

# A Second Order Sliding Mode Controller Applications in Industrial Process

B.J.Parvat<sup>1</sup>, S.D.Ratnaparkhi<sup>2</sup>

<sup>1</sup>Associate Professor, Instrumentation and Control Engineering, P.R.E.C.Loni, Ahmednagar-413736, India

<sup>2</sup>PG Student, Instrumentation and Control Engineering, P.R.E.C.Loni, Ahmednagar-413736, India

**Abstract**— This paper proposes a second order sliding mode controller (SOSMC) for industrial process control applications. This SOSMC overcomes the problem of chattering occurred in classical sliding mode controller (SMC). The chattering is the major drawback of SMC. So that proposed controller is designed to achieve smooth and suitable control input for practical systems. The performance of the designed controller is verified in simulation and compared with existing well known super twisting SOSMC to prove the performance of the proposed controller for the process control systems such as a level control of coupled tank system and a concentration control of chemical continuous stirred tank reactor (CSTR) system. The simulation is performed on the mathematical models of both mentioned systems to show the applicability of the proposed method for process control systems.

**Keywords**— Sliding mode control (SMC), Second order sliding mode controller (SOSMC), Higher order sliding mode controller (HOSMC), Variable Structure System (VSS), Coupled Tank System, Continuous Stirred Tank Reactor(CSTR)

## I. INTRODUCTION

Sliding modes control theory first described by scientists in the former Soviet Union [1] and was first published in Russian, by Prof. V. Utkin, one of the main contributors to the SMC. The SMC is widely recognized control strategy in the control system engineering which is to be insensitive to plant parametric uncertainties and external disturbances. For better understanding of SMC refer Fig.1, it gives two phase plane plots showing the behavior of SMC. The plant is defined

as  $\dot{x}_1 = x_2$ ,  $\dot{x}_2 = f + u$  with the sliding variable  $\sigma = \lambda x_1 + x_2$ . The simplest SMC controller  $u = k \text{sign}(\sigma)$  is applied. In the phase plot, after the initial reaching period, the controller is forcing the trajectories to stay on the line  $\sigma = \lambda x_1 + x_2$  which corresponds to exponential decay of  $x_1$  with the rate defined by the control parameter  $\lambda$ . The system has been reduced from being two-dimensional to be of just one dimension, i.e., the sliding surface  $\sigma = 0$  along which system slides. When the hysteresis tends to zero width, any bounded disturbance will be rejected if  $k$  is chosen sufficiently large. This disturbance rejection is known as the invariance property. These are the main advantages of SMC. The chattering in the sliding phase Deviations are due to the hysteresis which is an imperfection [2]. The strength of SMC lies in its robustness against plant parameter. High speed switching feedback gain is necessary to achieve these goals. In SMC, a high speed switching gain is attained using a control signal with a discontinuous element

(sign function). The simple discontinuous control law  $u = -k \text{sign}(\sigma)$  will switch the controller output from  $k$  to  $-k$  or vice versa for the slightest change across  $\sigma = 0$ . Such a control law is theoretically realizing an infinite feedback gain with a finite-valued control signal. The discontinuous element implements high theoretically infinite gain that is the conventional mean to suppress the influence of disturbances and uncertainties in system behaviour and unlike systems with continuous control. The invariance is attained using finite control actions. A typical trajectory of an uncertain system being controlled with SMC is shown in Fig. 1. It is also seen that SMC introduce chattering [3]. This effect can be overcome by introducing proposed SOSMC in this paper.

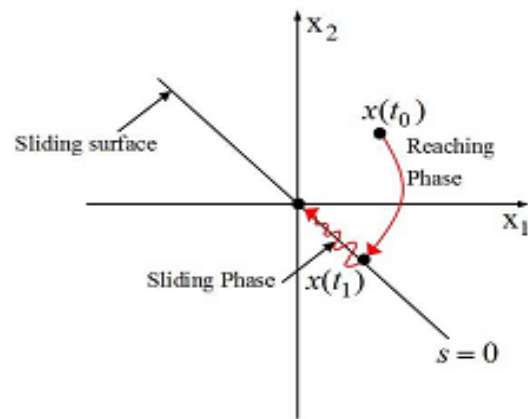


Fig. 1 State trajectory and sliding surface in SMC

Following are the major contributions of this paper:

- 1) A new method of SOSMC design is suggested and successfully implemented in simulation to show its effectiveness.
- 2) The main objective of the SOSMC design is chattering elimination in control.
- 3) The control laws are computed for SOSMC. These control is applied to industrial applications like level control and concentration control.
- 4) The obtained control in SOSMC is smooth and requires lower control effort as compared to SMC.

