Flow and Heat Transfer in a Laminar Two Phase Plane Wall Jet

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***Abstract*-** **Laminar mixing of a two-dimensional plane wall jet of particulate suspension in an incompressible carrier fluid has been considered. The basic equations are of the boundary layer type and include the diffusion equation for sub micron particles to investigate the flow field. The drag force due to slip, finite volume fraction ,heat due to conduction and viscous dissipation in the particle phase energy equation have been introduced to study their effect on skin friction & heat transfer. The governing equations are solved by taking perturbations on Schlirchting's model. Again, the effects of Prandtle number ,Eckrt number, Nusselt number, size of the particles ,Material density of the particles and diffusion parameters on the velocity and temperature field for both phases have shown through figures and tables. It is observed that Nusselt number always increases with the increase of the above parameters, and heat always transform from fluid to plate in all the cases.**

***Keywords*-** **Volume Fraction, Suspended Particulate Matter(SPM), Skin Friction, Heat Transfer**

NOMENCLATURE

$\vec{q}(u, v)$	$\rightarrow$	Velocity components for the fluid phase in x- and y- directions respectively
$\overrightarrow{q_p}(u_p, v_p)$	$\rightarrow$	velocity components for the particle phase in x-and y-directions respectively
(T, T_p)	$\rightarrow$	temperatures of fluid and particle phase
(T_w, T_∞)	$\rightarrow$	temperature at the wall and free-stream respectively
(ρ, ρ_p)	$\rightarrow$	density of fluid and particle phase respectively
ρ_s	$\rightarrow$	Material density of particles
Pr	$\rightarrow$	Prandtl number
Ec	$\rightarrow$	Eckret number
Nu	$\rightarrow$	Nusselt number
c_f	$\rightarrow$	Skin friction coefficient
φ	$\rightarrow$	Volume fraction of SPM
D	$\rightarrow$	Diameter of the particle
U	$\rightarrow$	free stream velocity
ρ_{p0}	$\rightarrow$	density of particle phase in free stream
T_∞	$\rightarrow$	Temperature of the fluid at free stream
α	$\rightarrow$	concentration parameter
ε	$\rightarrow$	diffusion parameter
F	$\rightarrow$	friction parameter between the fluid and the particle $(F = 18\mu/\rho_p D^2)$

Jmax → Maximum number of Grid points along Y – axis

L → Reference length

W → Dummy variable

r_y → Grid growth ratio

u_{pw} → Particle velocity on the plate

ρ_{pw} → Particle density distribution on the plate

T_{pw} → Temperature of particle phase on the plate

Superscripts

*→ Nondimensional quantities.

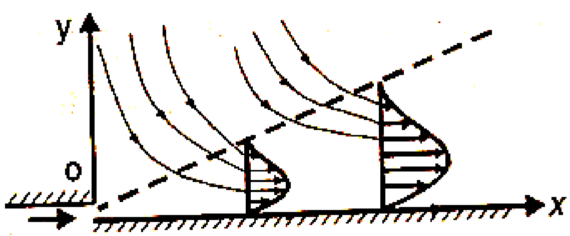

Fig. : Plane wall jet

INTRODUCTION

Gas-particle flows, dusty fluid flows and the flow of suspensions have received considerable attention due to the importance of these types of flow in various engineering applications. The influence of dust particles in both natural and industrial processes like sand dust storms, tornados, volcano eruptions, fluidized beds, coal classifiers, power conveyers, particle-laden jets, petroleum industry, purification of crude oil, manufacturing in the chemical, pharmaceutical, biomedical, mineral and new materials sectors, and increasingly grow in importance as new techniques and applications, such as functional nanomaterials, are developed. One important engineering application is the predication and prevention of dust fires and explosions in plants, storerooms and coal mines. It is well known that many organic or metallic powders like cornstarch, coal, aluminum and magnesium are suspended in air form explosive mixtures due to huge specific surface area of fine dispersed particles.

Schlichting's model [11] of a laminar jet consider a thin incompressible homogeneous jet issuing into a medium at rest. This model can be analyzed easily because the equations goverening the problem admit a similar solutions. Pozzi & Binachani [9] have found that the velocity distribution can be studied by a perturbation on the Schlichting's model [11] and obtained the first order perturbation solution in the closed form.

Bansaal and Tak [1] have studied Compressible laminar plane wall jet and solved the governing equations by a proper transformation of a similarity variable and obtained the solution for temperature distribution in a closed form, for viscous heating, wall heating, initial heating, for arbitrary values of prandtl number.

Bansal & Tak [2] have obtained approximate solution of heat and momentum transfer in a laminar plane wall jet. Mellivlle & Bray [3,4] have proposed a model of two phase Turbulent jet. Panda at.el [7,8] have studied two phase jet flow for incompressible and compressible fluids.

No consulted effort found in the literature for studying two phase wall jet problems. Here in the present study, we have considered two phase jet flow of an incompressible fluid be discharged through a narrow slit in the half space along a plane wall and mixed with the same surrounding fluid , Being initially at rest having a temperature T_∞ .

MATHEMATICAL MODELING

Let an incompressible fluid with SPM be discharged through a narrow slit in the half space along a plane wall and mixed with the same surrounding fluid being initially at rest having temperature T_∞. The wall is also maintained at the same constant temperature T_∞.Taking the origin in the slit and the co-ordinate axis x and y along and normal to the plane wall respectively, the boundary layer equations for the continuity, momentum and energy after Introducing the non-dimensional quantities like

$$x^* = \frac{x}{L}, \quad y^* = \frac{y}{L}\sqrt{Re}, \quad u^* = \frac{u}{U}, \quad v^* = \frac{v}{U}\sqrt{Re},$$
$$u_p^* = \frac{u_p}{U}, \quad v_p^* = \frac{v_p}{U}\sqrt{Re}, \quad \rho_p^* = \frac{\rho_p}{\rho_{p_0}}, \quad T^* = \frac{T - T_\infty}{T_\infty},$$
$$T_p^* = \frac{T_p - T_\infty}{T_\infty} \qquad (1)$$

