Design of Sampling Scheme Based on Kullback-Liebler Information for Compliance Testing

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Abstract: In this paper a possibility of designing the sampling scheme is discussed from the information theory. A design procedure for the Tightened-Normal-Tightened sampling scheme is established based on the Kullback – Liebler information and minimum sum of risk. A table giving the values of the parameter of TNT scheme of type TNTSS (n_T, n_N;c) indexed through the Average Quality Level and Limiting Quality Level is presented. The proposed procedure is simple and applicable to practical usage.

Key words: Tightened - Normal -Tightened sampling scheme, Kullback – Liebler information ,Compliance testing, AQL and LQL.

I. INTRODUCTION

Scientific sampling plans are the primary tools for quality and performance management in industry today. In an Industrial plant, sampling plans are used to decide either to accept or reject a received batch of items. With attribute sampling plans, these accept/reject decisions are based on a count of the number of nonconforming items and in traditional sampling plans, the sample size is assumed constant. Calvin (1977) has proposed a new type of sampling plan namely zero acceptance number sampling. This scheme utilizes two samples with different sample size and fixing c for both samples, together with switching rules, to construct the shoulders of the operating characteristic (OC) curve. If c can take values other than zero, the TNT scheme by attributes can be designated as TNTASS (n_T, n_N; c), which the normal and tightened single sampling plans have the same acceptance number c, but the sample size, n_T on tightened inspection is larger than the sample size, n_N on normal inspection. One-way of defining the sampling scheme is to say that the normal and tightened plans utilize the sample size n_T and $n_N$but with different acceptance number c. Ikuo Arizono, Hiroshi Ohta, Shigeo Kase (1985) have given the procedure for single sampling plans by attributes based on the Kullback – Leibler information. Raju and Jothikumar (1994) have presented a design procedure for chain sampling plan ChSP-1 based on Kullback-Leibler information and a minimum sum of risk. Soundararajan and Vijayaraghavan (1992) have constructed procedure for Tightened - Normal – Tightened sampling scheme of type TNT-(n_1, n_2; c) and they have studied the efficiency of TNT-(n_1, n_2; c) scheme over conventional single and double sampling plans. Muthuraj and Senthilkumar(2006) have designed and constructed tightened- normal-tightened variables sampling scheme as designated as TNTVSS(n_σ,k_T,k_N) based on AQL and LQL. Senthilkumar and Hareesh (2007) have presented the procedure for the selection of Tightened- Normal-Tightened sampling scheme based on Kullback-Leibler Information of type TNT (n; c_T,c_N). Senthilkumar and Muthuraj (2008) have constructedtables for Tightened-Normal-Tightened variables sampling scheme of type TNTVSS (n_T,n_N; k).

This paper proposed a designing procedure for Tightened-Normal-Tightened attribute sampling scheme (TNT-(n_T, n_N; c)) for given values of AQL and LQL, established based on Kullback - Liebler information. The proposed procedure is easy to implement in the situation for compliance testing. The conditions for application and operating procedure of TNT (n_T, n_N; c) are given below.

A. Conditions for Application

- Production is steady so that the result on current, preceding and succeeding lots are indicative of a continuing process.
- Submitted lots are expected to be of essentially the same quality.
- Successive lots are submitted for inspection in the order of their production.

B. Operating procedure

- Inspect under tightened inspection using the single sampling plan with sample size n_T and acceptance number c.

- If t lots in a row are accepted, switch to normal inspection (step3).
- Inspect under normal inspection using the single sampling plan with sample size n_N ($<n_T$) and acceptance number c.
- If an additional lot is rejected in the next s lots after a rejection, switch to tightened inspection

Here n_T andn_N are the sample sizes, under tightened and normal inspection respectively, c is the acceptance number, t is the criterion for switching from tightened to normal inspection and s is the criterion for switching from normal to tightened inspection.

II. Operating Characteristic Function

The OC function of the TNT is given by Calvin (1977) is

$$P_a(p) = \frac{P_T(1-P_N^s)(1-P_T^t)(1-P_N)+P_N P_T^t(1-P_T)(2-P_N^s)}{(1-P_N^s)(1-P_T^t)(1-P_N)+P_T^t(1-P_T)(2-P_N^s)} \quad (1)$$

where $P_T$is the probability of acceptance under tightened inspection and P_N is the probability of acceptance under normal inspection.

