybrid control strategy to designing a practical model of an inverted pendulum system. Simulation results and practical experiment results have been obtained, demonstrating the effectiveness and applicability of the inverted pendulum model integrated with a proper hybrid control strategy in the balancing problem. The future work raised from this study will be building intelligent control schemes such as fuzzy logic and neural network to obtain much better control performances. In addition, the practical model built in this work might need a little refinement to be able to testify various balancing control methodologies with higher correctness. In this promising scenario, the real, inverted pendulum model can be usefully applied in many possibly useful control strategies.

References

[1] E. Susanto, A. Surya Wibowo, and E. Ghiffary Rachman, Fuzzy swing-up control and optimal state feedback stabilization for the self-erecting inverted pendulum, IEEE Access, 8 (2020) 6496–6504. https://doi.org/10.1109/ACCESS.2019.2963399.

[2] N. Tahir, M. Muhammad, M. Idi, S. Buyamin, L. Maijama'a, and S. Yarima, Comparative analysis of observer-based lqr and lmi controllers of an inverted pendulum, Bulletin of Electrical Engineering and Informatics, 9(6) (2020) 2244–2252. https://doi.org/10.11591/eei.v9i6.2271.

[3] I. Siradjuddin, Z. Amalia, B. Setiawan, F. Ronilaya, E. Rohadi, A. Setiawan, C. Rahmad, and S. Adhisuwignjo, Stabilising a cart inverted pendulum with an augmented PID control scheme, MATEC Web of Conferences, 197 (2018) 11013. https://doi.org/10.1051/matecconf%2F201819711013.

[4] Roslan, A. A., Nasir, A. N. K., Jusof, M. F. M., Razak, A. A. A., & Ahmad, M. A. Sine-Based Barnacle Mating Optimization for PD Control of an Inverted Pendulum System. In Proceedings of the 6th International Conference on Electrical, Control and Computer Engineering, (2022) 429-439 2022. Springer, Singapore.

[5] Widjiantoro, B. L. & Wafi, M. K. Discrete-Time State-Feedback Controller with Canonical Form on Inverted Pendulum (on a Cart). International Journal of Science and Engineering Investigations (IJSEI), 11(120) (2022)16-21. http://www.ijsei.com/papers/ijsei-1112022-03.pdf

[6] S. Krafes, Z. Chalh, and A. Saka, Review: Linear, nonlinear and intelligent controllers for the inverted pendulum problem, 2016 International Conference on Electrical and Information Technologies (ICEIT), (2016) 136-141. doi: 10.1109/EITech.2016.7519577.

[7] Ngoc-Khoat Nguyen, Duy-Trung Nguyen. A Comparative Study on PI – and PD – Type Fuzzy Logic Control Strategies. International Journal of Engineering Trends and Technology, 69(7) (2021) 101-108. https://doi.org/10.14445/22315381/IJETT-V69I7P215

[8] D-. Trung N., N-. Khoat N. et al. Designing PSO-based PI-type fuzzy logic controllers: a typical application to load-frequency control strategy of an interconnected hydropower system. Proceedings of 2019 3rd International Conference on Automation, Control and Robots (ICACR 2019), Prague, Czech Republic, (2019) 61 – 66.

[9] Vinothkumar C, Esakkiappan C. Optimization of PI Controller on Level Control of Hopper Tank System with PSO Technique, International Journal of Engineering Trends and Technology, 69(10) (2021) 178-185. https://doi.org/10.14445/22315381/IJETT-V69I10P222

[10] S. D. Hanwate and Y. V. Hote, Design of PID controller for inverted pendulum using stability boundary locus, 2014 Annual IEEE India Conference (INDICON), (2014) 1-6, doi: 10.1109/INDICON.2014.7030563.

[11] Girish Bhide, Brijesh Iyer, Anil Nandgaonkar, Sanjay Nalbalwar, Abhay Wagh. Design and Analysis of Compact Giraffe-Shaped Patch Antenna for UWB Applications: A FDTD and Hybrid PSO Algorithm Approach. International Journal of Engineering Trends and Technology, 70(3) (2022) 13-21, 2022. https://doi.org/10.14445/22315381/IJETT-V70I3P202

[12] S. Nayak and A. Kumar, An Intelligent CSO – DBNN based Cyber Intrusion Detection Model for Smart Grid Power System, International Journal of Engineering Trends and Technology, 68(6) (2020) 50 – 57.

[13] Mr. Ashfaq Ur Rehman Mohamed Riazuddin, Dr.Mahabuba Abdulrahim, Dr.Jennathu Beevi Sahul Hameed, Dr. Jayashree Ramasubramaniam, Optimal Design of Controller for Automatic Generation Control in Multi Area Restructured Power System with Battery Energy Storage and Wind Energy Systems. International Journal of Engineering Trends and Technology, 69(12) (2021) 188-196, 2021.

[14] S. Handoyo and Y. P. Chen, The Developing of Fuzzy System for Multiple Time Series Forecasting with Generated Rule Bases and Optimized Consequence Part, International Journal of Engineering Trends and Technology, 68(12) (2020) 118 – 122.

[15] T-M-P. Dao, Y. Wang, N-K. Nguyen. Applying meta–heuristic optimization methods to design novel adaptive PI–type fuzzy logic controllers for load–frequency control in a large–scale power grid, Turkish Journal of Electrical.

[16] Engineering & Computer Sciences, 24(6) (2016) 4900-4914, 2016.

[17] Wei Zhang, Jiao Liu, Lv-bin Fan, Ya-hong Liu, Di Ma, Control strategy PSO, Applied Soft Computing, 38 (2016) 75-86. https://doi.org/10.1016/j.asoc.2015.09.030.

[18] Taizhou Wu, Cuicui Zhou, Zhe Yan, Huang Peng, Linzhang Wu, Application of PID optimization control strategy based on particle swarm optimization (PSO) for the battery charging system, International Journal of Low-Carbon Technologies, 15(4) (2020) 528–535. https://doi.org/10.1093/ijlct/ctaa020.

[19] Mohamed Abdul Raouf Shafei, Doaa Khalil Ibrahim, Mostafa Bahaa,
Application of PSO tuned fuzzy logic controller for LFC of two-area power system with redox flow battery and PV solar park, Ain Shams Engineering Journal13(5) (2022).
https://doi.org/10.1016/j.asej.2022.101710.

[20] Hossein Oghab Neshin, Manouchehr Jandaghi Semnani. Two-area power system stability analysis by frequency controller with UPFC synchronization and energy storage systems, International Journal of Engineering Trends and Technology, 68(1) (2021) 14-29. https://doi.org/10.14445/22315381/IJETT-V68I1P203