The governing equations

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (2)$$

$$\frac{\partial}{\partial x}(\rho_p u_p) + \frac{\partial}{\partial y}(\rho_p v_p) = 0 \qquad (3)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} - \alpha\frac{1}{1-\varphi}\frac{FL}{U}\rho_p(u - u_p) \qquad (4)$$

$$u_p\frac{\partial u_p}{\partial x} + v_p\frac{\partial u_p}{\partial y} = \epsilon\frac{\partial^2 u_p}{\partial y^2} + \frac{FL}{U}(u - u_p) \qquad (5)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{1}{Pr}\frac{\partial^2 T}{\partial y^2} + \frac{2\alpha}{3Pr}\frac{1}{1-\varphi}\frac{FL}{U}\rho_p(T_p - T)$$
$$+Ec\left(\frac{\partial u}{\partial y}\right)^2 + \frac{1}{1-\varphi}\frac{FL}{U}\alpha Ec\,\rho_p(u - u_p)^2 \qquad (6)$$

$$u_p\frac{\partial T_p}{\partial x} + v_p\frac{\partial T_p}{\partial y} = \frac{\epsilon}{Pr}\frac{\partial^2 T_p}{\partial y^2} - \frac{FL}{U}(T_p - T)$$
$$+\frac{3}{2}Pr.\epsilon Ec\left[\left(\frac{\partial u_p}{\partial y}\right)^2 + u_p\frac{\partial^2 u_p}{\partial y^2}\right]$$
$$-\frac{3}{2}\mathrm{Pr}\,Ec\,\frac{FL}{U}(u - u_p)^2 \qquad (7)$$

$$u_p\frac{\partial \rho_p}{\partial x} + v_p\frac{\partial \rho_p}{\partial y} = \epsilon\frac{\partial^2 \rho_p}{\partial y^2} \qquad (8)$$

Subjected to the boundary conditions

$$y = 0:\ u = 0, u_p = u_{p_w}, \rho_p = \rho_{p_w},$$
$$T = 0,\ T_p = T_{p_w},$$
$$y = \infty:\ u = u_p = 0,\ \rho_p = 0,\ T = 0,\ T_p = 0 \qquad (9)$$

and the integral conditions

$$\frac{\partial}{\partial x}\int_0^\infty\{u^2(\int_0^y u\,dy)\}dy$$
$$+\frac{1}{1-\varphi}\frac{FL}{U}\alpha\int_0^\infty\left\{u\int_y^\infty \rho_p(u - u_p)dy\right\}dy = 0 \qquad (10)$$

$$\frac{\partial}{\partial x}\int_0^\infty uT\left(\int_0^y u\,dy\right)dy$$
$$= \frac{1}{1-\varphi}\frac{2\alpha}{3Pr}\frac{FL}{U}\int_0^\infty\left\{u\int_y^\infty \rho_p(T_p - T)dy\right\}dy$$
$$-\frac{1}{1-\varphi}\frac{FL}{U}\alpha Ec\int_0^\infty\left\{u\int_y^\infty \rho_p(u - u_p)^2dy\right\}dy$$
$$+Ec\int_0^\infty\left\{u\int_y^\infty\left(\frac{\partial u}{\partial y}\right)^2dy\right\}dy \qquad (11)$$

SOLUTION FOR THE VELOCITY DISTRIBUTION

By taking a perturbation on the Schlichting [11] model by writing

$$u = u_0 + u_1,\ u_p = u_{p_0} + u_{p_1}, \rho_p = \rho_{p_0} + \rho_{p_1} \qquad (12)$$

where u_1, u_{p_1} and ρ_{p_1} are perturbation quantities and substituting in equations (2) to (5), we get two sets of equations as follows

1ST SET AND ITS SOLUTION

$$\frac{\partial u_0}{\partial x} + \frac{\partial v_0}{\partial y} = 0 \qquad (13)$$

$$\frac{\partial}{\partial x}(\rho_{p_0}u_{p_0}) + \frac{\partial}{\partial y}(\rho_{p_0}v_{p_0}) = 0 \qquad (14)$$

$$u_0\frac{\partial u_0}{\partial x} + v_0\frac{\partial u_0}{\partial y} = \frac{\partial^2 u_0}{\partial y^2} - \frac{1}{1-\varphi}\frac{FL}{U}\alpha\,\rho_{p_0}(u_0 - u_{p_0}) \qquad (15)$$

$$u_{p_0}\frac{\partial u_{p_0}}{\partial x} + v_{p_0}\frac{\partial u_{p_0}}{\partial y} = \epsilon\frac{\partial^2 u_{p_0}}{\partial y^2} + \frac{FL}{U}(u_0 - u_{p_0}) \qquad (16)$$

$$u_{p_0}\frac{\partial \rho_{p_0}}{\partial x} + v_{p_0}\frac{\partial \rho_{p_0}}{\partial y} = \epsilon\frac{\partial^2 \rho_{p_0}}{\partial y^2} \qquad (17)$$

Subjected to the boundary condition

$$y = 0:\ u_0 = 0,\quad u_{p_0} = u_{p_{w0}},\ \rho_{p_0} = \rho_{p_{w0}}$$
$$y = \infty:\ u_0 = u_{p_0} = 0,\ \rho_{p_0} = 0 \qquad (18)$$

Together with the integral condition

$$\frac{\partial}{\partial x}\int_0^\infty\{u_0^2(\int_0^y(u_0)dy)\}\,dy$$

$$+\frac{1}{1-\varphi}\frac{FL}{U}\,\alpha\int_0^\infty\left\{u_0\int_y^\infty\rho_{p_0}\left(u_0-u_{p_0}\right)dy\right\}dy=0 \quad (19)$$

Since we are considering the case of a dilute suspension of particles, the velocity distribution in the fluid is not significantly affected by the presence of the particles. Therefore the drag force term [i.e. 2nd term in the R.H.S. of equation (15)] is dropped. But for the submicron particles, Brownian motion can be significant, the concentration distribution equation (14) above will then be modified by Brownian diffusion equation (17). With the above consideration the equations (15) and (19) become

$$u_0\frac{\partial u_0}{\partial x}+v_0\frac{\partial u_0}{\partial y}=\frac{\partial^2 u_0}{\partial y^2} \quad (20)$$

$$\frac{\partial}{\partial x}\int_0^\infty\{u_0^2(\int_0^y(u_0)dy)\}\,dy=0$$

Or, $\int_0^\infty\{u_0^2(\int_0^y(u_0)dy)\}\,dy=E(say)$ (21)

A similar solution of the equation (20) under the present boundary and integral conditions is possible if we take

$$\Psi=(Ex)^{1/4}f(\eta),\ \eta=(\tfrac{E}{1})^{1/4}yx^{-\frac{3}{4}} \quad (22)$$

and $u_0=\frac{\partial\Psi}{\partial y}=(\frac{E}{x})^{\frac{1}{2}}f'(\eta)$,

$$v_0=-\frac{\partial\Psi}{\partial x}=\frac{1}{4}(\frac{E}{x^3})^{\frac{1}{4}}\{3\eta f'(\eta)-f(\eta)\} \quad (23)$$

Where a prime denotes differentiation w.r.t. 'η', and the equation of continuity is satisfied identically.