In the special case, the OC function equation (1) of the TNT Scheme becomes the scheme OC function of MIL-STD-105D, involving only tightened and normal inspection were derived by Dodge (1965), Hald and Thyregod (1965) and Stephens and Larson (1967)

A. The Design concept of sampling plans based on the Kullback – Leibler Information approach

Let p_1(AQL) be the specified acceptable quality level and p_2 (LQL) the unsatisfactory quality level. The Kullback-Leibler information for discrimination in favour of the probability distribution P with the fraction defective p against the probability distribution P_1 with the fraction defective p_1 is

$$I(P : P_1) = p\log\frac{p}{p_1}+(1-p)\log\frac{(1-p)}{(1-p_1)}$$

and against the probability distribution P_2 with the fraction defective p_2 is

$$I(P : P_2) = p\ \log\frac{p}{p_2}+(1-p)\log\frac{(1-p)}{(1-p_2)}$$

The discrimination based on the Kullback-Leibler information results in the following rule for acceptance or rejection of the lot:

If $I(P : P_1) \le I(P : P_2)$, then accept the lot, otherwise, reject the lot.

This rule is reduced to the following rule: if $p \le p^*$, then accept the lot, otherwise, reject the lot, where

$$P^* = \left\{\log\frac{1-p_1}{1-p_2}\right\} \Big/ \left\{\log\frac{p_2(1-p_1)}{p_1(1-p_2)}\right\}$$

and p^* is easily obtained from the relationship

$$I(P: P_1) = I(P : P_2)$$

When two quality levels p_0 , p_1 and producer's risk α and consumer's risk β are given, let $F(p_0, p_1, \alpha, \beta)$ denote the minimum information required for deciding whether one has to accept the inspected lot or not. Allowing n and c to be non-integral n^* and c^*, the required minimum sample size n^* is obtained from

$$n^* = \frac{F(p_1, p_2, \alpha, \beta)}{I^*} \quad (2)$$

where $I^* = I(P^*: P_1) = I(P^* : P_2)$ is the mean information per observation for deciding the acceptance or rejection of the lot,and

$$F(p_1,p_2,\alpha,\beta)=\frac{1}{2}\left\{\frac{(p_2-p^*)}{(p_2-p_1)}F_1(\alpha,\beta)+\frac{p^*-p_1}{p_2-p_1}F_2(\alpha,\beta)\right\}$$

Is the minimum information required for deciding the acceptance or rejection of the lot. Here, we have

$$F_1(\alpha,\beta)=\alpha\log\left[\frac{\alpha}{1-\beta}\right]+(1-\alpha)\log\left[\frac{(1-\alpha)}{\beta}\right]$$

$$F_2(\alpha,\beta)=\beta\log\left[\frac{\beta}{1-\alpha}\right]+(1-\beta)\log\left[\frac{(1-\beta)}{\alpha}\right]$$

The fact that n^* is the minimum sample size which gives the required information and enables us to determine the sample size n as follows:

- if n^* is an integer, then $n = n^*$;
- otherwise, $n = [n^*] +1$, where $[n^*]$ is the integer part of n^*.

III. Design Procedure

The design procedure is presented in this paper combines Kullback-Leibler information (Arizono et al., 1985) and Golub's (1953) procedure .At first we determined the required sample size from the viewpoint of information so its procedure is different from conventional one. Then we are using Golub's approach to determine the acceptance number c_T and c_N

A. Steps.

- For the desire (p_1, 1-α) and (p_2, β) the sample size n is obtained using Kullback-Leibler information
- For the sample size so obtained, the parameters c_T and c_N are determine such that the resulting plan minimizes the some of the risks, ($\alpha+\beta$).

B. Advantages of this Procedure

- Provide a strong basis to fix a sample size, which is the minimum sample size providing the required information to accept or reject the lot for given two quality levels (AQL, 1- α) (LQL, β)
- Plans determined by this method have the minimum sum of risks ($\alpha+\beta$) i.e. the plan has good discrimination between good and bad lot

C. Procedure for Selection

Using the procedure outlined above the values of n_T, n_N and c are obtained from chosen pairs of AQL and LQL values for the case when α = 0.05 and β = 0.10. The results are presented in Table 1.

Example 1

For AQL = 0.5% (0.005) and LQL = 2% (0.02), let us obtain a TNT from Table 1, corresponding to AQL = 0.5% (0.005) and LQL = 2% (0.02), the parameters are n_T =836, (n_T = 2 x n_N), n_N = 418, c = 11 for combination of s = 4, t = 5.

IV. OC curve of TNT-(n_T, n_N; c) Schemes

Figure 1 show the OC curves of tightened and normal inspection plans, and the composite OC curve of single-sampling (normal),TNT and single-sampling (tightened) schemes : (1) SSP (normal), n_N = 615, c = 6 ; (2) TNT, n_T =1230, n_N = 615, c = 6, s = 4, t = 5; (3) SSP (tightened), n_T= 1230, c = 6.

