Dynamic d-q Model of Induction Motor Using Simulink

Anand Bellure[#1], Dr. M.S Aspalli[#2],

[#1,2] Electrical and Electronics Engineering Department, Poojya Doddappa Appa College of Engineering, Gulbarga, India

Abstract-*This paper describes a dynamic d-q model of a three phase induction motor in state space form and its computer simulation in MATLAB/SIMULINK. The details on the construction of sub models for the induction motor are given and their implementation in SIMULINK delineated. The required equations are stated at the beginning and then a d-q model of induction motor is developed. This plan could be led to other engineering systems.*

Keywords - *Induction Motors, Torque, Speed.*

I. INTRODUCTION

In an adjustable speed drive, the transient behaviour of induction motor has to be taken into consideration. In addition, high performance drive control such as vector or field oriented control is based on dynamic d-q model of the induction motor. The dynamic model in state space form is important for transient analysis of induction motor particularly for computer simulation study in MATLAB/SIMULINK. Thus the SIMULINK software for dynamic modelling of induction motor is used here. The motor can be modelled in rotating reference frame and stationary frame. The electrical variables in the model can be chosen as fluxes, currents, or a mixture of both. In this paper, state space equations of the machine in rotating frame are considered with flux linkages as the main variables. Since SIMULINK is a model operation programmer, the simulation model can be easily developed by addition of new sub models to cater for various control functions. As a sub model, the induction motor could be incorporated in a complete electric motor drive system.

II. ANALYSIS

Three phase voltages supplied to the motor are as follows,

$$V_{as} = V_m \sin(\omega_e t) \quad (1)$$

$$V_{bs} = V_m \sin\left(\omega_e t - \frac{2\pi}{3}\right) \quad (2)$$

$$V_{cs} = V_m \sin\left(\omega_e t + \frac{2\pi}{3}\right) \quad (3)$$

Where, V_m is the amplitude of terminal voltage, ω_e is the supply frequency.
To develop dynamic model of induction motor, the three phase to two axis transformation is needed.

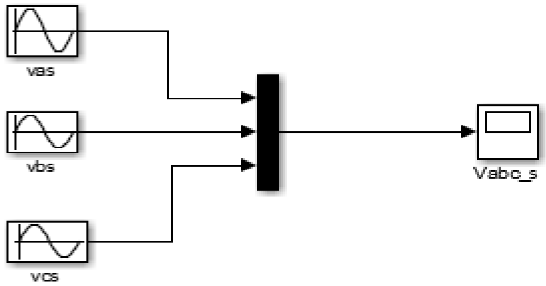

Fig.1.Model for power supply (three phase stator voltages)

This is done as follows.

A three phase stationary reference frame variables V_{as},V_{bs} , V_{cs}) are transformed into two phase stationary reference frame variables ($V_{qs}{}^s$ and $V_{ds}{}^s$). This is done by following equations.

$$V_{qs}{}^s = V_{as} \quad (4)$$

$$V_{ds}{}^s = -\frac{1}{\sqrt{3}}V_{bs} + \frac{1}{\sqrt{3}}V_{cs} \quad (5)$$

Fig.2 shows sub model to transform (V_{as} , V_{bs} , V_{cs}) to ($V_{qs}{}^s$,$V_{ds}{}^s$).

Then the two phase stationary reference frame variables ($V_{qs}{}^s$,$V_{ds}{}^s$) are transformed into two phase synchronously rotating frame variables (V_{qs} and V_{ds}).

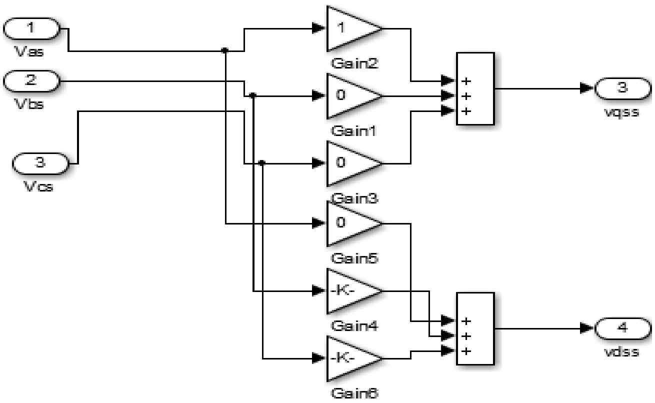

Fig.2.V_{as},V_{bs},V_{cs} to ($V_{qs}{}^s$,$V_{ds}{}^s$) transformation

This is achieved by following equations and the Simulink model is as shown in fig.3.

$$V_{qs} = V_{qs}{}^{s} \cos\theta_a - V_{ds}{}^{s} \sin\theta_e \quad (6)$$

$$V_{ds} = V_{qs}{}^{s} \sin\theta_a + V_{ds}{}^{s} \cos\theta_e \quad (7)$$

Where, ω_e, is the synchronous speed at which the rotating frame rotates and $\theta e= \omega_e t$ is the angle of rotating frame with respect to stationary frame.