Substituting in the equation (20), we get

$$4f'''+ff''+2f'^2=0 \quad (24)$$

and the boundary conditions are

$$\eta=0;f=0,f'=0;\quad \eta=\infty:f'=o \quad (25)$$

and integral condition

$$\int_0^\infty ff'^2\,d\eta=1 \quad (26)$$

Multiplying by f(Integrating factor) throughout and integrating the equation (24) gives,

$$4ff''-2f'^2+f^2f'=0 \quad (27)$$

Where the constant of integration is zero by using boundary condition (25).

The differential equation (27) can be linearized if we substitute $f'=\phi$, and considering the function f as the independent variable, we get $f''=\phi\frac{d\phi}{df}$ and the linearized form of the equation (27) is

$$\frac{d\phi}{df}-\frac{1}{2f}\phi=-\frac{f}{4}\ ,\quad \text{as}\quad \phi\neq 0 \quad (28)$$

The solution of (28) is given by

$$\phi=f'=C\sqrt{f}-\frac{1}{6}f^2 \quad (29)$$

Where C is arbitrary constant to be determined.

Assuming at $\eta=\infty, f=f_\infty$, then in view of boundary condition (25) we get

$$C=\frac{1}{6}f_\infty^{\frac{3}{2}} \quad (30)$$

The value of f_∞ is yet to be determined and for this we use the integral condition (26) which may be written as

$$\int_0^{f_\infty}ff'df=1 \quad (31)$$

From (29) , we get

$$\int_0^{f_\infty}f\left(C\sqrt{f}-\frac{f^2}{6}\right)df=1$$

Or, $f_\infty=40^{\frac{1}{4}}=2.515$ (32)

Now to solve the differential equation (31) we substitute

$$F = \frac{f}{f_{\infty,}} \quad (33)$$

So that it becomes

$$\frac{dF}{d\eta} = \frac{f_{\infty,}}{6}\left(\sqrt{F} - F^2\right)$$

Solving we get

$$\eta = \frac{2}{f_{\infty,}}\left(ln\frac{1+\sqrt{F}+F}{\left(1-\sqrt{F}\right)^2} + 2\sqrt{3}arctg\,\frac{\sqrt{3F}}{2+\sqrt{F}}\right) \quad (34)$$

To develop a computational algorithm with non-uniform-grid, finite difference expressions are introduced for the various terms in equations (14) and (16) as,

$$\frac{\partial W}{\partial x} = \frac{1.5\,W_j^{n+1} - 2W_j^n + 0.5\,W_j^{n-1}}{\Delta x} + o(\Delta x^2) \quad (35)$$

$$\frac{\partial W}{\partial y} = \frac{W_{j+1}^{n+1} - (1 - r_y^2)W_j^{n+1} - r_y^2\,W_{j-1}^{n+1}}{r_y\,(r_y + 1)\,\Delta y} + o(\Delta y^2) \quad (36)$$

$$\frac{\partial^2 W}{\partial y^2} = 2\,\frac{W_{j+1}^{n+1} - (1 + r_y)W_j^{n+1} + r_y\,W_{j-1}^{n+1}}{r_y\,(r_y + 1)\,\Delta y^2} + o(\Delta y^2) \quad (37)$$

$$W_j^{n+1} = 2W_j^n - W_j^{n-1} \quad (38)$$

and

$$y_{j+1} - y_j = r_y\left(y_j - y_{j-1}\right) = r_y\Delta y_j \quad (39)$$

Now the equations (16) and (17/14) reduced to

$$a_j^* W_{j-1}^{n+1} + b_j^* W_j^{n+1} + c_j^* W_{j+1}^{n+1} = d_j^* \quad (40)$$

Where W stands for either u_{p_0} or ρ_{p_0}.

2ND SET AND ITS SOLUTION

$$\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} = 0 \quad (41)$$

$$u_{p_1}\frac{\partial \rho_{p_0}}{\partial x} + \rho_{p_0}\frac{\partial u_{p_1}}{\partial x} + u_{p_0}\frac{\partial \rho_{p_1}}{\partial x} + \rho_{p_1}\frac{\partial u_{p_0}}{\partial x} + v_{p_1}\frac{\partial \rho_{p_0}}{\partial y} + \rho_{p_0}\frac{\partial v_{p_1}}{\partial y} + v_{p_0}\frac{\partial \rho_{p_1}}{\partial y} + \rho_{p_1}\frac{\partial v_{p_0}}{\partial y} = 0 \quad (42)$$

$$u_0\frac{\partial u_1}{\partial x} + u_1\frac{\partial u_0}{\partial x} + v_0\frac{\partial u_1}{\partial y} + v_1\frac{\partial u_0}{\partial y} = \frac{\partial^2 u_1}{\partial y^2} - \frac{1}{1-\varphi}\frac{FL}{U}\alpha\,\rho_{p_1}\left(u_0 - u_{p_0}\right) - \frac{1}{1-\varphi}\frac{FL}{U}\alpha\,\rho_{p_0}\left(u_1 - u_{p_1}\right) \quad (43)$$

$$u_{p_0}\frac{\partial u_{p_1}}{\partial x} + u_{p_1}\frac{\partial u_{p_0}}{\partial x} + v_{p_0}\frac{\partial u_{p_1}}{\partial y} + v_{p_1}\frac{\partial u_{p_0}}{\partial y} = \epsilon\frac{\partial^2 u_{p_1}}{\partial y^2} + \frac{FL}{U}\left(u_1 - u_{p_1}\right) \quad (44)$$

$$u_{p_0}\frac{\partial \rho_{p_1}}{\partial x} + u_{p_1}\frac{\partial \rho_{p_0}}{\partial x} + v_{p_0}\frac{\partial \rho_{p_1}}{\partial y} + v_{p_1}\frac{\partial \rho_{p_0}}{\partial y} = \epsilon\frac{\partial^2 \rho_{p_1}}{\partial y^2} \quad (45)$$