From Figure.1, it can be observed that, for good quality, i.e. for larger values of fraction defective, the OC curve of the TNT – (n_T, n_N; c) schemes coincides with the OC curve of the tightened inspection plan. As quality deteriorates, the scheme OC curve moves towards that for tightened inspection and comes close to it beyond the indifference quality level p_2.Using Table 1, one can obtain a TNT plan for any desired combination of AQL and LQL.

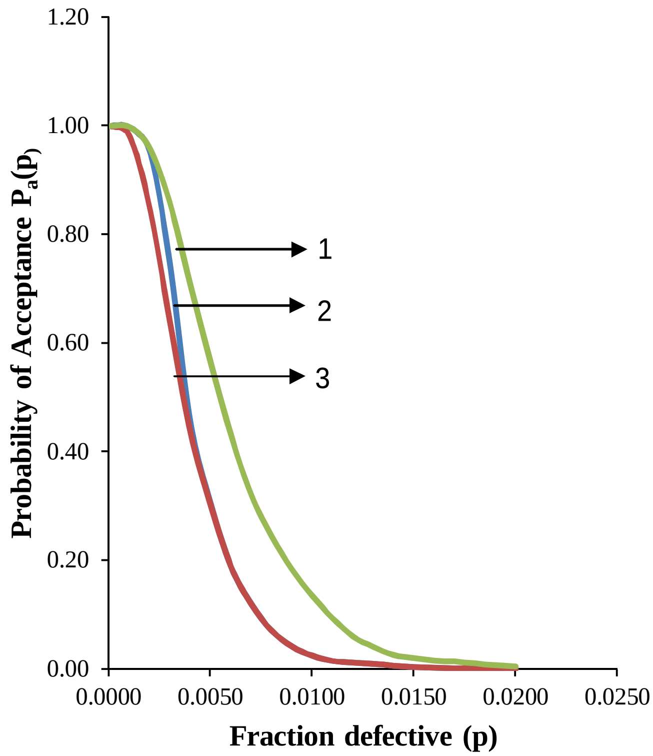

Figure. 1. OC curve of single-sampling (normal),TNT and single-sampling (tightened) schemes : (1) SSP (normal), n_N = 615, c = 6 ; (2) TNT, n_T =1230, n_N = 615, c = 6, s = 4, t = 5; (3) SSP (tightened), n_T= 1230, c = 6.

V. Procedure for Construction of Table

The values of the sample size n_T and n_N are obtained for chosen paired of values of p_1 and p_2 using Kullback-Leibler information. The other parameter c is obtained by minimizing ($\alpha+\beta$). The sum of risks ($\alpha+\beta$) is minimized for given values of n_T and n_N and c. Minimizing ($\alpha+\beta$) is the same as maximizing (1-α + 1- β). That is maximizing $P_a(p_1)$ +1- $P_a(p_2)$. where $P_a(p)$ is given by

$$P_a(p_1)=\frac{P_{N1}(1-P_{N1}^S)(1-P_{T1}^t)(1-P_{N1})+P_{N1}P_{T1}^t(1-P_{T1})(2-P_{N1}^S)}{(1-P_{N1}^S)(1-P_{T1}^t)(1-P_{N1})+P_{T1}^t(1-P_{T1})(2-P_{N1}^S)}=(1-\alpha), \quad (3)$$

$$P_a(p_2)=\frac{P_{N2}(1-P_{N2}^S)(1-P_{T2}^t)(1-P_{N2})+P_{N2}P_{T2}^t(1-P_{T2})(2-P_{N2}^S)}{(1-P_{N2}^S)(1-P_{T2}^t)(1-P_{N2})+P_{T2}^t(1-P_{T2})(2-P_{N2}^S)}=(\beta), \quad (4)$$

where P_T is the probability of acceptance under tightened inspection and P_N is the probability of acceptance under normal inspection.

