On Nano Generalized A-Closed Sets & Nano A-Generalized Closed Sets in Nano Topological Spaces

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***Abstract*— The Objective of this paper is to introduce and investigate the Nano generalized α-closed sets and Nano α-generalized closed sets in a Nano Topological Space. Also inevestigate the Nano generalized α-Closure and Nano generalized α-Interior.**

Keywords: Nano closed set, Nano α – closed set, Nano generalized closed set, Nano generalized α-closed set, Nano α-generalized closed set, Nano generalized α-Closure and Nano generalized α-Interior.

1. INTRODUCTION

In 1970, Levine [2] introduced the concept of generalized closed sets as a generalization of closed sets in Topological Spaces. This concept was found to be useful and many results in general topology were improved. One of the generalizations of closed set is generalized α-closed sets which was defined by [5] R.Devi et.al, investigated some applications and related topological properties regarding generalized α-closed sets. Lellis Thivagar [4] introduced the weak form of Nano open sets namely Nano α-open sets, Nano semi open sets and Nano pre open sets. Based on this notation in this certain generalized form of weak form of Nano open sets is introduced and based on the new set, the intudine between the new set with the existing set is discussed.

2. PRELIMINARIES

Definition: 2.1 A subset A of a space (X, τ) is called

(i) Semi open[2] if $A \subseteq Cl(Int(A))$

(ii) Pre-open[4] if $A \subseteq Int(Cl(A))$

(iii) α – open[7] if $A \subseteq Int(Cl(Int(A)))$

Definition: 2.2[4] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with in another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$

(i) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = U\{R(x): R(x) \subseteq X\}$, Where R(x) denotes the equivalence class determined by x.

(ii) The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = U\{R(x): R(x) \cap X \neq \varphi\}$.

(iii) The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.

Property: 2.3[4] If (U, R) is an approximation space and X, Y $\subseteq$ U, then

(i) $L_R(X) \subseteq X \subseteq U_R(X)$.

(ii) $L_R(\varphi) = U_R(\varphi) = \varphi$ & $L_R(U) = U_R(U) = U$.

(iii) $U_R(XUY) = U_R(X)\ U\ U_R(Y)$.

(iv) $L_R(XUY) \supseteq L_R(X)\ U\ L_R(Y)$.

(v) $U_R(X\cap Y) \subseteq U_R(X) \cap U_R(Y)$.

(vi) $L_R(X\cap Y) = L_R(X) \cap L_R(Y)$.

(vii) $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$, whenever $X \subseteq Y$.

(viii) $U_R(X^C) = [L_R(X)]^C$ and $L_R(X^C) = [U_R(X)]^C$.

(ix) $U_R\ U_R\ (X) = L_R\ U_R(X) = U_R(X)$.

(x) $L_R\ L_R(X) = U_R\ L_R(X) = L_R(X)$.

Definition: 2.4[4] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then by property 2.3, $\tau_R(X)$ satisfies the following axioms:

(i) U and $\phi \in \tau_R(X)$

(ii) The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

(iii) The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ is a topology on U called the Nano topology on U with respect to X. We call (U, $\tau_R(X)$) as the Nano topological space. The elements of $\tau_R(X)$ are called as Nano open sets.

Remark: 2.5[4] If $\tau_R(X)$ is the Nano topology on U with respect to X, the the set B = {U, $L_R(X)$, $U_R(X)$} is the basis for $\tau_R(X)$.

Definition: 2.6[4] If (U, $\tau_R(X)$) is a Nano topological space with respect to X where $X \subseteq U$ and if $A \subseteq U$ then the Nano interior of A is defined as the union of all Nano open subsets of A and it is denoted by NInt(A). That is, NInt(A) is the largest Nano open subset of A. The Nano closure of A is defined as the intersection of all Nano closed sets containing A and it is denoted by NCl(A). That is, NCl(A) is the smallest Nano closed set containing A.

Definition: 2.7[4] A Nano topological space (U, $\tau_R(X)$) is said to be extremely disconnected, if the Nano closure of each Nano open set is Nano open.