Subjected to the boundary condition

$$y = 0:\ u_1 = 0,\quad u_{p_1} = u_{p_{w1}},\ \rho_{p_0} = \rho_{p_{w1}}$$

$$y = \infty:\ u_1 = u_{p_1} = 0,\ \rho_{p_1} = 0 \quad (46)$$

and the integral condition

$$\frac{\partial}{\partial x}\int_0^\infty\left\{u_0^2\left(\int_0^y(u_1)dy\right)\right\}dy + \frac{\partial}{\partial x}\int_0^\infty\left\{2u_0u_1\left(\int_0^y(u_1)dy\right)\right\}dy + \frac{1}{1-\varphi}\frac{FL}{U}\alpha\begin{bmatrix}\int_0^\infty\left\{u_0\int_y^\infty\rho_{p_0}\left(u_1 - u_{p_1}\right)dy\right\}dy\\ +\int_0^\infty\left\{u_0\int_y^\infty\rho_{p_1}\left(u_0 - u_{p_0}\right)dy\right\}dy\\ +\int_0^\infty\left\{u_1\int_y^\infty\rho_{p_0}\left(u_0 - u_{p_0}\right)dy\right\}dy\end{bmatrix} = 0 \quad (47)$$

is identically satisfied.

Using equations (35) to (39) in (41) to (45), we get

$$v_{1j}^{n+1} = v_{1j-1}^{n+1} - 0.5\frac{\Delta y}{\Delta x}\begin{bmatrix}\left(1.5\,u_{1j}^{n+1} - 2\,u_{1j}^{n} + 0.5\,u_{1j}^{n-1}\right) + \\ \left(1.5\,u_{1j-1}^{n+1} - 2\,u_{1j-1}^{n} + 0.5\,u_{1j-1}^{n-1}\right)\end{bmatrix} \quad (48)$$

$$a_j W_{j-1}^{n+1} + b_j W_j^{n+1} + c_j W_{j+1}^{n+1} = d_j \quad (49)$$

Where W stands for either u_1 or u_{p_1} or ρ_{p_1}

SOLUTION FOR TEMPERATURE DISTRIBUTION

By taking a perturbation on the Schlichting [11] model by writing

$u = u_0 + u_1, v = v_0 + v_1, u_p = u_{p_0} + u_{p_1},$

$v_p = v_{p_0} + v_{p_1}, \rho_p = \rho_{p_0} + \rho_{p_1},$

$T = T_0 + T_1,\ T_p = T_{p_0} + T_{p_1}$

Where $u_1, v_1, u_{p_1}, v_{p_1}, \rho_{p_1}, T_1$ and T_{p_1} are perturbation quantities,

and substituting in equations (6) and (7), we get the following two sets of equations.

SET-1 AND ITS SOLUTION

$$u_0 \frac{\partial T_0}{\partial x} + v_0 \frac{\partial T_0}{\partial y} = \frac{1}{Pr}\frac{\partial^2 T_0}{\partial y^2} - \frac{2}{3}\frac{1}{1-\varphi}\frac{FL}{U}\frac{\alpha}{Pr}\rho_{p_0}\left(T_0 - T_{p_0}\right) + Ec\left(\frac{\partial u_0}{\partial y}\right)^2 + \frac{1}{1-\varphi}\frac{FL}{U}\alpha\, Ec\, \rho_{p_0}\left(u_0 - u_{p_0}\right)^2 \quad (50)$$

$$u_{p_0}\frac{\partial T_{p_0}}{\partial x} + v_{p_0}\frac{\partial T_{p_0}}{\partial y} = \frac{\epsilon}{Pr}\frac{\partial^2 T_{p_0}}{\partial y^2} + \frac{FL}{U}\left(T_0 - T_{p_0}\right) - \frac{3}{2}\frac{FL}{U}Pr Ec\left(u_0 - u_{p_0}\right)^2 + \frac{3}{2}Pr.Ec\,\epsilon\left[u_{p_0}\frac{\partial^2 u_{p_0}}{\partial y^2} + \left(\frac{\partial u_{p_0}}{\partial y}\right)^2\right] \quad (51)$$

Subjected to the boundary condition

$y = 0:\ T_0 = 0,\ \ T_{p_0} = T_{p_{w0}}$

$$y = \infty:\ T_0 = 0,\ T_{p_0} = 0 \quad (52)$$

and the integral condition

$$\frac{\partial}{\partial x}\int_0^\infty \{u_0 T_0 \int_0^y u_0\, dy\} dy = \frac{1}{1-\varphi}\frac{2\,\alpha}{3\,Pr}\frac{FL}{U}\int_0^\infty\left\{u_0\int_y^\infty \rho_{p_0}\left(T_{p_0} - T_0\right)dy\right\}dy + \frac{1}{1-\varphi}\frac{FL}{U}\alpha\, Ec\int_0^\infty\left\{u_0\int_y^\infty \rho_{p_0}\left(u_0 - u_{p_0}\right)^2 dy\right\}dy + Ec\int_0^\infty\left\{u_0\int_y^\infty\left(\frac{\partial u_0}{\partial y}\right)^2 dy\right\}dy \quad (53)$$

Since we are considering the case of a dilute suspension of particles, the temperature distribution in the fluid is not significantly affected by the presence of the particles. Therefore the 2nd and 4th term in the R.H.S. of equation (50), and 1st and 2nd term of R.H.S. in equation (53) are dropped.