This paper is organized as: The classical SMC design theory and its implementation is explained in Section 2. The SOSMC theory is discussed in Section 3. The proposed SOSMC control law and its implementation is given in Section 4. Industrial applications of proposed SOSMC and its results are presented in Section 5. Conclusions from results are drawn in Section 6.

I. CLASSICAL SLIDING MODE CONTROL (SMC)

The use of sliding mode control is a simple design procedure. It is shown how a stabilizing controller can be designed for a linear plant using sliding mode control. Further, it is shown that the system can be reached into sliding mode in finite time in the presence of bounded plant uncertainties. During sliding, the system dynamics is independent of the control and is governed by the properties of the chosen sliding surface. Consider a SISO system with its state space model as:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -a_1x_1 - a_2x_2 + bu \end{aligned} \tag{1}$$

Where

and  $0 < a_i < \bar{a}_i, i = 1,2$  and  $0 < \underline{b} < b < \bar{b}$  are the known constraints. For design of SMC of above mentioned system, let us choose sliding surface as:

$$\sigma = \lambda x_1 + x_2 \tag{2}$$

Where  $\lambda$  is positive.

For above sliding surface, a control law should be designed so that the sliding surface is reached to zero, i.e. made attractive towards the origin. This can be achieved by Lyapunov stability technique. Let us consider a Lyapunov function as:

$$V = \frac{1}{2} \sigma^2 \tag{3}$$

which implies that

$$\begin{aligned} \dot{V} &= \sigma \dot{\sigma} \\ &= \sigma[-a_1x_1 - (a_2 - \lambda)x_2 + bu] \\ &= \rho|\sigma| \\ u &= b^{-1}[a_1x_1 + (a_2 - \lambda)x_2 - \rho \frac{|\sigma|}{\sigma}] \\ &= b^{-1}[a_1x_1 + (a_2 - \lambda)x_2 - \rho sign(\sigma)] \\ &= u_{eq} + u_{sw} \end{aligned} \tag{4}$$

where

$$sign(\sigma) = \frac{|\sigma|}{\sigma} \tag{5}$$

$$u_{eq} = b^{-1}[a_1x_1 + (a_2 - \lambda)x_2] \tag{6}$$

$$u_{sw} = -b^{-1} \rho sign(\sigma) \tag{7}$$

The simulation results of system given in (1) using above control law (4) are plotted in Fig.2.

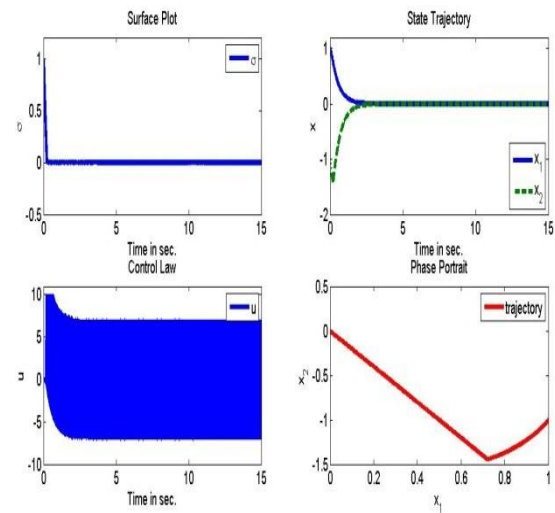


Fig.2. Output of system(1) for SMC ( $a_1 = 5, a_2 = 2, b = 1$  and  $\lambda=4$ )

In the figure, sliding surface, control input and state trajectories are plotted with respective to time. From this the convergence of sliding surface is confirmed. For control input, it shows that control input is not smooth and contains chattering. It also shows that state trajectories are reached to zero in finite time. From phase portrait it is observed that the existence of sliding mode control is occurred and clearly shows reaching and sliding phases. But control with chattering is not suitable for practical systems. To overcome this problem SOSMC is one of the perfect solution [4]. The SOSMC theory and proposed SOSMC are explained in next section.

