Optimization Speed Control of DC Separately Excited Motor Using Tuning Controller of Linear Quadratic Regulator (LQR) Technique

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Abstract— **The tuning of Linear Quadratic regulator (LQR) controllers is a challenge for researchers and plant operators. This paper presents a optimization and comparison of time response specification between Traditional ZN Tuning & Modified ZN Tuning controllers with Linear Quadratic Regulator (LQR) for a speed control of a separately excited DC motor. The goal is to determine which control strategy delivers better performance with respect to DC motor's speed. Both these method are compared on the basis of output response, less rise time, less setting time and less over shoot for speed demand of DC motor.**

Keywords— **Separately excited DC motor, Linear-Quadratic Regulator (LQR), Traditional ZN Tuning , Modified ZN Tuning, Optimization.**

I. Introduction

DC motor are widely used in applications requiring adjustable speed, good speed regulations and frequent starting, braking and reversing. Due to its excellent speed control characteristics, the DC motor has been widely used in industry even though its maintenance costs are higher than the induction motor [1]. As a result, Speed control of DC motor has attracted considerable research and several methods have evolved. Some important applications are rolling mills, paper mills, mine winders, hoists, machine tools, traction, printing presses, textile mills, excavators and cranes. Although, it is being predicted that AC drives will replace DC drives, however, even today the variable speed applications are dominated by DC drives because of lower cost, reliability and simple control. As per the control of DC motor, there is lot of methods to control the speed control of motor. Separately excited dc motors have industrial applications. They are often used as actuators. This type of motors is used in trains and for automatic traction purposes. The purpose of a motor speed controller is to take a signal representing the demanded speed and to drive a motor at that speed [2]. Several approaches have been documented in literatures for determining the PID parameters of such controllers which is first found by Ziegler- Nichols tuning [3]. Genetic Algorithm, neural network, fuzzy based approach [4, 5], particle swarm optimization techniques [6] are just a few among these numerous works. In 1942, Ziegler-Nichols presented a tuning formula [7, 8].

The other type of control methods can be developed such as Linear- Quadratic Regulator (LQR) optimal control, linear quadratic regulator design technique is well known in modern optimal control theory and has been widely used in many applications. It has a very nice robustness property. It is attractive property appeals to the practicing engineers. Thus, the linear quadratic regulator theory has received considerable attention since 1950s. The liner quadratic regulator technique seeks to find the optimal controller that minimizes a given cost function (performance index). This cost function is parameterized by two matrices, Q and R, that weight the state vector and the system input respectively. These weighting matrices regulate the penalties on the excursion of state variables and control signal. One practical method is to Q and R to be diagonal matrix. The value of the elements in Q and R is related to its contribution to the cost function. To find the control law, Algebraic Riccati Equation (ARE) is first solved, and an optimal feedback gain matrix, which will lead to optimal results evaluating from the defined cost function is obtained [9-13].

In this paper a new method of optimal speed control of dc separately excited motor by using of Linear Quadratic Regulator (LQR) technique. The results of this method compared with traditional ZN tuning and modified ZN tuning method .The LQR controller which applied to control the speed of DC separately excited motor. The rest of the paper is presented at first the dc separately excited motor mathematical model is described. The next section describes and designs the LQR technique.

II.DC Separately Excited Motor Math Mathematical Model

As reference we consider a DC separately excited motor as is shown in figure 1. A separately excited DC motor. This paper focuses on the study of DC motor linear speed control, therefore, the separately excited DC motor is adopted. Make use of the armature voltage control method to control the DC motor velocity, the armature voltage controls the distinguishing feature of method as the flux fixed, is also a field current fixedly [8,9,15].

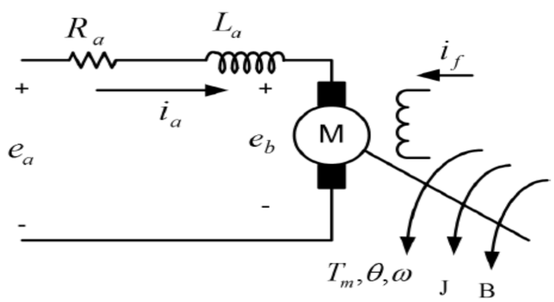

Fig 1 DC separately excited motor [8]

Assuming constant field excitation the armature circuit electrical equation is written (1) ,(2)and (3).