Hence equation (50) and (53) reduces to,

$$u_0\frac{\partial T_0}{\partial x} + v_0\frac{\partial T_0}{\partial y} = \frac{1}{Pr}\frac{\partial^2 T_0}{\partial y^2} + Ec\left(\frac{\partial u_0}{\partial y}\right)^2 \quad (54)$$

and

$$\frac{\partial}{\partial x}\int_0^\infty \{u_0 T_0 \int_0^y u_0\, dy\} dy = Ec\int_0^\infty\left\{u_0\int_y^\infty\left(\frac{\partial u_0}{\partial y}\right)^2 dy\right\}dy \quad (55)$$

The equation (54) is a linear differential equation in T_0. So we can solve the equation by the principle of superposition of solutions T_{00} and T_{01} such that T_{00} is the solution of the equation

$$u_0\frac{\partial T_{00}}{\partial x} + v_0\frac{\partial T_{00}}{\partial y} = \frac{1}{Pr}\frac{\partial^2 T_{00}}{\partial y^2} \quad (56)$$

and T_{01} is the solution of the equation

$$u_0\frac{\partial T_{01}}{\partial x} + v_0\frac{\partial T_{01}}{\partial y} = \frac{1}{Pr}\frac{\partial^2 T_{01}}{\partial y^2} + Ec\left(\frac{\partial u_0}{\partial y}\right)^2 \quad (57)$$

So that $T_0 = T_{00} + T_{01}$ (58)

T_{00} and T_{01} satisfies the boundary and the integral conditions are given by

$y = 0: T_{00} = 0,\ T_{01} = 0$

$$y = \infty: T_{00} = 0,\ T_{01} = 0 \quad (59)$$

and

$$\int_0^\infty T_{00} u_0 \left(\int_0^y u_0 dy\right) dy = const = I \ (say) \qquad (60)$$

Further,

$$\frac{\partial}{\partial x}\int_0^\infty T_{01} u_0 \left(\int_0^y u_0 dy\right) dy$$

$$= Ec\int_0^\infty \left\{u_0 \int_y^\infty \left(\frac{\partial u_0}{\partial y}\right)^2 dy\right\} dy \qquad (61)$$

is identically satisfied.

It implies the constancy of the product of volume and heatflux, for a given prandtl number, through any cross-section of the boundary layer perpendicular to the wall.

For arbitrary value of Pr, it is assumed that

$$T_{00} = \frac{l}{E}\left(\frac{E}{x}\right)^{1/2} h(\eta) \qquad (62)$$

Substituting u_0 and v_0 from(23) and T_{00} from (62) in equation (57), the function $h(\eta)$ satusfies the differential equation

$$4h'' + Pr(fh' + 2f'h) = 0 \qquad (63)$$

With the boundary conditions

$$\eta=0;\ h=0; \qquad \eta\to\infty, h = 0 \qquad (64)$$

and the Integral condition

$$\int_0^\infty hff' \, d\eta = 1 \qquad (65)$$

Introducing the transformation $s = [F(n)]^{3/2}$ and $H(s) = \frac{2}{3} f_\infty^2\, h(\eta)$ in equation (63).

The transformed equation is

$$s(1-s)\frac{d^2H}{ds^2} + \left\{\frac{2}{3} - \left(\frac{2}{3} - Pr + 1\right)s\right\}\frac{dH}{ds} + \frac{4}{3}PrH = 0 \qquad (66)$$

With the boundary condition

$$s = 0: H = 0\ ;\ s \to 1: H = 0 \qquad (67)$$

and the integral condition

$$\int_0^1 Hs^{1/3}\, ds = 1 \qquad (68)$$

Equation (66) is a hyper-geometric equation, whose solution is given by

$$H(s) = A\ {}_2F_1(a,b,c;s) + $$

$$+Bs^{1/3}\ {}_2F_1(a-c+1, b-c+1; 2-c; s) \qquad (69)$$

Where

$$a + b = \frac{2}{3} - Pr, ab = -\frac{4}{3}\mathrm{Pr},\ \text{and}\quad c = \frac{2}{3} \qquad (70)$$

and

$${}_2F_1(a,b,c;s) = \sum_{r=0}^{\infty}\frac{a_r b_r}{c_r}\cdot\frac{s^r}{r!} = \frac{a_0 b_0}{c_0}\frac{s^0}{0!} + \frac{a_1 b_1}{c_1}\frac{s^1}{1!} + \cdots \ldots \ldots$$

Since the prandtl number Pr of a fluid is always +ve integer the series is absolutely converges.

Now by boundary condition (67), for $s = 0$, A= 0 (71)

and $$H(s)=Bs^{1/3}\ {}_2F_1(a-c+1, b-c+1; 2-c; s) \qquad (72)$$

In (72) 'B' is still an unknown constant which will be determined by the integral condition (68) which is given by

$$B = \frac{5}{3}\left\{\ {}_3F_2\left(a-c+1, b-c+1, \frac{5}{3}, 2-c, \frac{8}{3}; 1\right)\right\}^{-1} \qquad (73)$$

The solution of the equation (57) is solved by using finite difference technique.

Using equations (35) to (39) in equation (57), we get

$$a_j^{\blacksquare\blacksquare} W_{j-1}^{n+1} + b_j^{\blacksquare\blacksquare} W_j^{n+1} + c_j^{\blacksquare\blacksquare} W_{j+1}^{n+1} = d_j^{\blacksquare\blacksquare} \qquad (74)$$

Where W stands for T_{01}

2ND SET AND ITS SOLUTION

$$u_0\frac{\partial T_1}{\partial x}+u_1\frac{\partial T_0}{\partial x}+v_0\frac{\partial T_1}{\partial y}+v_1\frac{\partial T_0}{\partial y}=\frac{1}{Pr}\frac{\partial^2 T}{\partial y^2}$$
$$+\frac{2}{3}\frac{\alpha}{1-\varphi}\frac{1}{Pr}\frac{FL}{U}\{\rho_{p0}(T_{p_1}-T_1)+\rho_{p1}(T_{p_0}-T_0)\}$$
$$+2Ec\frac{\partial u_0}{\partial y}\frac{\partial u_1}{\partial y}$$
$$+\frac{\alpha}{1-\varphi}\frac{FL}{U}Ec\begin{Bmatrix}\rho_{p_1}(u_0-u_{p_0})^2\\ +2\rho_{p_0}(u_0-u_{p_0})(u_1-u_{p_1})\end{Bmatrix} \quad (75)$$

$$u_{p_0}\frac{\partial T_{p_1}}{\partial x}+u_{p_1}\frac{\partial T_{p_0}}{\partial x}+v_{p0}\frac{\partial T_{p_1}}{\partial y}+v_{p_1}\frac{\partial T_{p_0}}{\partial y}$$
$$=\frac{\epsilon}{Pr}\frac{\partial^2 T_{p_1}}{\partial y^2}-\frac{FL}{U}(T_{p_1}-T_1)$$
$$+\frac{3}{2}\mathrm{Pr}\,Ec\,\epsilon.\left(u_{p_0}\frac{\partial^2 u_{p_1}}{\partial y^2}+u_{p_1}\frac{\partial^2 u_{p_0}}{\partial y^2}+2\frac{\partial u_{p0}}{\partial y}\frac{\partial u_{p_1}}{\partial y}\right)$$
$$-3\frac{FL}{U}PrEc\,(u_0-u_{p_0})(u_1-u_{p_1}) \quad (76)$$