II. A SECOND ORDER SLIDING MODE CONTROL (SOSMC)

Some advantages of first-order sliding mode control have been motivated in the previous section. In particularly, the invariance property, the simplicity of implementation of the controller itself using approximate plant model and the order reduction when the system is on the sliding surface. In all practical implementations, there is one important potential problem to be faced when using SMC such as, chattering which is undesirable fast switching of the system trajectories on the sliding surface. It needs to be considered in SMC implementation. SMC control law is discontinuous in nature and due to this , it is not suitable for actuators. A smooth control signal might not create this problem. Chattering is also caused when the control law is implemented in a discrete time system. Also finite-time stabilization of the sliding variable at the origin,  $\sigma=0$ , is only guaranteed with a SMC algorithm if the algorithm can use the sign of  $\sigma$  directly. Means the input must appear explicitly in the first derivative  $\dot{\sigma}$ . The system must have relative degree one with respect to  $\sigma$  [5].

There are different ways to overcomes above mentioned disadvantages and among them is selection of second order

sliding mode controllers. A SOSMC controller is a special case of higher order sliding mode control which preserve the desirable properties ,particularly invariance and order reduction but they achieve better accuracy and guarantee finite-time stabilization of relative degree two systems. But in SOSMC it is complicated to prove the stability in comparison with SMC while Lyapunov direct method can be used for SMC designs.

A variety of SOSMC control algorithms have been proposed by different authors. The algorithms differ with respect to the following aspects:

Some are formulated in continuous time which must be discrete and others explicitly in discrete time. The drift algorithm is formulated in discrete-time. Most SOSMC algorithms are able to stabilize a sliding constraint  $\sigma$  with relative degree  $r=2$ .The algorithms differ in the number of necessary parameters to tune. Most algorithms require

knowledge of  $\sigma$  and  $\dot{\sigma}$  , but some require less real-time plant information such as the super-twisting algorithm and some require other information such as the sub-optimal algorithm [6],[12]. In this paper super-twisting control algorithm is used for comparison of proposed algorithm. The super-twisting algorithm, was introduced by Levant in 1993 [9],[10].

The SOSM controller is defined as the point  $(x, \dot{x}) = (0,0)$  is called a second order sliding mode point if it is a solution of system given in(1)(Refer Fig.3).

$$\dot{x}_2 = f(t, x_1, x_2) + g(t, x_1, x_2)u \tag{8}$$

where  $x_1, x_2, t \in \mathcal{R}$

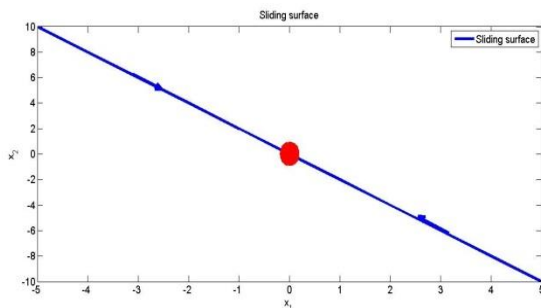


Fig.3. Existence of SMC on sliding surface and SOSM control at the origin.

The SOSMC was also defined the control  $u = u(x_1, x_2)$  with a non-linear sliding manifold that drives the controlled output  $x_1$  and its derivative  $\dot{x}_1 = x_2$  to zero in finite time  $t \leq t_r$  ( $t_r$  \_reaching time) and keeps them thereafter in the presence of a bounded disturbance  $f(x_1, x_2, t)$  is called SOSM control and an ideal SOSM is said to be taking place in system for all  $t \leq t_r$  [7]

### III. THE PROPOSED SOSMC

The SOSMC which is to be proposed is directly given by equation (9) as:

$$u = -\rho_1 \sqrt{|\sigma|} \text{sign}(\sigma) - \rho_2 \sqrt{|-2\lambda x_1 + \sigma|} \text{sign}(-2\lambda x_1 + \sigma) \tag{9}$$

Where,  $\rho_1 = a_1 + a_2$  and  $\rho_2 = 0.5\rho_1$

For convergence of sliding surface and system stability,  $\rho_1 > \rho_2$  is the required condition [11]. The control can be made smooth by adding non-linear component (square root term) in the control law equation. The simulation results of the proposed algorithm for system represented by equation (1) are shown in figure 3 and figure 4. By observing these results it is clear that the proposed algorithm of SOSMC yields smooth control hence it is chattering free. The simulated results for control law (9) are represented in Fig.4 and Fig.5. From these figures it is observed that proposed controller is robust for plant parameter variations It is also shows that there are convergence of sliding surface.