The OC curves from the above OC function will be of type B, based on probabilities of sampling from an infinite universe or process. For a given value of the process fraction defective p, the Poisson distribution as an approximation to binomial distribution can be utilized in the computation of probability. Thus, under Poisson conditions, P_T and P_N are defined as

$$P_T = \sum_{r=o}^{c} \frac{\exp(-n_T p)(n_T p)^r}{r!} \qquad (5)$$

and $$P_N = \sum_{r=o}^{c} \frac{\exp(-n_N p)(n_N p)^r}{r!} \qquad (6)$$

Let $P_a(p_1) + 1 - P_a(p_2) = f(n_T, n_N, c)$ (7)

The values n_T and n_N are obtained by using Kullback-Leibler information and the values of c is substituted in equation (5) through (7) subject to the condition that $(\alpha+\beta)$ is minimized.
That is maximizing,

$$P_a(P_1) + 1 - P_a(P_2) = (1-\alpha) + (1-\beta)$$

The value throughout the discussion in this paper, α and β were taken as 0.05 and 0.10 respectively have been singled out and furnished in Table1. Proceeding similar way for various values of p_1 and p_2 combinations, the entries of Table 1 have been obtained (ie. n_N and c, here $n_T=2n_N$). Values of p_1 and p_2 are taken from the table of Arizono et aI., (1985)

VI. Conclusion

The TNT attributes sampling scheme is designated as TNT (n_T, n_N; c). The method of designing the scheme based on Kullback-Leibler information involving a minimum sum of risk is indicated. A table giving the values of the parameters n_N and c, indexed by the Acceptable Quality Level and Limiting Quality Level is presented, from which one can select a scheme which gives a desired AQL and LQL when the producer's risk α = 0.05 and the consumer's risk β = 0.10. It is advantageous to apply this plan in industries for reducing the inspection cost and time, particularly in the area of compliance testing and especially for safety – related product.

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Table 1: Parameters (n_N, c) of TNT–(n_T, n_N ; c) Scheme based on Kullback-Leibler Information (where $n_T = 2n_N$)

(p_1%) AQL	Parameters	(p_2%) LQL																
		0.8	**1.00**	**1.25**	**1.60**	**2.00**	**2.50**	**3.15**	**4.00**	**5.00**	**6.30**	**8.00**	**10.00**	**12.50**	**16.00**	**20.00**	**25.00**	**31.50**
0.1	n_N	615	435	313	221	163	122	90	67	51	38	28	22	17	12	9	7	5
	c	6	5	4	3	3	3	2	2	2	2	2	1	1	1	1	1	1
0.13	n_N	728	503	355	246	179	132	97	71	54	40	30	23	355	13	10	7	5
	c	7	6	5	4	3	3	3	2	2	2	2	2	1	1	1	1	1
0.16	n_N	855	575	398	271	195	142	104	75	56	42	31	24	18	13	10	8	6
	c	9	7	6	5	4	4	3	3	2	2	2	2	2	1	1	1	1
0.2	n_N	1052	683	460	306	217	156	112	81	60	44	33	25	19	14	10	8	6
	c	11	9	7	6	5	4	4	3	3	3	2	2	2	2	1	1	1
0.25	n_N		841	546	353	244	173	123	87	64	47	35	26	19	14	11	8	6
	c		11	9	7	6	5	4	4	3	3	2	2	2	2	1	1	1
0.32	n_N			688	425	286	198	138	97	70	51	37	28	21	15	11	8	6
	c			12	9	7	6	5	4	4	3	3	3	2	2	1	1	1
0.4	n_N				524	340	228	156	107	77	55	40	29	22	16	12	9	6
	c				11	9	7	6	5	4	4	3	3	2	2	2	2	1
0.5	n_N					418	271	179	121	85	60	43	31	23	17	12	9	7
	c					11	9	7	6	5	4	4	3	3	2	2	2	1
0.63	n_N						336	214	140	97	67	47	34	25	18	13	10	7
	c						11	9	7	6	5	4	3	3	3	2	2	1
0.8	n_N							268	168	113	76	52	37	27	19	14	10	7
	c							12	9	7	6	5	4	3	3	2	2	1
1.00	n_N								206	133	88	59	41	30	21	15	11	8
	c								11	9	7	5	4	4	3	2	2	2
1.25	n_N								267	164	104	68	47	33	22	16	12	8
	c								13	11	8	6	5	4	3	3	2	2
1.60	n_N										131	82	55	37	25	18	13	9
	c										11	8	7	5	4	4	3	2
2.00	n_N											101	65	43	28	20	14	10
	c											11	8	5	5	4	3	3
2.50	n_N											130	79	51	32	22	15	10
	c											13	11	8	6	5	4	3
3.15	n_N												104	63	38	25	17	12
	c												13	11	8	6	5	4
4.00	n_N													83	48	30	20	13
	c													13	10	8	6	4
5.00	n_N															37	23	15
	c															10	7	5
6.30	n_N																29	18
	c																10	7
8.00	n_N																	22
	c																	9
10.00	n_N																	28
	c																	12