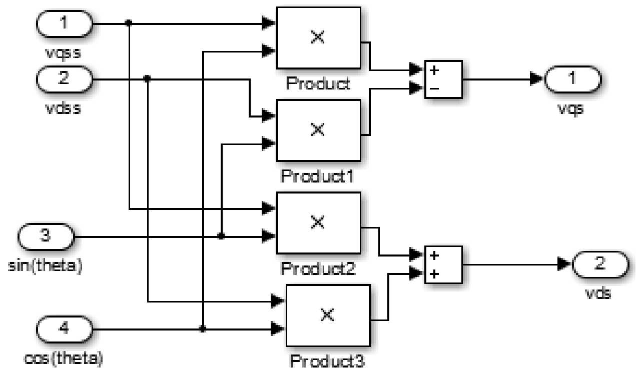

Fig.3. $V_{qs}{}^{s}$, $V_{ds}{}^{s}$ to V_{qs}, V_{ds} transformation

Now let us represent both stator and rotor circuits (d^s-q^s and d^r-q^r) and their variables in a synchronously rotating frame d^e-q^e frame. The stator circuit equations are given as follows.

$$V_{qs}{}^{s} = R_s\, i_{qs}{}^{s} + \frac{d\Psi qs^s}{dt} \quad (8)$$

$$V_{ds}{}^{s} = R_s i_{qs}{}^{s} + \frac{d\Psi ds^s}{dt} \quad (9)$$

Where Ψqs^s and Ψds^s are stator flux linkages of q-axis and d-axis respectively. After converting above equations to synchronously rotating d^e-q^e frame, we get following equations.

$$V_{qs} = R_s i_{qs} + \frac{d\Psi_{qs}}{dt} + \omega_e\, \Psi_{ds} \quad (10)$$

$$V_{ds} = R_s i_{ds} + \frac{d\Psi_{ds}}{dt} - \omega_e\, \Psi_{qs} \quad (11)$$

The rotor circuit equations are given by

$$V_{qr} = R_r i_{qr} + \frac{d\Psi_{qr}}{dt} + \omega_e \Psi_{dr} \quad (12)$$

$$V_{dr} = R_r i_{dr} + \frac{d\Psi_{dr}}{dt} - \omega_e \Psi_{qr} \quad (13)$$

The rotor actually moves at speed ω_r. Therefore the d-q axes fixed on the rotor move at speed $(\omega_e - \omega_r)$ relative to synchronously rotating frame. Therefore in d^e-q^e frame, the actual rotor equations are written as follows.

$$V_{qr} = R_r i_{qr} + \frac{d\Psi_{qr}}{dt} + (\omega_e - \omega_r)\, \Psi_{dr} \quad (14)$$

$$V_{dr} = R_r i_{dr} + \frac{d\Psi_{dr}}{dt} - (\omega_e - \omega_r)\, \Psi_{qr} \quad (15)$$

As earlier said, in this paper dynamic model of induction motor in state space form is going is developed. Therefore it is necessary to define flux linkage variables and are given as follows

$$F_{qs} = \omega_b \Psi_{qs} \quad (16)$$

$$F_{qr} = \omega_b \Psi_{qr} \quad (17)$$

$$F_{ds} = \omega_b \Psi_{ds} \quad (18)$$

$$F_{dr} = \omega_b \Psi_{dr} \quad (19)$$

Where, ω_b is base frequency of the machine. Substituting the relations (16) to (19) in stator and rotor equations (8), (9), (14) and (15), we get the following equations assuming $V_{qr} = V_{dr} = 0$

$$V_{qs} = R_s i_{qs} + \frac{1}{\omega_b}\frac{dF_{qs}}{dt} + \frac{\omega_e}{\omega_b} F_{ds} \quad (20)$$

$$V_{ds} = R_s i_{ds} + \frac{1}{\omega_b}\frac{dF_{ds}}{dt} - \frac{\omega_e}{\omega_b} F_{qs} \quad (21)$$

$$0 = R_r i_{qr} + \frac{1}{\omega_b}\frac{dF_{qr}}{dt} + \frac{(\omega_e - \omega_r)}{\omega_b} F_{dr} \quad (22)$$

$$0 = R_r i_{dr} + \frac{1}{\omega_b}\frac{dF_{dr}}{dt} - \frac{(\omega_e - \omega_r)}{\omega_b} F_{qr} \quad (23)$$