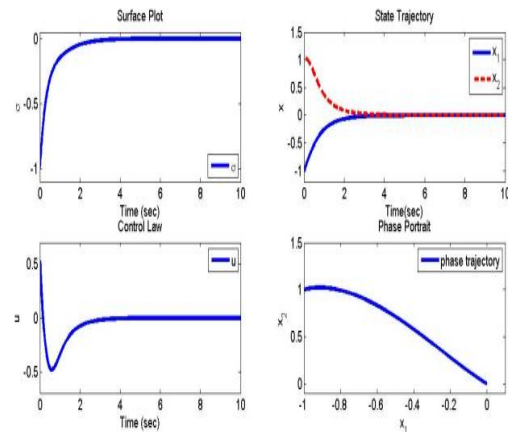


Fig.4. Simulated results for  $(a_1 = 2, a_2 = 2, b = 1$  and  $\lambda=2)$

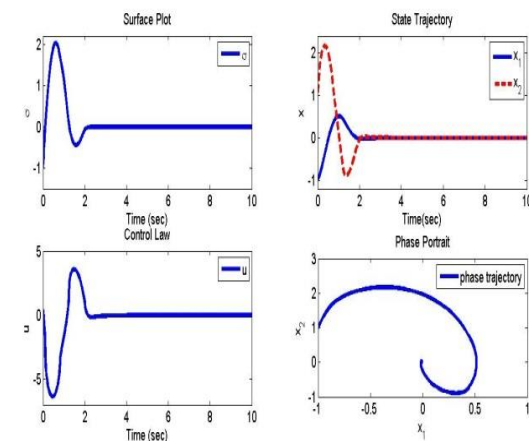


Fig. 5 Simulated results for  $(a_1 = 5, a_2 = -2, b = 1, \lambda=2)$

IV. INDUSTRIAL APPLICATIONS OF PROPOSED SOSMC

The most important step of SMC design is the construction of the sliding surface  $\sigma(t)$ . A sliding mode is first order sliding mode if  $\sigma(t) = 0$  and  $\sigma(t)\dot{\sigma}(t) < 0$  is the fundamental condition for sliding mode. In SMC, the aim is to force the state or error to move on the switching surface  $\sigma(t)$ . In the classical SMC sliding surface depends on the tracking error,  $e(t)$  and its derivative  $\dot{e}(t)$  [8]. Hence

$$\sigma = \lambda e(t) + \dot{e}(t) \tag{10}$$

where  $e = x_1 - x_{desired}$ ,  $\lambda > 0$

By referring equation (9), the control law is directly written as 
$$u = -\rho_1 \sqrt{|\sigma|} \text{sign}(\sigma) + \rho_2 \sqrt{|-2\lambda e + \dot{\sigma}|} \text{sign}(-2\lambda e + \dot{\sigma}) \tag{11}$$

This control law applied for following two applications and its results are presented in below.

- 1) Level control of Coupled Tank system
- 2) Product concentration control of CSTR

A. Level Control of Coupled Tank System

The coupled tank system consists of two small tanks of equal areas and are coupled with an orifice. A pump is used to supply water to the first tank providing flow rate  $Q_{in}$ . In this case pump is only applicable for increases the liquid level and not responsible for pumping out water. The arrangement is shown in Figure 5.

