# Higher Order Partial Differential Equation Based Method for Image Enhancement

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***Abstract*— Many color image enhancing techniques have been proposed to eliminate noise and uninteresting details from an image, without blurring semantically important structures such as edges. PDE-based methods have been proposed to tackle the problem of image denoising with a good preservation of edges and also to explicitly account for intrinsic geometry. The existing method can be viewed as generalization of the Bettahar – Stambouli filter to multivalued images. The proposed algorithm is based on using single vectors of gradient magnitude and the fourth order derivatives as a manner to relate different color components of the image. The Partial Differential Equation based algorithm is more efficient than existing models and some previous works at color images denoising and sharpens the edges efficiently without creating false colors.**

***Index Terms*— Blur, diffusion, fourth order PDEs, image smoothing.**

## I. INTRODUCTION

Capturing an image with sensors is an important step in many areas. The captured image is used in several applications, which all have their own requests on the quality of the captured image. Acquired images are often degraded with blur, noise, or both blur and noise simultaneously.

Noise reduction is usually the first process that is used in the analysis of digital images. In any image denoising algorithm, it is very important that the denoising process has no blurring effect on the image, and makes no changes or relocation to image edges.

There are various methods for image denoising. Using simple filters, such as average filter, median filter and Gaussian filter, are some of the techniques employed for image denoising. These filters reduce noise at the cost of smoothing the image and hence softening the edges.

To overcome the above-mentioned problems, the partial differential equations (PDEs) based methods have been introduced in the literature. These methods assume the intensity of illumination on edges varies like geometric heat flow in which heat transforms from a warm environment to a cooler one until the temperature of the two environments reaches a balanced point.

It was shown that these changes are in the form of a Gaussian function. Consequently, sudden changes in edges might be due to the existence of noise. In fact, an image includes a series of regions in which different regions might have different standard deviations.

## II. BACKGROUND

A large number of PDE-based methods have been proposed to tackle the problem of image denoising with a good preservation of edges and also to explicitly account for intrinsic geometry. Hence, PDEs based on diffusion methods, and a shock filter have recently dominated image processing research as a very good tool for noise elimination, image enhancement, and edge detection. Then, many solutions have been proposed in the processing of gray-level images by coupling diffusion to the shock filter.

The extension of these methods to multivalued images can be achieved in two ways. The first one consists in using a marginal approach that enhances each color component of the multivalued image separately using a scalar method.

The second way consists in using a single vector processing, where different components of the image are enhanced by considering the correlation between them. Originating from a well-known physical heat-transfer process, the PDE-based approaches consist in evolving in time the filtered image u(t) under a PDE.

When coupling diffusion and a shock filter, the PDE is a combination of three terms, i.e.,

$$\frac{\partial u}{\partial t} = c_\eta u_{\eta\eta} + c_\varepsilon u_{\varepsilon\varepsilon} - c_{sk} F(u_{\eta\eta}) \left| \nabla u \right| \quad (1)$$

Where u(t=0) = u is the input image $I\nabla uI$ is the gradient magnitude, $\eta$ is the gradient direction, and $\varepsilon$ is the direction perpendicular to the gradient; therefore, $u_{\eta\eta}$ and $u_{\varepsilon\varepsilon}$ represent the diffusion terms in gradient and level-set directions, respectively.

Where $c_\eta$ and $c_\varepsilon$ are some flow control coefficients. The first kind of diffusion smooth's edges, whereas the second one smooth's parallel to the edge on both sides.

The last term in (1), which is weighted by $c_{sk}$, represents the contribution of the shock filter in the enhancement of the image. Function F(s) should satisfy conditions F (0) = 0 and F(s).s ≥0. The choice of F(s) = sign(s) gives the classical shock filter.

Hence, by considering adaptive weights $c_\eta$,$c_\varepsilon$, and $c_{sk}$ as functions of the local contrast, we can favour the smoothing process under diffusion terms in homogeneous parts of the image or enhancement operation under the shock filter at edge locations.

In the first model of coupling diffusion and the shock filter, where the image is diffused only in the direction perpendicular to the gradient eliminating fluctuations and developing shocks with the production of false piecewise constant images. This model is given by

$$\frac{\partial u}{\partial t} = c_\varepsilon u_{\varepsilon\varepsilon} - sign(G_\sigma * u_{\eta\eta}) \left| \nabla u \right| \quad (2)$$

Where * denotes the convolution operator, $G_\sigma$ denotes the Gaussian function with the standard deviation $\sigma$ and $c_\varepsilon$ is a constant.