$$e_a = R_a i_a + L_a \frac{di_a}{dt} + e_b \ldots\ldots\ldots\ldots\ldots(1)$$

$$e_a = R_a i_a + L_a \frac{di_a}{dt} + K_b\ \omega \ldots\ldots\ldots(2)$$

Equation (2) and equation (3) are rearranged to obtain the Equation (4) and (5)

$$Tm = K_T i_a = J\frac{d\omega}{dt} + B\omega \ldots\ldots\ldots\ldots(3)$$

$$\frac{d\,i_a}{dt} = -\frac{R_a}{L_a} i_a - \frac{K_b}{L_a}\omega + \frac{e_a}{L_a} \ldots\ldots(4)$$

$$\frac{d\omega}{dt} = \frac{K_T}{J} i_a - \frac{B}{J}\omega \ldots\ldots\ldots\ldots(5)$$

In the state space model of a separately excited DC motor, Equations (4) and (5) can be expressed by choosing the angular speed (ω) and armature current (ia) as state variables and the armature voltage (Va) as an input. The output is chosen to be the angular speed [9,15].

$$\begin{bmatrix} \frac{di_a}{dt} \\ \frac{d\omega}{dt} \end{bmatrix} = \begin{bmatrix} \frac{-R_a}{L_a} & \frac{-K_b}{L_a} \\ \frac{K_T}{J} & \frac{-B}{J} \end{bmatrix} \begin{bmatrix} i_a \\ \omega \end{bmatrix} = \begin{bmatrix} \frac{1}{L_a} \\ 0 \end{bmatrix} + [e_a] \ldots\ldots \quad (6)$$

$$y = [0 \quad 1]\begin{bmatrix} i_a \\ \omega \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots(7)$$

obtaining the transfer function of the motor using the state space model by formula G(s)= C (s I - A)-1 B + D [17] in the equation (6) and (7) and obtain the equation 14.and show the Block Diagram of separately excited DC Motor in the fig 2.

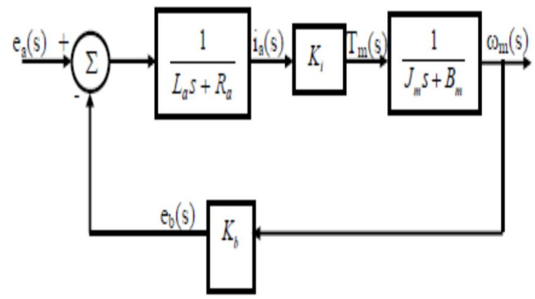

Fig. 2- Block Diagram of armature voltage control of separately excited DC Motor

$$G(s) = \frac{\omega(s)}{e_a(s)} = \frac{K_T}{(L_a(s) + R_a)(JS + B) + (K_b K_T)} \ldots(14)$$

Where

- Ra: Armature resistance in ohm
- e_a: Armature voltage in volts
- e_b: Back emf voltage in volts
- La :Armature inductance in H
- K**b** :Back emf constant in volt/ (rad/sec)
- ω: angular speed in rad/s
- □i_a: Armature current in ampere
- □J: Moment of inertia of motor and load in Kg.m2/s^2
- Tm : Motor□Torque constant in N.m
- B Viscous Frictional coefficient in Nms
- K_T :□Torque factor constant in N.m/Ampere

III DESIGN OF LINEAR QUADRATIC REGULATOR CONTROLLER

LQR is a method in modern control theory that used state-space approach to analyse such a system. Using state space methods it is relatively simple to work with Multi- Input Multi-Output

(MIMO) system. Linear quadratic regulator design technique is well known in modern optimal control theory and has been widely used in many applications, Linear-Quadratic Regulator (LQR) optimal control problems have been widely investigated in the literature. The performance measure is a quadratic function composed of state vector and control input. If the linear time-invariant system is controllable, the optimal control law will be obtained via solving the algebraic Ricci equation optimal control. The function of Linear Quadratic Regulator (LQR) is to minimize the deviation of the speed of the motor. The speed of the motor is specifying that will be the input voltage of the motor and the output will be compare with the input.

In general, the system model can be written in state space equation as follows:

$$= Ax + Bu \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(8)$$

A is the state matrix of order $n \times n$ B is the control matrix of order $n \times m$. Also, the pair (A, B) is assumed to be such that

the system is controllable. The linear quadratic regulator controller design is a method of reducing the performance index to a minimize value. The minimization of it is just the means to the end of achieving acceptable performance of the system. For the design of a linear quadratic regulator controller, the performance index (J)
is given by:

$$J = \int_0^{\infty} (x^T Q_x + u^T R_u) dt \quad \text{.............. (9)}$$

Where Q is symmetric positive semi-definite state weighting matrix of order $n \times n$ and R is symmetric positive definite control weighting matrix of order $n \times m$ The choice of the element Q and R allows the relative weighting of individual state variables and individual control inputs as well as relative weighting state vector and control vector against each other. The weighting matrices Q and R are important components of an LQR optimization process. The compositions of Q and R elements have great influences of system performance. The designer is free to choose the matrices Q and R, but the selection of matrices Q and R is normally based on an iterative procedure using experience and physical understanding of the problems involved. Commonly, a trial and error method has been used to construct the matrices Q and R elements. This method is very simple and very familiar in linear quadratic regulator application. However, it takes long time to choose the best values for matrices Q and R. The number of matrices Q and R elements are dependent on the number of state variable (n) and the number of input variable (m), respectively. The diagonal-off elements of these matrices are zero for simplicity. If diagonal matrices are selected, the quadratic performance index is simply a weighted\ integral of the squared error of the states and inputs. The term in the brackets in equation (9) above are called quadratic forms and are quite common in matrix algebra. Also, the performance index will always be a scalar quantity, whatever the size of Q and R matrices .The conventional linear quadratic regulator problem is to find the optimal control input law $u*$ that minimizes the performance index under the constraints of Q and R matrices. The closed loop optimal control law is defined as:

$$\overset{*}{u} = -Kx \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots . (10)$$

Where K is the optimal feedback gain matrix, and determines the proper placement of closed loop poles to minimize the performance index in equation (9). The feedback gain matrix K depends on the matrices A, B, Q, and R. There are two main equations which have to be calculated to achieve the feedback gain matrix K. Where P is a symmetric and positive definite matrix obtained by solution of the ARE is
defined as:

$$A^T P + PA - PBR^{-1}B^T \quad P + Q = 0 \ldots\ldots\ldots\ldots (11)$$

Then the feedback gain matrix K is given by:

$$K = R^{-1} B^T P \ldots\ldots\ldots\ldots\ldots\ldots\ldots . . (12)$$

Substituting the above equation (10) into
Equation (8) gives:

$$\dot{x} = AX - BKx = (A - BK)x \quad \ldots\ldots\ldots..(13)$$

If the eigen values of the matrix (A-BK) have negative real parts, such a positive definite solution Always exits [18,19,20].

IV Analysis of Result of dc motor

Analysis of Result of dc motor, The performance of linear quadratic regulator controller has been investigated and compared with the traditional ZN tuning method and Modified ZN Tuning. The parameters of dc separately excited motor show in table 1.

Table 1 separately excited Dc motor parameters [9]

S.no	Parameters of DC motor	Motor 1	Motor 2	Motor 3
1	Armature Resistance Ra in ohm	2	2	1
2	Armature Inductance La (H)	0.5	0.5	0.5
3	Moment of Inertia J (Kgm2)	0.02	1.2	0.01
4	Friction constant B (Nms)	0.2	0.2	0.00003
5	Torque constant KT (Nm/A)	0.015	0.2	0.023
6	EMF constant KB (Vs/rad)	0.01	0.2	0.023

Show the transfer functions for the separately excited motor 1

$$G1(s) = \frac{0.015}{0.01\, S^2 + 0.14S + 0.40015}$$

Determine the optimal speed control using LQR method. The better result show in the table 2 and Fig. 3 by compare the Traditional ZN Tuning and Modified ZN Tuning so the better result rise time, better setting and better over shoot in obtaining by Linear-Quadratic Regulator (LQR) technique ,the tuning of LQR parameter Q and R : Q=[0.0005;0.2] and R=[0.0000003].

Table 2 Comparison Of Parameter Traditional Zn, Modified Zn Tuning And LQR Technique

Methods	Rise time (Tr)	Setting time (Ts)	Maximum overshoot (%)
Traditional ZN Tuning [8]	0.312	2.27	27.9
Modified ZN Tuning[8]	0.074	0.439	14.5
LQR Technique	**0.0708**	**0.108**	**1.3**

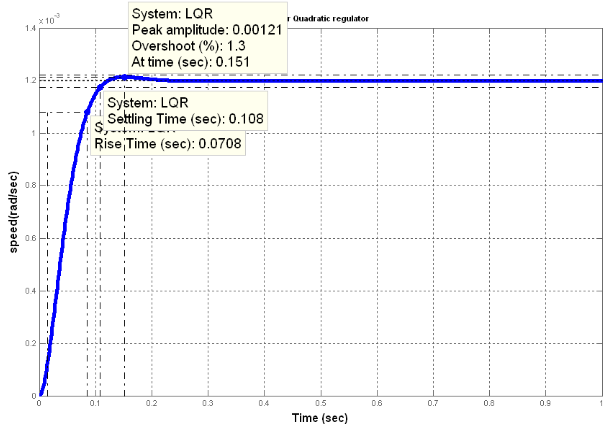

Fig 3 Speed Response of Linear Quadratic Regulator Controller

In this case using the transfer function of motor 2 compare Traditional ZN Tuning and Modified ZN Tuning show in the table 3 and fig. 4 so the better result rise time 0.307, better setting 0.555 and better over shoot only 0 by obtaining in Linear-Quadratic Regulator (LQR) technique ,the tuning of LQR parameter Q and R : Q=[0.0005;0.1] and R=[0.0000003].