Using equations (35) to (39) in equation (75) and (76), we get

$$a_j^{++}W_{j-1}^{n+1}+b_j^{++}W_j^{n+1}+c_j^{++}W_{j+1}^{n+1}=d_j^{++} \quad (77)$$

Where W stands for T_1 , T_{p1}

DISCUSSION

We choose the following parameters involved

ρ = 0.913 kg/m^3; ρ_p = 8010 kg/m^3; α = 0.1; D = 50μm, 100μm ; U = 0.45 m/ sec; L = 0.044 m; Ec = 0.1 ; Pr = 0.71, 1.0, 7.0; $\mu = 22.26\times10^{-6}$ kg/m sec; $\nu = 2.43\times10^{-5}\ m^2/sec$

Fig.-1 shows the velocity distribution u_0 ,is plotted against y. The velocity distribution near the plate is of Blasius type and away from it resembles with the distribution of plane free jet.

Fig.-2 & Fig.-3 depicts velocity profile u_1 without and with viscous heating respectively for the presence of particles of above micron size. The conclusion is that the nature of the curve is of Balsius type near the plate and away it resembles like the distribution of plane free jet. The magnitude in both the cases is same .But inclusion of viscous heating decrease the magnitude of u_1.

Fig.-4 shows the distribution of the perturbed velocity u_1 for submicron size particles. Here also the magnitude of u_1 is less in case of viscous heating. The Pattern of u_1 near the plate is of Balsius type and away is like that of free jet.

Fig.5 & Fig.6 – Shows the velocity pattern of particle velocity u_{p_1} incase of without and with viscous heat respectively in the presence of particles of above micron size. Magnitude of u_{p_1} is greater when coarser particles are present. The velocity pattern is of Blasius type near the plate and away is that of free jet.

Fig.- 7 Demonstrates the pattern of velocity u_{p_1} for submicron Particles with and without viscous heating. The magnitude of u_{p_1} is less incase of Viscous heating is considered. Here also the pattern is of Blasius type near the plate and of free jet type at away from plate.

Fig.-8. Shows the pattern of fluid phase temperature T_0 is of Balasius type near the plate and like free jet at away from plate.

Fig.-9 depicts the particle phase temperature T_{p_0} for various material densities of the particles. The temperature is more in case of particles with less material density and here also the temperature distribution is of Blasius type.

Fig.-10 Shows the temperature distribution T_1 for with and without viscous heating and the pattern is of Balasius type near the plate and of free type away from the plate.

To show the heat transfer in the wall jet, the nusselts number is calculated for different values of the parameter. The Value of Nu is given in Table-1 to Table-5. Here Nu $=Nu_0 + Nu_1$, where Nu_0 is the nusselt number when the carrier fluid is not affected by the presence of particle and Nu_1 is calculated based on perturbation temperature T_1 . In table-1 values of Nu_0 for initial heating and viscous heating for different values of Pr. Similarly Fig-2 shows the dependence of Nu_1 on Pr. From Table-1 and Table-2 , it can be observed that Nu_0 and Nu_1 is increasing in both for initial heating and for viscous heating. Further it is observed that Nu_0 and Nu_1 increases when x increases i.e in the down stream direction of the plate. In the Table -3, the dependence of Nu_1 on Pr for initial heating as well as viscous heating is shown. In this case also Nu_1 increases with the increase of size of the particle. Table-4 shows the dependence of Nu_1 on ρ_s i.e material density of the particle. Here also Nu_1 increases with the ρ_s. .Table -5 shows the dependence of Nu_1 with the diffusion parameter ϵ for initial heating and viscous heating. It is observed that Nu_1 increases with the increase of ϵ . From numerical calculation, it has been observed that dependence of Nu_0 on the size of the particles, material density of particles and diffusion parameter is negligible.

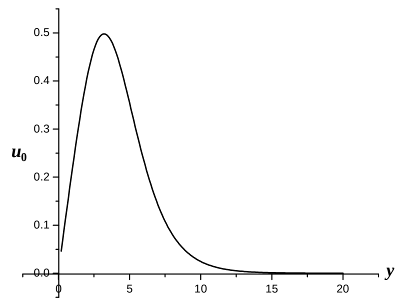

Fig. 1: Velocity distribution (u_0) with y

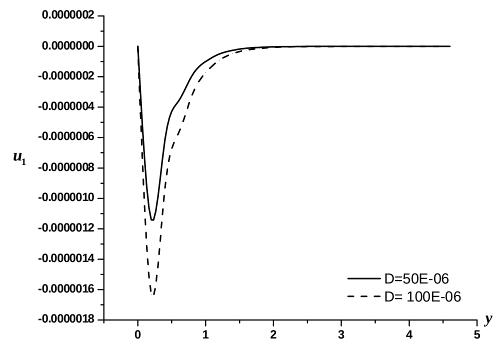

Fig. 2: Variation of u_1 with y for different size of the particles(Initial Heating)

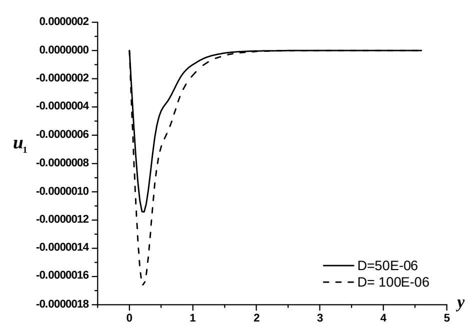

Fig. 3: Variation of u_1 with y for different size of the particles (Viscous Heating)