**Existing Methods**

Several enhancement methods have been proposed for the gray level images. Only a very few works tackle the shock diffusion coupling using an approach specifically dedicated to multi-valued images. So, to avoid the effect of the apparition of false colors, the processing applied to the image must be driven in a common and coherent manner for all image components. This type of approach is denoted as "vector processing", in opposition to the marginal processing which is a multi-scalar processing.

Thus, in order to describe vector-valued image variations and structures, Di Zenzo and Lee have proposed to use the local variation of a vector gradient norm that detects edges and corners when its value becomes high.

It can be computed using the eigen-values of a symmetric and semi-positive matrix that can be computed. Hence, by using multi-valued geometrical description of, Tschumperlé and Deriche proposed a new form of diffusion shock filter coupling especially for enhancement of colour images. The Kornprobst model is seen also to produce false piecewise constant images. However, Gilboa developed a complex-diffusion–shock-filter coupling model that smooths the image with an enhancement of weak edges.

The imaginary value of the solution, which is an approximated smoothed second derivative, is used as an edge detector. In the other hand, Fu developed a region-based shock–diffusion scheme, where directional diffusion and shock terms are factored by adaptive weights.

It is based on the following equation:

$$\frac{\partial u}{\partial t} = c_\eta u_{\eta\eta} + c_\varepsilon u_{\varepsilon\varepsilon} - \omega(u_\eta) sign(G_\sigma * u_{\eta\eta}) \left| \nabla u \right| \quad (3)$$

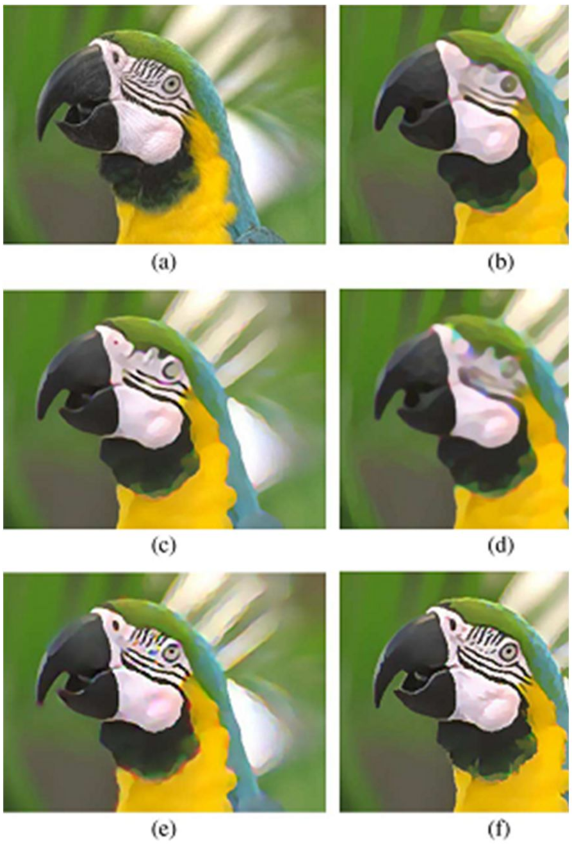


Fig. 1. Enhancement of the Parrot image. (a) Original image. (b) Al-varez–Mazorra filter. (c) Kornprobst filter. (d) Gilboa filter. (e) Fu filter. (f) Bettahar–Stambouli filter.

Where

$c_\varepsilon$ is used to prevent excess smoothness to smaller details under gradient direction smoothing.

$c_\eta$, $c_\varepsilon$ and $\omega(u_\eta)$ are computed by

| | $c_\eta$ | $c_\varepsilon$ | $\omega(u_\eta)$ |
|---|---|---|---|
| $\lvert\nabla u_\sigma\rvert > T_1$ | 0 | $1/(1+l_1 u_{\varepsilon\varepsilon}^2)$ | 1 |
| $T_2 < I\nabla u_\sigma I \leq T_1$ | 0 | $1/(1+l_1 u_{\varepsilon\varepsilon}^2)$ | $\lvert \mathrm{th}(l_2 u_{\eta\eta}) \rvert$ |
| else | 1 | 1 | 0 |

The thresholds $T_1$ and $T_2$ are used to select which edge to be enhanced or smoothed $l_1$ and $l_2$ are constants. This scheme uses the hyperbolic tangent membership function $\lvert \mathrm{th}(l_2 u_{\eta\eta}) \rvert$ to guarantee a natural smooth transition, by controlling softly the changes of gray levels of the image.

However, the Fu filter creates a strong shock between the regions of the image, which is due to the oscillations at big edges that make this scheme instable.