The flux linkage expressions in terms of currents can be written as follows

$$\Psi_{qs} = L_{ls}\, i_{qs} + L_m\, (i_{qs} + i_{qr}) \quad (24)$$

$$\Psi_{qr} = L_{lr}\, i_{qr} + L_m\, (i_{qs} + i_{qr}) \quad (25)$$

$$\Psi_{qm} = L_m\, (i_{qs} + i_{qr}) \quad (26)$$

$$\Psi_{ds} = L_{ls}\, i_{ds} + L_m\, (i_{ds} + i_{dr}) \quad (27)$$

$$\Psi_{dr} = L_{lr}\, i_{dr} + L_m\, (i_{ds} + i_{dr}) \quad (28)$$

$$\Psi_{dm} = L_m\, (i_{ds} + i_{dr}) \quad (29)$$

Multiplying equations (24) to (28) by ω_b on both sides, the flux linkage expressions can be written as

$$F_{qs} = \omega_b \Psi_{qs} = X_{ls} i_{qs} + X_m (i_{qs} + i_{qr}) \quad (30)$$

$$F_{qr} = \omega_b \Psi_{qr} = X_{lr}\, i_{qr} + X_m\, (i_{qs} + i_{qr}) \quad (31)$$

$$F_{qm} = \omega_b \Psi_{qm} = X_m (i_{qs} + i_{qr}) \quad (32)$$

$$F_{ds} = \omega_b \Psi_{ds} = X_{ls} i_{ds} + X_m (i_{ds} + i_{dr}) \quad (33)$$

$$F_{dr} = \omega_b \Psi_{dr} = X_{lr}\, i_{dr} + X_m\, (i_{ds} + i_{dr}) \quad (34)$$

$$F_{dm} = \omega_b \Psi_{dm} = X_m (i_{ds} + i_{dr}) \quad (35)$$

Where $X_{ls} = \omega_b L_{ls,}$ = Stator leakage reactance

$X_{lr} = \omega_b L_{lr,}$= Rotor leakage reactance

$X_m = \omega_b L_{m,}$ = Magnetizing Reactance

Equations (30), (31), (33) and (34) are written as,

$$F_{qs} = X_{ls} i_{qs} + F_{qm} \quad (36)$$

$$F_{qr} = X_{lr}\, i_{qr} + F_{qm} \quad (37)$$

$$F_{ds} = X_{ls} i_{ds} + F_{dm} \quad (38)$$

$$F_{dr} = X_{lr}\, i_{dr} + F_{dm} \quad (39)$$

Now, currents can be expressed in terms of flux linkages as follows

$$i_{qs} = \frac{F_{qs} - F_{qm}}{X_{ls}} \quad (40)$$

$$i_{qr} = \frac{F_{qr} - F_{qm}}{X_{lr}} \quad (41)$$

$$i_{ds} = \frac{F_{ds} - F_{dm}}{X_{ls}} \quad (42)$$

$$i_{dr} = \frac{F_{dr} - F_{dm}}{X_{lr}} \quad (43)$$

Using equations (40) to (43) in equations (36) to (39), F_{qm} and F_{dm} are written as

$$F_{qm} = X_m \left(\frac{F_{qs} - F_{qm}}{X_{ls}} + \frac{F_{qr} - F_{qm}}{X_{lr}} \right) \quad (44)$$

$$F_{qm} = \frac{X_{m1}}{X_{ls}} F_{qs} + \frac{X_{m1}}{X_{lr}} F_{qr} \quad (45)$$

Also,

$$F_{dm} = \frac{X_{m1}}{X_{ls}} F_{ds} + \frac{X_{m1}}{X_{lr}} F_{dr} \quad (46)$$

Where, $X_{m1} = \frac{X_m\, X_{ls}\, Xlr}{Xls\, Xlr + Xlr\, Xm + Xls\, Xm}$

Finally, the stator and rotor voltage equations can be written after substituting equations (40) to (43) in equations (20) to (23) as follows.