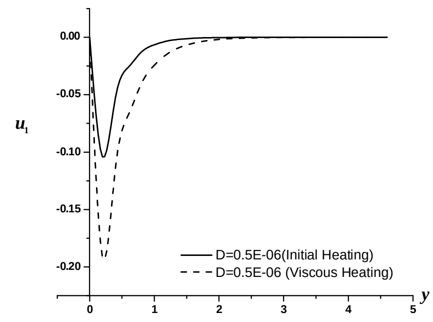

Fig. 4: Variation of $\boldsymbol{u_1}$ with $\boldsymbol{y}$ for sub-micron particles

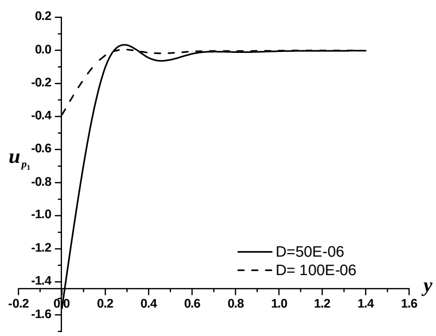

Fig. 5: Variation of $\boldsymbol{u_{p_1}}$ with $\boldsymbol{y}$ for different size of the particles (Initial Heating)

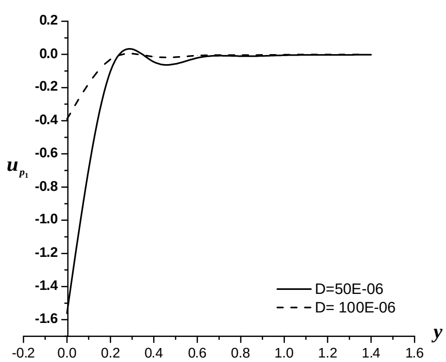

Fig. 6: Variation of $\boldsymbol{u_{p_1}}$ with $\boldsymbol{y}$ for different size of the particles (Viscous Heating)

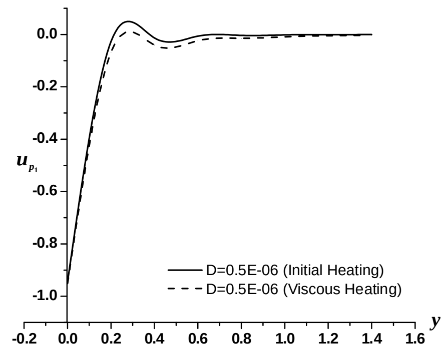

Fig. 7: Variation of $\boldsymbol{u_{p_1}}$ with $\boldsymbol{y}$ for sub-micron particles

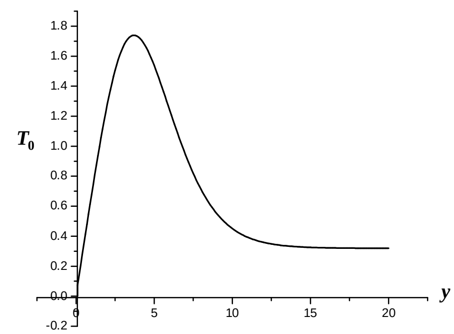

Fig. 8: Variation of T_0 with y

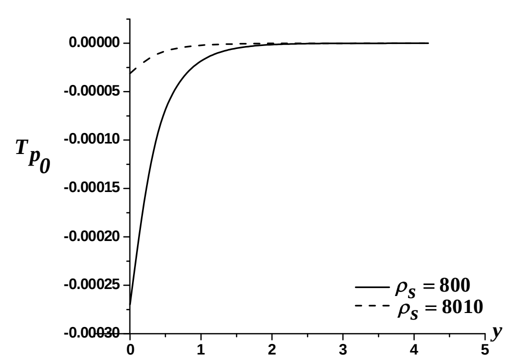

Fig. 9: Variation of T_{p_0} with y for different material density of particles (For Viscous Heating)

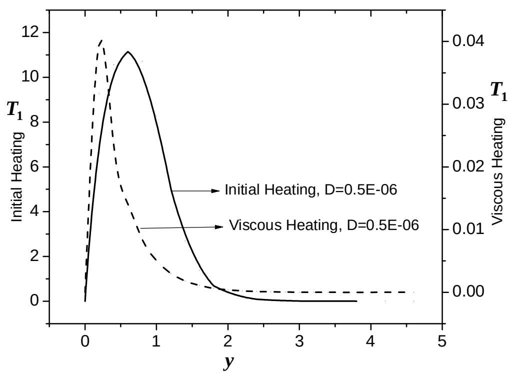

Fig.10: Variation of T_1 with y for submicron particles

Table 1: Variation of Nu_0 with x for different Prandtl Number (Pr)

	Initial Heating			Viscous Heating		
x	Pr= 0.71	Pr=1.0	Pr=7.0	Pr= 0.71	Pr=1.0	Pr=7.0
1.20	-1.44E+02	-2.02E+02	-6.02E+03	-1.44E+02	-2.02E+02	-6.02E+03
1.40	-1.34E+02	-1.87E+02	-5.57E+03	-1.34E+02	-1.87E+02	-5.57E+03
1.60	-1.25E+02	-1.75E+02	-5.21E+03	-1.25E+02	-1.75E+02	-5.21E+03
1.80	-1.18E+02	-1.65E+02	-4.91E+03	-1.18E+02	-1.65E+02	-4.91E+03
2.00	-1.12E+02	-1.56E+02	-4.66E+03	-1.12E+02	-1.56E+02	-4.66E+03
2.40	-1.02E+02	-1.43E+02	-4.25E+03	-1.02E+02	-1.43E+02	-4.25E+03
2.80	-9.44E+01	-1.32E+02	-3.94E+03	-9.44E+01	-1.32E+02	-3.94E+03
3.20	-8.83E+01	-1.24E+02	-3.68E+03	-8.83E+01	-1.24E+02	-3.68E+03
3.60	-8.33E+01	-1.16E+02	-3.47E+03	-8.33E+01	-1.16E+02	-3.47E+03
4.00	-7.90E+01	-1.11E+02	-3.30E+03	-7.90E+01	-1.11E+02	-3.30E+03
4.40	-7.53E+01	-1.05E+02	-3.14E+03	-7.53E+01	-1.05E+02	-3.14E+03
4.80	-7.21E+01	-1.01E+02	-3.01E+03	-7.21E+01	-1.01E+02	-3.01E+03
5.00	-7.07E+01	-9.88E+01	-2.95E+03	-7.07E+01	-9.88E+01	-2.95E+03