In the system  $Q_{in}$ ,  $Q_{12}$  and  $Q_{out}$  are the inlet volumetric

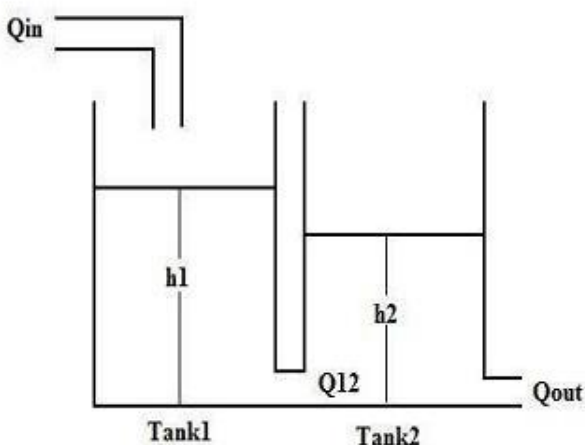


Fig. 6 Coupled Tank System

flow rate to tank1, volumetric flow rate from tank1 to tank2 and outlet flow rate from tank2 respectively in  $cm^3/sec$ . Whereas  $h_1$  and  $h_2$  are the levels of tank1 and tank2 respectively in cm. Let A is the cross section area of tank1 and tank2,  $a_{12}$  is the area of the coupling orifice between tank1 and tank2,  $a_2$  is the cross section area of the outlet pipe of tank2 and g is the gravitational constant. Then the system dynamical equations can be written as [13],[14]:

$$\frac{dh_1}{dt} = \frac{(Q_{in} - Q_{12})}{A} \tag{12}$$

$$\frac{dh_2}{dt} = \frac{(Q_{12} - Q_{out})}{A} \tag{13}$$

Where

$$Q_{out} = a_2 \sqrt{2gh_2} \tag{14}$$

$$Q_{12} = a_{12} \sqrt{2g(h_1 - h_2)} \tag{15}$$

By using above equations, simplified form can be written as:

$$\frac{dh_1}{dt} = \frac{Q_{in}}{A} - k_1 \sqrt{h_1 - h_2} \tag{16}$$

$$\frac{dh_2}{dt} = k_1 \sqrt{h_1 - h_2} - k_2 \sqrt{h_2} \tag{17}$$

Where,

$$k = \frac{a_{12} \sqrt{2g}}{A} \tag{18}$$

$$k_2 = \frac{a_2 \sqrt{2g}}{A}$$

For simulation of above equation take  $A = 208cm^2$ ,  $g = 981cm/s^2$ ,  $0cm^3/s < Q_{in} < 50cm^3/s$ ,  $h_{2desired} = 5cm$ ,  $a_2 = 0.24cm^2$  and  $a_{12} = 0.58cm^2$ , and  $a_{12} = 0.58cm^2$

And computed control law u is as given below.

$$u = -30 \sqrt{|\sigma|} \text{sign}(\sigma) - 15 \sqrt{|-2\lambda e + \dot{\sigma}|} \text{sign}(-2\lambda e + \dot{\sigma}) \tag{19}$$

Where  $\sigma = \lambda e(t) + \dot{e}(t)$  and  $\lambda = 10$ . The output results of coupled tank system are presented in Fig.8. for control law given in(19). Fig.7 gives output results for super twisting SOSMC. Hence from this it observed that results achieved by proposed controller is very much closer to super twisting controller. The advantage of proposed controller is simple and easy for implementation as compared to super twisting control.