$$Vqs = \frac{Rs}{Xls}(Fqs - Fqm) + \frac{1}{\omega b}\frac{dFqs}{dt} + \frac{\omega e}{\omega b} Fds \quad (47)$$

$$Vds = \frac{Rs}{Xls}(Fds - Fdm) + \frac{1}{\omega b}\frac{dFds}{dt} - \frac{\omega e}{\omega b} Fqs \quad (48)$$

$$0 = \frac{Rr}{Xlr}(Fqr - Fqm) + \frac{1}{\omega b}\frac{dFqr}{dt} + \frac{(\omega e - \omega r)}{\omega b} Fdr \quad (49)$$

$$0 = \frac{Rr}{Xlr}(Fdr - Fdm) + \frac{1}{\omega b}\frac{dFqr}{dt} - \frac{(\omega e - \omega r)}{\omega b} Fqr \quad (50)$$

Now, the above equations (47) to (50) can be modified to express in state space form as follows

$$\frac{d\,Fqs}{dt} = wb\left[Vqs - \frac{we\,Fds}{wb} + \frac{Rs}{Xls}\left\{\frac{Fqr\,Xm}{xlr} + Fqs\left(\frac{Xm}{Xls} - 1\right)\right\}\right] \quad (51)$$

$$\frac{d\,Fds}{dt} = wb\left[Vds + \frac{We\,Fqs}{Wb} + \frac{Rs}{Xls}\left\{\frac{Fdr\,Xm}{xlr} + Fds\left(\frac{Xm}{Xls} - 1\right)\right\}\right] \quad (52)$$

$$\frac{d\,Fqr}{dt} = wb\left[\left(\frac{we - wr}{-wb}\right)Fdr + \frac{Rr}{Xlr}\left\{\frac{Fqs\,Xm}{Xls} + Fqr\left(\frac{Xm}{Xlr} - 1\right)\right\}\right] \quad (53)$$

$$\frac{d\,Fds}{dt} = wb\left[\left(\frac{we - wr}{wb}\right)Fqr + \frac{Rr}{Xlr}\left\{\frac{Fds\,Xm}{Xls} + Fdr\left(\frac{Xm}{Xlr} - 1\right)\right\}\right] \quad (54)$$

Equations (51) to (54) are used to develop sub models for obtaining F_{qs}, F_{ds}, F_{qr} and F_{dr}. Figures 4, 5, 6, and 7 show the sub models which have been implemented.