Definition: 2.8[4] Let (U, $\tau_R(X)$) be a Nano topological space and $A \subseteq U$. Then A is said to be

(i) Nano Semi open if $A \subseteq NCl(NInt(A))$

(ii) Nano Pre-open if $A \subseteq NInt(NCl(A))$

(iii) Nano α–open if $A \subseteq NInt(NCl(NInt(A)))$

NSO(U, X), NPO(U, X) and $\tau_R^{\alpha}(X)$ respectively, denote the families of all Nano semi open, Nano pre open and Nano α – open subsets of U.

Definition: 2.9 A subset A of a topological space (X, τ) is called a generalized closed set [2] if $Cl(A) \subseteq V$ and V is open in (X, τ).

3. NANO GENERALIZED α-CLOSED SET & NANO α-GENERALIZED CLOSED SET

In this section, we define and study the forms of Nano generalized α-closed sets and Nano α-generalized closed sets.

Definition: 3.1 A subset A of (U, $\tau_R(X)$) is called an Nano generalized α-closed set (briefly Ngα-closed) if $N\alpha Cl(A) \subseteq V$ whenever $A \subseteq V$ and V is Nano α-open in (U, $\tau_R(X)$). The complements of Nano generalized α-closed set is Nano generalized α-open set in (U, $\tau_R(X)$).

Definition: 3.2 A subset A of (U, $\tau_R(X)$) is called a Nano α-generalized closed set if $N\alpha Cl(A) \subseteq V$ whenever $A \subseteq V$ and V is Nano open in (U, $\tau_R(X)$). The complements of Nano α-generalized closed set is Nano α-generalized open set in (U, $\tau_R(X)$).

Theorem: 3.3 If A is Nano closed set in (U, $\tau_R(X)$), then it is Nano generalized α-closed set but converse is not true.

Proof: Let $A \subseteq V$ and V be a nano open in $\tau_R(X)$. Since, A is Nano closed and $N\alpha Cl(A) \subseteq NCl(A)$. Also, NCl(A)=A. Thus $N\alpha Cl(A) \subseteq NCl(A) = A \subseteq V$. Since every Nano open set is Nano α-open set. Hence, $N\alpha Cl(A) \subseteq V$. Therefore, A is a Nano generalized α-closed set. The converse of the above theorem is need not be true as seen from the following example.

Example: 3.4 Let U = {a, b, c, d} with U/R = {{a}, {c}, {b, d}} and X = {a, b}. Then the Nano topology, $\tau_R(X)$ = {U, ϕ, {a}, {a, b, d}, {b, d}}. Then in a space (U, $\tau_R(X)$), a subset {a, c, d} is a Nano generalized α-closed set but it is not a Nano α-closed set.

Theorem: 3.5 If A is Nano closed set in (U, $\tau_R(X)$), then it is Nano α-generalized closed set but converse is not true.

Proof: Let $A \subseteq V$ and V be a nano open in $\tau_R(X)$. Since, $N\alpha Cl(A) \subseteq NCl(A)$ and A is Nano closed. Since, NCl(A)=A. Thus $N\alpha Cl(A) \subseteq NCl(A) = A \subseteq V$. Hence, $\alpha Cl(A) \subseteq V$. Therefore, A is a Nano α-generalized closed set. The converse of the above theorem is need not be true as seen from the following example.

Example: 3.6 Let U = {a, b, c, d} with U/R = {{a}, {c}, {b, d}} and X = {a, b}. Then the Nano topology, $\tau_R(X)$ = {U, ϕ, {a}, {a, b, d}, {b, d}}. Then in a space (U, $\tau_R(X)$), a subset {a, b, c} is a Nano generalized closed set but it is not a Nano closed set.

Theorem: 3.7 If A is Nano generalized closed set in (U, $\tau_R(X)$), then it is Nano generalized α-closed set.