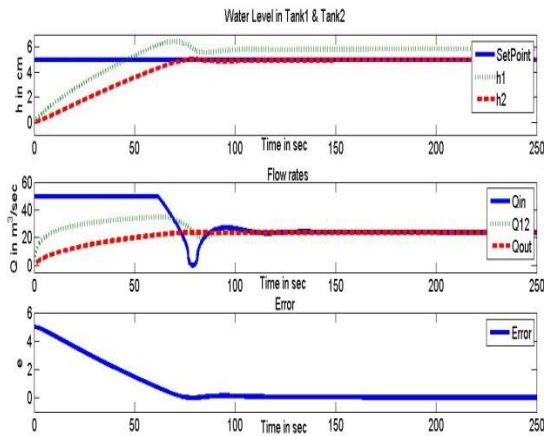


Fig. 7. Level control of coupled tank using Super-twisting Algorithm

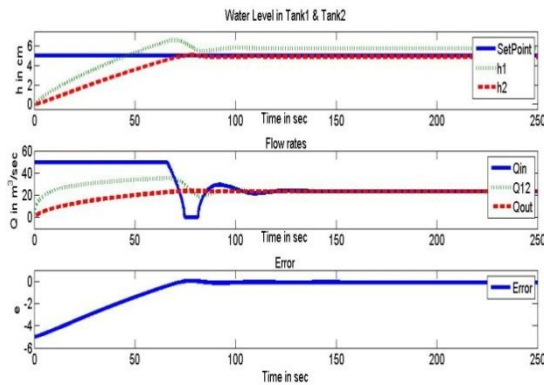


Fig. 8. Level control of coupled tank using proposed algorithm

**B Concentration control in continuous stirred tank reactor (CSTR)**

Continuous stirred tank reactors have widespread application in industry. A CSTR model provides a convenient way of illustrating modelling principles for chemical reactors. A mathematical model of the CSTR is implemented in the simulink to demonstrate the SOSMC controller. In this example product concentration in CSTR is taken as controlling variable. A diagram of the process is shown in the Fig.9.

The dynamic model of the system is [15]

$$\begin{aligned} \frac{dh}{dt} &= Q_{in}(t) + Q_2(t) - 0.2\sqrt{h(t)} \frac{dC_b}{dt} \\ &= (C_{b1} - C_b(t)) \frac{Q_1(t)}{h(t)} + (C_{b2} - C_b(t)) \frac{Q_2(t)}{h(t)} \\ &\quad - \frac{k_1 C_b(t)}{(1 + k_2 C_b(t))^2} \end{aligned} \tag{20}$$

Where  $h(t)$  is the liquid level,  $C_b(t)$  is the product concentration at the output of the process,  $Q_1(t)$  is the flow

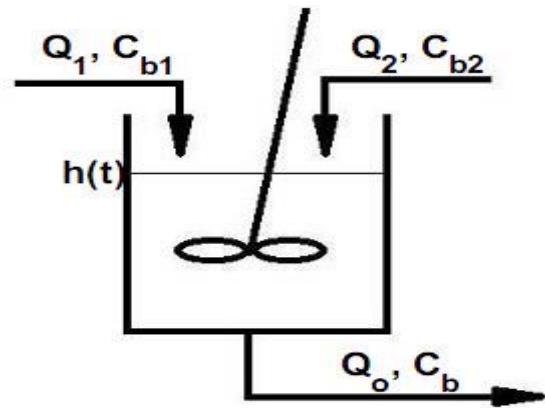


Fig. 9 Catalytic Continuous Stirred Tank Reactor

rate of the concentrated feed  $C_{b1}$ , and  $Q_2(t)$  is the flow rate of the diluted feed  $C_{b2}$ . The input concentrations are set to  $C_{b1} = 24.9$  and  $C_{b2} = 0.1$ . The constants associated with the rate of consumption are  $k_1 = 1$  and  $k_2 = 1$ . The objective of the controller is to maintain the product concentration by adjusting the flow  $Q_1(t)$ . To simplify the demonstration, set  $Q_2(t) = 0.1$ . The level of the tank  $h(t)$  doesn't control for this experiment. The computed control law  $u$  for this CSTR system is given by equation (21).

$$u = -2\sqrt{|\sigma|} \text{sign}(\sigma) - \sqrt{-2\lambda e + \sigma} \text{sign}(-2\lambda e + \sigma) \tag{21}$$

where  $\sigma = \lambda e(t) + \dot{e}(t)$  and  $\lambda = 10$ . The output results of CSTR are shown in Fig.10 and Fig.11 for super twisting and proposed controller respectively. Again it is clear that output results of proposed controller are closer to output results of super twisting controller.