Table 2: Variation of Nu_1 with x for different Prandtl Number (Pr)

	Initial Heating			Viscous Heating		
x	Pr = 0.71	Pr = 1.0	Pr = 7.0	Pr = 0.71	Pr = 1.0	Pr = 7.0

1.20	1.18E+12	1.18E+12	1.18E+12	1.89E+05	1.79E+05	2.51E+04
1.40	3.04E+13	2.47E+13	7.43E+12	4.22E+05	4.22E+05	5.62E+05
1.60	2.02E+14	1.46E+14	3.83E+13	1.21E+06	1.23E+06	6.75E+05
1.80	4.49E+14	3.94E+14	1.61E+14	9.49E+05	9.59E+05	2.23E+06
2.00	1.57E+14	1.37E+14	5.68E+13	6.95E+05	6.79E+05	3.90E+06
2.40	8.98E+11	7.78E+11	2.46E+11	1.13E+05	1.49E+05	1.63E+06
2.80	1.03E+10	8.48E+09	3.51E+09	7.94E+03	6.93E+03	3.36E+05
3.20	8.41E+09	7.09E+09	2.69E+09	7.20E+04	9.11E+04	4.53E+05
3.60	4.79E+09	4.04E+09	1.53E+09	8.92E+04	1.14E+05	5.86E+05
4.00	2.86E+09	2.41E+09	9.17E+08	1.03E+05	1.32E+05	6.75E+05
4.40	1.78E+09	1.50E+09	5.69E+08	1.18E+05	1.50E+05	7.68E+05
4.80	1.13E+09	9.55E+08	3.63E+08	1.32E+05	1.69E+05	8.61E+05
5.00	9.12E+08	7.69E+08	2.92E+08	1.39E+05	1.78E+05	9.07E+05

Table 3: Variation of Nu_1 with x for different size of the particles

x	Initial Heating			Viscous Heating		
	D=0.5E-06	D=50E-06	D= 100E-06	D=0.5E-06	D=50E-06	D= 100E-06
1.20	-1.18E+12	1.04E+05	1.04E+04	-1.89E+05	-4.40E+05	-4.42E+04
1.40	3.04E+13	-7.14E+04	-5.11E+08	-4.22E+05	-1.05E+04	-8.50E+06
1.60	-2.02E+14	-3.93E+04	-2.43E+08	-1.21E+06	-2.25E+03	4.46E+06
1.80	4.49E+14	-1.61E+03	-1.75E+05	-9.49E+05	-1.41E+03	5.44E+03
2.00	-1.57E+14	-4.79E+02	-3.01E+03	-6.95E+05	1.57E+03	-7.67E+02
2.40	8.98E+11	-2.18E+06	-6.90E+03	1.13E+05	-5.64E+04	-3.65E+02
2.80	-1.03E+10	1.78E+04	-1.72E+02	7.94E+03	-4.11E+03	-5.54E+02
3.20	-8.41E+09	-1.08E+03	-5.65E+02	7.20E+04	-1.08E+03	-5.46E+02
3.60	-4.79E+09	-1.09E+03	-5.31E+02	8.92E+04	-1.12E+03	-3.47E+02
4.00	-2.86E+09	-1.12E+03	-5.26E+02	1.03E+05	-1.11E+03	-5.37E+02
4.40	-1.78E+09	-1.11E+03	-5.23E+02	1.18E+05	-1.11E+03	-5.34E+02
4.80	-1.13E+09	-1.10E+03	-5.20E+02	1.32E+05	-1.11E+03	-5.31E+02
5.00	-9.12E+08	-1.10E+03	-5.18E+02	1.39E+05	-1.11E+03	-5.29E+02

Table 4: Variation of Nu_1 with x for different material density

	Initial Heating		Viscous Heating	
	ρ_s=800	ρ_s=8010	ρ_s=800	ρ_s=8010
1.20	-1.18E+12	2.40E+13	-1.89E+05	-2.59E+04
1.40	3.04E+13	-4.68E+13	-4.22E+05	-3.61E+04
1.60	-2.02E+14	-5.51E+14	-1.21E+06	3.17E+04
1.80	4.49E+14	-5.30E+14	-9.49E+05	-1.03E+04
2.00	-1.57E+14	-5.88E+13	-6.95E+05	2.33E+03
2.40	8.98E+11	-5.19E+10	1.13E+05	9.14E+03
2.80	-1.03E+10	-1.06E+11	7.94E+03	6.92E+03
3.20	-8.41E+09	-5.45E+10	7.20E+04	8.73E+03
3.60	-4.79E+09	-2.81E+10	8.92E+04	1.03E+04
4.00	-2.86E+09	-1.58E+10	1.03E+05	1.18E+04
4.40	-1.78E+09	-9.54E+09	1.18E+05	1.34E+04
4.80	-1.13E+09	-6.01E+09	1.32E+05	1.51E+04
5.00	-9.12E+08	-4.84E+09	1.39E+05	1.59E+04

Table 5: Variation of Nu_1 with x for different diffusion parameter (ϵ)

	Initial Heating		Viscous Heating	
x	ϵ =0.05	ϵ =0.1	ϵ =0.05	ϵ =0.1
1.20	-1.18E+12	1.19E+08	-1.89E+05	-1.18E+06
1.40	3.04E+13	2.64E+11	-4.22E+05	-8.04E+04
1.60	-2.02E+14	-3.86E+12	-1.21E+06	-2.86E+05
1.80	4.49E+14	-2.18E+11	-9.49E+05	-3.59E+05
2.00	-1.57E+14	-7.58E+12	-6.95E+05	-2.00E+05
2.40	8.98E+11	-2.50E+09	1.13E+05	3.98E+05
2.80	-1.03E+10	4.58E+06	7.94E+03	1.16E+05
3.20	-8.41E+09	1.55E+06	7.20E+04	2.84E+05
3.60	-4.79E+09	6.72E+05	8.92E+04	3.48E+05
4.00	-2.86E+09	3.06E+05	1.03E+05	4.04E+05
4.40	-1.78E+09	1.32E+05	1.18E+05	4.61E+05
4.80	-1.13E+09	4.42E+04	1.32E+05	5.19E+05
5.00	-9.12E+08	1.71E+04	1.39E+05	5.47E+05

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