Proof: Let A be Nano generalized closed set, then $NCl(A) \subseteq V$ whenever $A \subseteq V$ and V be a nano open in $\tau_R(X)$. Also since, $N\alpha Cl(A) \subseteq NCl(A)$ and every Nano open set is Nano α open set . Then $N\alpha Cl(A) \subseteq NCl(A) = NCl(A) \subseteq V$. Hence, $N\alpha Cl(A) \subseteq V$. Therefore, A is a Nano generalized α-closed set.

Theorem: 3.8 If A is Nano generalized closed set in (U, $\tau_R(X)$), then it is Nano α-generalized closed set.

Proof: Let $A \subseteq V$ and V be a nano open in $\tau_R(X)$. Since, $NCl(A) \subseteq V$. Also since, $N\alpha Cl(A) \subseteq NCl(A)$ and A is Nano closed set. Thus $N\alpha Cl(A) \subseteq NCl(A) = NCl(A) \subseteq V$. Hence, $N\alpha Cl(A) \subseteq V$. Therefore, A is a Nano α-generalized closed set.

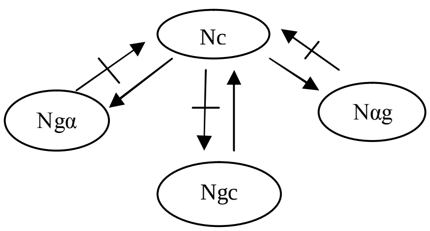
Theorem: 3.9 If A is Nano α-generalized closed set in (U, $\tau_R(X)$), then it is Nano generalized α-closed set.

Proof: Let A⊆V and V be an Nano open in $\tau_R(X)$, then NαCl(A)⊆V. Since, every Nano open set is Nano α-open set. Therefore, V is Nano α-open in $\tau_R(X)$ and A⊆V. Hence NαCl(A)⊆V. Therefore A is a Nano α-generalized closed set.

Theorem: 3.10 If A is Nano generalized α-closed set in (U, $\tau_R(X)$), then it is Nano generalized semi closed set.

Proof: Let A be a nano generalized α closed set and A⊆V; V is nano open in U. We have Nαcl(A)⊆V. Also for A⊆V, V is nano open, we have Nscl(A) ⊆ Nαcl(A). Then Nscl(A)⊆V as Nαcl(A)⊆V. Hence Nscl(A) ⊆V whenever A⊆V, V is nano open. So A is nano generalized-semi closed set.

Remark: 3.11 From the above theorems and examples shows that the following diagram of implications.



4. CHARACTERIZATIONS OF NANO GENERALIZED A-CLOSED SET & NANO A-GENERALIZED CLOSED SET

Theorem: 4.1 The Union of Two Nano generalized α-closed sets in (U, $\tau_R(X)$) are also Nano generalized α-closed set in (U, $\tau_R(X)$).

Proof: Assume that A and B are two Nano generalized α-closed sets in (U, $\tau_R(X)$). Let V be a Nano open in (U, $\tau_R(X)$) such that (AUB)⊂V. Then A⊂V and B⊂V. Since, A and B are Nano generalized α-closed sets in $\tau_R(X)$. NαCl(A)⊂V and NαCl(B)⊂V. Hence, NαCl(AUB)=NαCl(A)UNαCl(B)⊂V. That is NαCl(AUB)⊂V. Hence (AUB) is an Nano generalized α-closed set.

Remark: 4.2 The intersection of two Nano generalized α-closed sets in (U, $\tau_R(X)$) are also Nano generalized α-closed set in (U, $\tau_R(X)$) as seen from the following example.

Example: 4.3 Let U = {a, b, c, d} with U/R = {{a}, {c}, {b, d}} and X = {a, b}. Then the Nano topology, $\tau_R(X)$ = {U, ϕ, {a}, {a, b, d}, {b, d}}. Then the Nano generalized α-closed sets are {c}, {a, c}, {b, c}, {c, d}, {a, b, c}, {a, c, d} & {b, c, d}. Here {b, c, d}∩{a, b, c}={b, c} is also Nano generalized α-closed set.