In a more recent work, Bettahar and Stambouli proposed a new reliable and stable scheme, which is a kind of coupling diffusion to a shock filter with a reactive term.
This model is based on the following set:

$$\frac{\partial u}{\partial t} = \nabla u \, div\left(\frac{g(\nabla u_\sigma)\nabla u}{\nabla u I}\right) - \alpha I\nabla(f(I\nabla u_\sigma))^2(u - v) \quad (5)$$

$$\frac{\partial v}{\partial t} = \beta(1 - I\nabla f(\nabla u_{\sigma I))I^2)} u_{\eta\eta} - sign(G_\sigma * u_{\eta\eta})\vert \nabla u I \quad (6)$$

Where $u_\sigma$ is the smoothed image using the Gaussian kernel and v(t) is the just previous evolution of u(t). In discrete time, v(t) is the last value of u(t) and u(0) = $u_0 . g(I\nabla u_\sigma)$ and $f(\nabla u_\sigma)$. are decreasing functions having the same form with free parameters $k_d$ for g and $k_c$ for f respectively; therefore

$$g(s) = 1/(1 + \frac{s^2}{k_d^2}) \quad (7)$$

The first function $g(I\nabla u_\sigma)$ is used to assure an anisotropic behavior and to select "small edges" to be smoothed according to parameter $k_d$ .

However, $f(\nabla u_\sigma)$ is introduced to select which "big edges" have to be improved according to Parameters $k_c$. α and β are positive balance constants.

All mentioned models have been developed for enhancement of gray-level images. The natural way to apply them on multivalued images is to process each color component independently of the others in a marginal way. Such a way is well known to produce false colors as it can be observed on Fig. whatever filter is used. Only a very few works tackle the shock diffusion coupling using an approach specifically dedicated to color images.

### Tschumperlé–Deriche Model

To avoid the effect of the apparition of false colors, the processing applied to the image must be driven in a common and coherent manner for all image components. This type of approach is denoted as "vector processing," in opposition to the marginal processing, which is a multiscalar processing. Thus, in order to describe vector-valued image variations and structures, Di Zenzo and Lee have proposed to use the local variation of the vector gradient norm $\nabla u$ that detects edges and corners when its value becomes high. It can be computed using the eigenvalues $\lambda_+$ and $\lambda_-$ ($\lambda_+ > \lambda_-$) of the symmetric and semipositive matrix as follows:

$$G = \begin{pmatrix} g11 & g12 \\ g21 & g22 \end{pmatrix}$$

$$= \begin{pmatrix} u_{1x}^2 + u_{2x}^2 u_{3x}^2 & u_{1x}u_{1y} + u_{2x}u_{2y} + u_{3x}u_{3y} \\ u_{1x}u_{1y} + u_{2x}u_{2y} + u_{3x}u_{3y} & u_{1y}^2 + u_{2y}^2 u_{3y}^2 \end{pmatrix}$$

Where ($u_{pl}$(p=1, 2, 3) and $l = x, y$) represents the derivative in direction of the red, green, and blue (RGB) components of the color image, and

$$\lambda_{\pm} = \frac{g11 + g22 \pm \sqrt{(g11 - g22)^2 - 4g_{12}^2}}{2} \quad (8)$$

Hence, by using the multivalued geometrical description, Tschumperlé and Deriche proposed a new form of diffusion–shock-filter coupling particularly for the enhancement of color images.

### Proposed Method

The proposed method is based on fourth order PDE associated with anisotrophic diffusion. Let U denote the image intensity function, t the time, and C (.) the diffusion coefficient the anisotropic diffusion as formulated may be presented as

$$\frac{\partial u}{\partial t} = div(c(|\nabla u|\nabla u) \quad (9)$$

This equation was associated with the following energy functional

$$E(u) = \int_{\Omega} f(|\nabla u|)d\Omega \quad (10)$$

where is the image support, and is an increasing function associated with the diffusion coefficient as

$$c(s) = f'(S)/S \quad (11)$$

### Experimental Results

We evaluate performances of our model comparing to marginal channel by channel methods of Alvarez-Mazorra, Kornprobst, Gilboa, Fu and Bettahar-Stambouli, while we consider vector regularization of Tschumperlé-Deriche. For this comparison, we choose the parameters that give better results for each filter, except for the number of iterations which must be the same for an objective comparison. All models are applied to blurry and noised images. In the production of artificially blurry images, we use the Gaussian convolution of original test images. As objective criterions, we use is the SNR.. All results are obtained with 1500 iterations. We can see how our model has efficiently removed noise in smoother regions with edges sharpening referring to Alvarez-Mazorra, Kornprobst, Gilboa, Bettahar-Stambouli and Tschumperlé-Deriche filters.