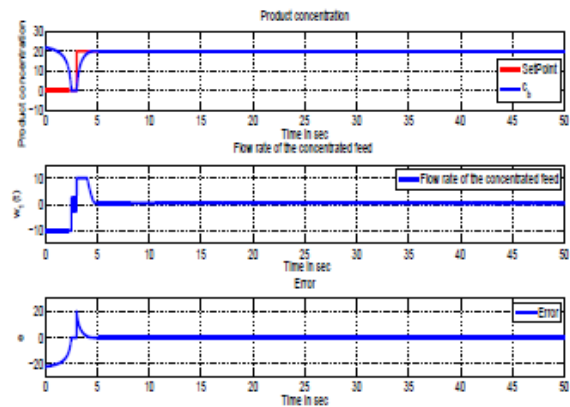


Fig. 10. Product concentration control of CSTR using Super-twisting algorithm

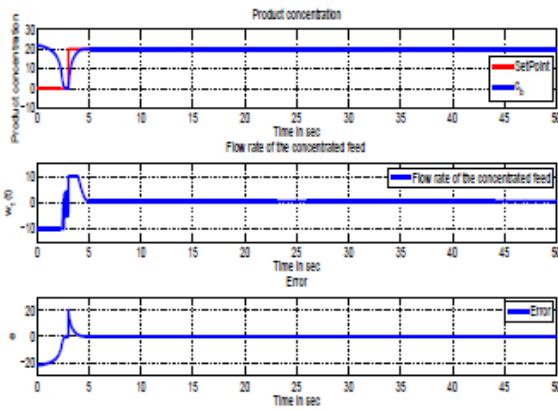


Fig. 11. Product concentration control of CSTR using Proposed algorithm

## V. CONCLUSION

In this work, a proposed second-order sliding mode controller has been designed and implemented for two different process control applications. The simulated results of these applications has been represented in above using proposed and existing super twisting SOSMC. From these results it has been concluded that results of proposed SOSMC is closer to super twisting SOSMC. The proposed controller gives smooth control, suitable for actuators in the process control. For design of proposed algorithm there is no need to find the higher derivative(s) of sliding surface. It is a perfect solution of the control problem in continuous control of processes. It would be a major contribution to the chemical industry where coupled tank and CSTR are common. A mathematical models for the coupled tank system and CSTR has been built up according to the theoretical considerations. The theoretical considerations did not lead to usable rules for tuning of the controller parameters. Finally from the results it is concluded

that the SOSMC algorithm which is to be proposed is more simpler and easy for implementation as compared with super-twisting algorithm.

## REFERENCES

- [1]. C. Edwards, and S. Spurgeon, Sliding Mode Control: Theory and Applications, Taylor & Francis, 1998.
- [2]. V. Utkin, Variable structure systems with sliding modes, IEEE Transactions on Automatic Control, 22(2): 212-222, 1977.
- [3]. V. Utkin, Sliding mode control design principles and applications to electric drives, IEEE Transactions on Industrial Electronics, 40(1):23-36, 1993.
- [4]. L. Fridman, J. Moreno, and R. Iriarte, Sliding Modes after the First Decade of the 21<sup>st</sup> Century, Lecture notes in Control and Information Sciences 412, Springer, Mexico City, 2010.
- [5]. J. E. Slotin, W. Li, Applied nonlinear control, Prentice Hall, New Jersey, 1991.
- [6]. A. Levant, Sliding order and sliding accuracy in sliding mode control, International Journal of Control, Vol.58, Issue 6, 1247-1263, 1993.
- [7]. Y. Shtessel, C. Edwards, L. Fridman and A. Levant, Sliding Mode Control and Observation, Control Engineering series, Springer, New York, 2013.
- [8]. I. Eker, Second-order sliding mode control with experimental application, ISA Transactions, Volume 49, Number 394-405, 2010..
- [9]. A. Levant Chattering Analysis, IEEE Transactions on Automatic Control, 55(6): 1380-1389, 2010.
- [10]. L. Fridman and A. Levant, Higher order sliding modes, Sliding Mode Control in Engineering, Marcel Dekker: New York, 2002
- [11]. P. Trivedi, B. Bandyopadhyay, A simple reaching law based design method for 2-sliding mode control, IEEE International Conference on Industrial Technology (ICIT), 2010.
- [12]. Bartolini G, Orani N, Pisano A, Usai E. Higher-order sliding mode approaches for control and estimation in electrical drives, AdvVarStruct, 423-45, 2006.
- [13]. H. Delavari, R. Ghaderi, A. Ranjbar, and S. Momani, Fuzzy fractional order sliding mode controller for nonlinear systems, Commun Nonlinear Sci Numer Simulat 15, 963-978, 2010.
- [14]. M. Khan, and S. Spurgeon, Robust MIMO water level control in interconnected twin-tanks using second order sliding mode control, Control Engineering Practice 14, 375-386, 2006.
- [15]. D.E. Seborg, T.F. Edgar and D.A. Mellichamp, Process Dynamics and Control, Second edition, Jon Wiley and sons, Inc, 2004.