Theorem: 4.4 The Union of two Nano α-generalized closed sets in (U, $\tau_R(X)$) are also Nano α-generalized closed set in (U, $\tau_R(X)$).

Proof: Assume that A and B are two Nano α-generalized closed sets in (U, $\tau_R(X)$). Let V be a Nano open in (U, $\tau_R(X)$) such that (AUB)⊂V. Then A⊂V and B⊂V. Since, A and B are Nano α-generalized closed sets in $\tau_R(X)$. NαCl(A)⊂V and NαCl(B)⊂V. Hence, NαCl(AUB)=NαCl(A)UNαCl(B)⊂V. That is NαCl(AUB)⊂V. Hence (AUB) is an Nano α-generalized closed set.

Remark: 4.5 The intersection of two Nano α-generalized closed sets in (U, $\tau_R(X)$) are not Nano α-generalized closed set in (U, $\tau_R(X)$) as seen from the following example.

Example: 4.6 Let U = {a, b, c, d} with U/R = {{b}, {d}, {a, c}} and X = {a, b}. Then the Nano topology, $\tau_R(X)$ = {U, ϕ, {b}, {a, b, c}, {a, c}}. Then the Nano α-generalized closed sets are {d}, {a, d}, {b, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d} & {b, c, d}. Here {b, d}∩{a, b, c}={b} is not Nano α-generalized closed set.

Theorem: 4.7 If a set A is Nano generalized α-closed set then αCl(A)-A contains no non-empty closed set.

Proof: Suppose that A is Nano generalized α-closed set. Let S be a Nano α-closed subset of NαCl(A)-A. Then, A⊆S^C, S^C is open and hence Nano α-open. Since, A is Nano generalized α-closed set, NαCl(A)⊆S^C. Consequently, S⊆$(\text{N}\alpha\text{Cl}(A))^C$. Since, Every Nano closed set is Nano α-closed set. Hence, S is Nano α-closed set. Therefore S⊆NαCl(A). S⊆NαCl(A)∩$(\text{N}\alpha\text{Cl}(A))^C$=ϕ. Hence S is empty.

Theorem: 4.8 Let A be a Nano generalized α-closed subset of X. If A⊆B⊆NαCl(A) then B is also a Nano generalized α-closed subset of $\tau_R(X)$.

Proof: Let V be a Nano α-open set of $\tau_R(X)$ such that B⊆V. Then A⊆V. Since, A is a Nano generalized α-closed set NαCl(A)⊆V. Also, B⊆NαCl(A). Then NαCl(B)⊆NαCl(A)⊆V. Hence B is also a Nano generalized α-closed subset of $\tau_R(X)$.

5. NANO GENERALIZED α-INTERIOR AND NANO GENERALIZED α-CLOSURE

Definition: 5.1 Let U be a Nano Topological space and let x∈U. A subset N of U is said to be Ngα-neighbourhood of x if there exists a Ngα-open set G such that x∈G⊂N.

Definition: 5.2 Ngα-Int(A) = U{B : B is Nano generalized α-open set and B ⊂ A}

Definition: 5.3 Ngα-Cl(A) = ∩{B : B is Nano generalized α-closed set and A ⊂ B}

Theorem: 5.4 If A be a subset of U. Then Ngα-Int(A) = U{B : B is Nano generalized α-open set and B ⊂ A}.
Proof: Let A be a subset of U. x∈ Ngα-Int(A) ⇔ x is a Ngα-Interior point of A.
⇔ A is a Ngα-neighbourhood of point x.
⇔ there exists Ngα-open set B such that x∈B ⊂ A.
⇔ x∈U{B : B is Ngα-open set and B ⊂ A}.
Hence, Ngα-Int(A) = U{B : B is Nano generalized α-open set and B ⊂ A}.