Only the last two models have successfully smoothed noise in homogeneous parts of the images. The other models have produced false piecewise constant images with residual noise, where artificial blobs have been created. Furthermore, we notice here how Alvarez-Mazorra, Kornprobst, Gilboa and Bettahar-Stambouli filters have introduced false colours as it appears on the different parts of the enhanced image.

Only Tschumperlé-Deriche model presents satisfactory results, with, however, a fine production of false colours at localized edges. On the contrary, in the proposed filter, edges can well be distinguished without any false colours. Our algorithm doesn't create false colours referring to other models with an effective selective smoothing of the sunglass according to the other parts of the image.

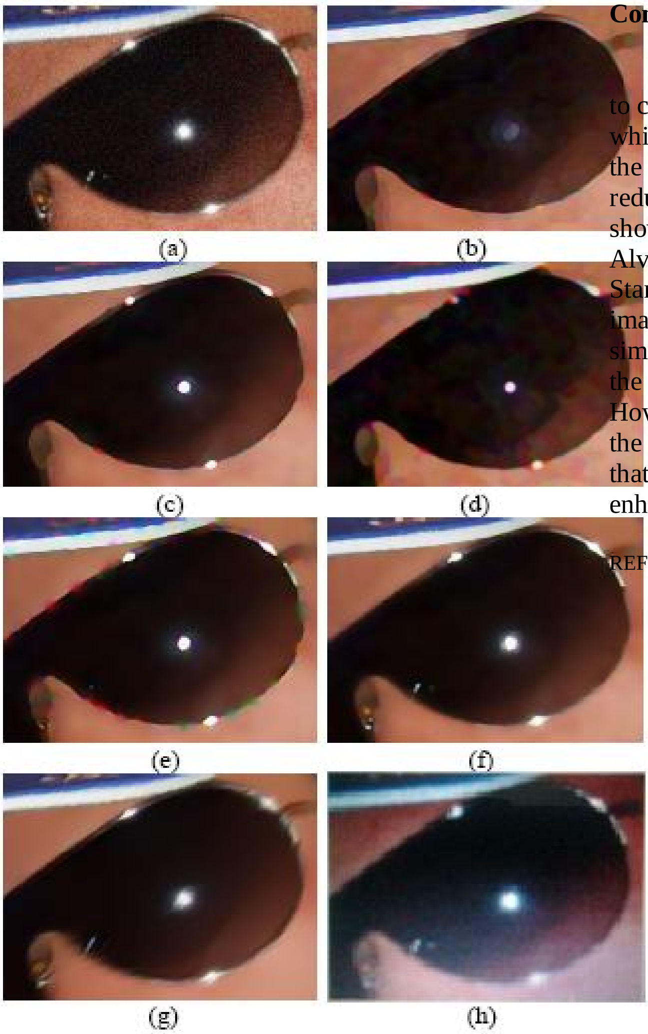

Figure - Enhancement of Sunglasses image: Blurry and noised image; (b) Alvarez-Mazorra filter; (c) Kornprobst filter; (d) Gilboa filter; (e) Fu filter; (f) Bettahar-Stambouli filter; (g) Tschumperlé-Deriche filter; (f) Proposed filter.

In the table-2, we present the SNR values for the different approaches. It can be noted that the SNR of our solution is bigger than the SNR of other models. This can be deep-rooted by the SNR values which confirm the best visual quality of our solution.

| Method | SNR |
|---|---|
| Alvarez-Mazorra filter | 12.870 |
| Kornprobst filter | 14.611 |
| Gilboa filter | 13.292 |
| Fu filter | 14.117 |
| Bettahar-Stambouli filter | 19.063 |
| Tschumperlé-Deriche filter | 16.464 |
| Proposed filter | 23.797 |

Table 2 – SNR values of different approaches

## Conclusion

We have proposed a new filter of coupling shock filter to curvature diffusion for multi-valued image enhancement, which is based on using single vectors for all components of the image. This filter produces a selective smoothing reducing efficiently noise and sharpens edges. Our analysis shows that the proposed method is more efficient than Alvarez-Mazorra, Kornprobst, Gilboa, Fu, Bettahar-Stambouli and Tschumperlé-Deriche models at multi-valued image restoration in presence of blur and noise simultaneously. In that it denoises homogeneous parts of the multi-valued image, while it keeps edges enhanced. However, due to the fact of using single vectors with the specific reaction, our filter doesn't create false colours that can appear when each component of the image is enhanced separately.

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