Transfer function for separately excited motor 2

$$G2(s) = \frac{0.2}{0.1\,S^2 + 2.5S + 0.44}$$

Table 3 Comparison Of Parameter Traditional ZN, Modified ZN Tuning And LQR Technique

Methods	Rise time (Tr)	Setting time (Ts)	Maximum overshoot (%)
Traditional ZN Tuning [8]	0.377	5.68	51.8
Modified ZN Tuning[8]	0.359	1.07	1.07
LQR Technique	0.307	0.555	0

the transfer function of motor 3 compare Traditional ZN Tuning and Modified ZN Tuning show fig 5 in the table 4 and fig. 5 .so the better result minimum rise time 0.0347, less setting time 0.0617 and better over shoot only 0.02 by obtaining in Linear-Quadratic Regulator (LQR) technique, the tuning of LQR parameter Q and R : Q=[0.02;0.02] AND R=[0.00002]. So finally tuning controller LQR is better then other technique for speed control of motor.

Transfer function for separately excited motor 3

$$G3(s) = \frac{0.023}{0.005\,S^2 + 0.010015S + 0.000559}$$

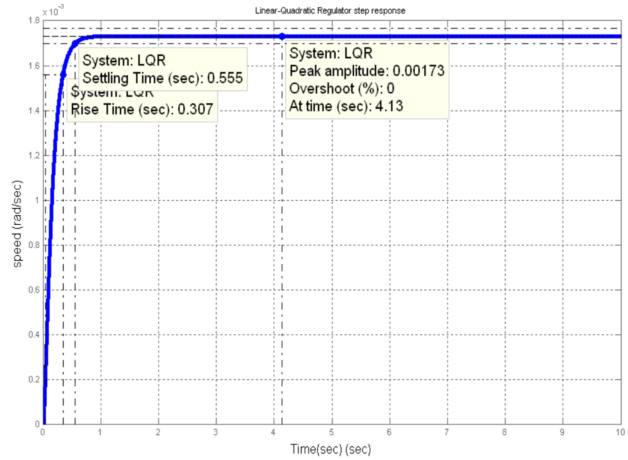

Fig 4 Speed Response of Linear Quadratic Regulator Controller

Table 4 Comparison Of Parameter Traditional ZN, Modified ZN Tuning And LQR Technique

Methods	Rise time (Tr)	Setting time (Ts)	Maximum overshoot (%)
Traditional ZN Tuning [8]	0.0508	0.527	5.84
Modified ZN Tuning[8]	0.786	2.33	6.99
LQR Technique	0.0347	0.0647	0.02

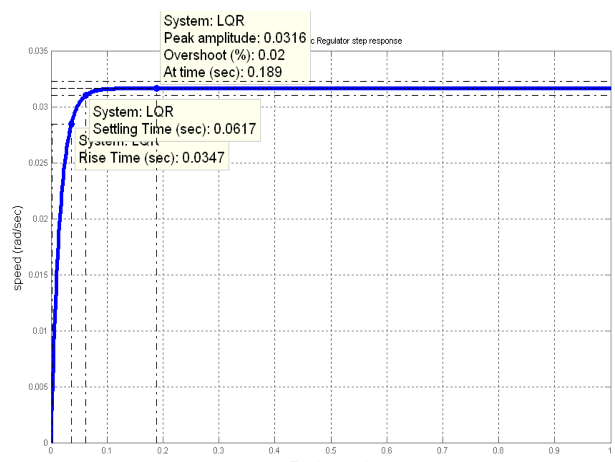

Fig 5 Speed Response of Linear Quadratic Regulator Controller

V Conclusions

DC separately excited motor the optimization of speed control and comparison between the speed control of the separately excited DC motor by linear quadratic regulator technique and Traditional ZN Tuning or Modified ZN Tuning controller shows clearly that the linear quadratic regulator technique gives better performances than Ziegler-Nichols(ZN) controller against parameter variations, the results so obtained show that the Tuning controller LQR controller gives greatest value . We observe overshoot and settling time and rise time and final value are improved in proposed controller of LQR. The separately excited DC motor with a rapid settling time, no overshoot, and zero steady state error.

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