Theorem: 5.5 Let A and B be subsets of U. Then (i) Ngα-Int(U) = U and Ngα-Int(ϕ) = ϕ.
(ii) Ngα-Int(A) ⊂ A.
(iii) If B is any Ngα-open set contained in A, then B ⊂ Ngα-Int(A).
(iv) If A⊂B, then Ngα-Int(A) ⊂Ngα-Int(B).
(v) Ngα-Int(Ngα-Int(A)) = Ngα-Int(A).
Proof: (i) Since, U and ϕ are Ngα-open sets, by theorem 5.4 Ngα-Int(U) = U{B : B is Ngα-open and G ⊂U} = U U {A : A is a Ngα-open set} = U. That is, Ngα-Int(U) = U. Since, ϕ is the only Ngα-open set contained in ϕ, Ngα-Int(ϕ) = ϕ.
(ii) Let x∈Ngα-Int(A) ⇒ x is a Ngα-interior point of A.
⇒ A is a Ngα neighbourhood of x.
⇒ x∈A.
Thus, x∈Ngα-Int(A) ⊂ A.
(iii) Let B be any Ngα-open sets such that B⊂A. Let x∈B, then since, B is a Ngα-open set contained in A, x is a Ngα interior point of A. That is B is a Ngα-Int(A).Hence, B⊂Ngα-Int(A).
(iv) Let A and B be subsets of U such that A⊂B. Let x∈Ngα-Int(A). Then x is a Ngα interior point of A and so A is Ngα neighbourhood of x. This implies that x∈Ngα-Int(B). Thus we have shown that x∈Ngα-Int(B). Hence, Ngα-Int(A) ⊂ Ngα-Int(B).
(v) Let A be any subset of U. By definition of Ngα interior, Ngα-Int(A) = ∩{A⊂F∈gαC(U)}, if A⊂F∈NgαC(U), then Ngα-Int(A) ⊂F. Since F is a Ngα closed set containing Ngα-Int(A). By(iii), Ngα-Int(Ngα-Int(A)) ⊂F. Hence, Ngα-Int(Ngα-Int(A)) ⊂∩{A⊂F∈NgαC(U)} = NgαCl(A).
That is, Ngα-Int(Ngα-Int(A)) = Ngα-Int(A).

Theorem: 5.6 If a subset A of a space U is Ngα-open then Ngα-Int(A) = A.
Proof: Let A be Ngα-open subset of U. We know that Ngα-Int(A) ⊂A. Also A is Ngα-open set contained in A. From theorem 5.5(iii), A⊂ Ngα-Int(A). Hence, Ngα-Int(A) = A.

Theorem: 5.7 If A and B are subsets of U, then Ngα-Int(A) U Ngα-Int(B) ⊂ Ngα-Int(AUB).
Proof: We know that A⊂ AUB and B⊂ AUB. We have by theorem 5.5(iv), Ngα-Int(A) ⊂ Ngα-Int(AUB) and Ngα-Int(B) ⊂ Ngα-Int(AUB). This implies that Ngα-Int(A) U Ngα-Int(B) ⊂ Ngα-Int(AUB).

Theorem: 5.8 If A and B are subsets of space U, then Ngα-Int(A∩B) = Ngα-Int(A) ∩ Ngα-Int(B).
Proof: We know that A∩B⊂ A and A∩B⊂ B. We have, by theorem5.5 (iv), Ngα-Int(A∩B) ⊂ Ngα-IntA and Ngα-Int(A∩B) ⊂ Ngα-IntB. This implies that Ngα-Int(A∩B) ⊂ Ngα-Int(A) ∩ Ngα-Int(B).-----(1) Again, let x∈ Ngα-Int(A) ∩ Ngα-Int(B). Then x∈ Ngα-Int(A) and x∈Ngα-Int(B). Hence, x is a Ngα interior point of each sets A and B. It follows that A and B is Ngα neighbourhood of x, so that their intersection A∩B is also Ngα neighbourhood of x. Hence, x∈ Ngα-Int(A∩B). Therefore, Ngα-Int(A) ∩ Ngα-Int(B) ⊂ Ngα-Int(A∩B)-------(2). From (1) & (2), we get Ngα-Int(A∩B) = Ngα-Int(A) ∩ Ngα-Int(B).

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