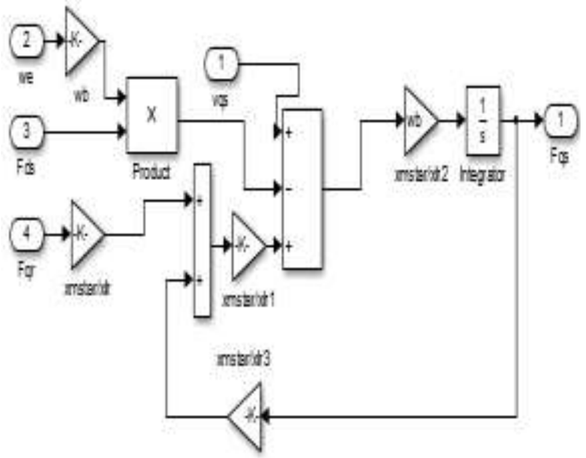

Fig.4. Model to find F_{qs}

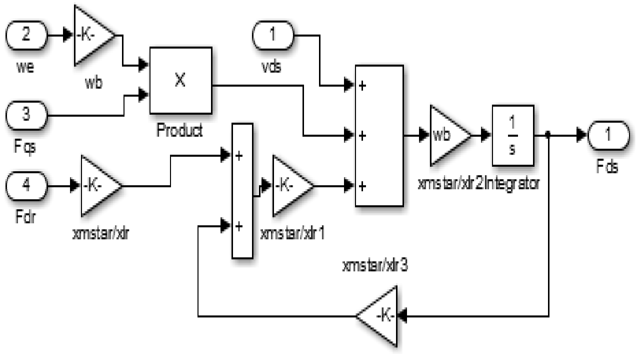

Fig.5. Model for finding F_{ds}

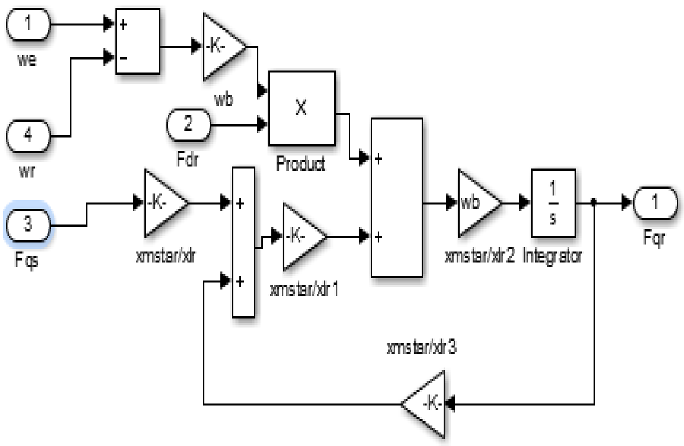

Fig.6. Model for finding F_{qr}

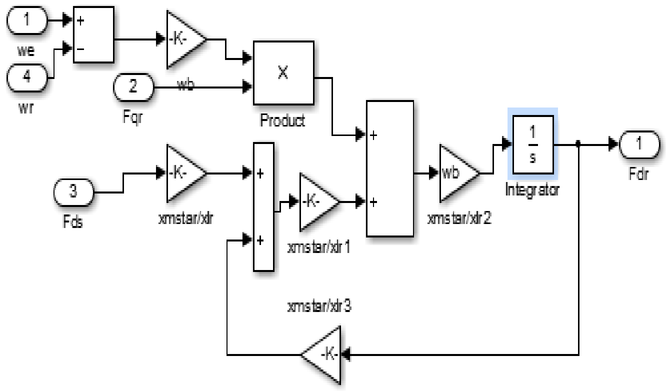

Fig.7. Sub model to find F_{dr}

Once the state variables F_{qs}, F_{ds}, F_{qr} and F_{dr} are calculated, further the variables F_{qm}, F_{dm} are found which in turn are used to find q-axis stator and rotor currents i.e. i_{qs}, i_{qr} and d-axis stator and rotor currents i.e. i_{ds}, i_{dr} by using equations (40) to (43). The sub models for finding F_{qm}, F_{dm} , i_{qs}, i_{qr} , i_{ds}, i_{dr} are shown in following figures.

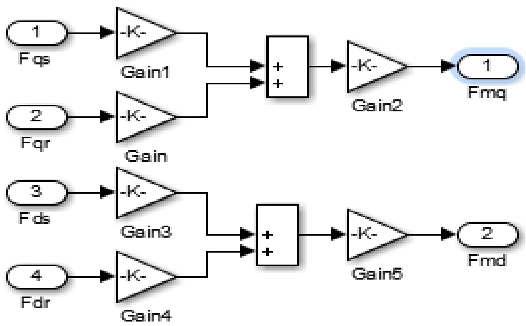

Fig.8. Sub model to find F_{qm}, F_{dm}

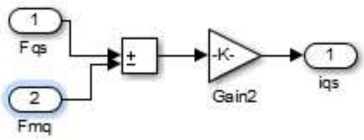

Fig.9. sub model to find i_{qs}

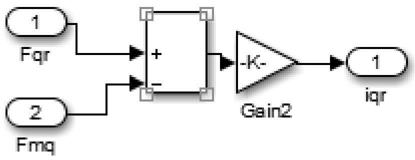

Fig.10. sub model to find i_{qr}

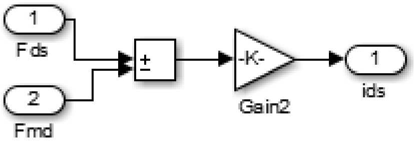

Fig.11. sub model to find i_{ds}

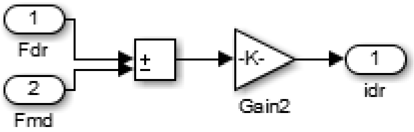

Fig.12. sub model to find i_{dr}

After finding state variables and currents, finally electromagnetic torque produced by the motor is found by following equation.

$$Te = \frac{3}{2}\left(\frac{P}{2}\right)\frac{1}{wb}\left(F_{ds}\, i_{qs} - F_{qs}\, i_{ds}\right) \qquad (55)$$

Once the torque is found, the speed of the motor can be calculated by the equation,

$$Te = TL + \frac{2}{P} J \frac{dwr}{dt} \qquad (56)$$

Where, TL is load torque which is given by a step signal in this simulation, P is the number of poles, J is the rotor inertia and wr is the rotor speed in rad/sec. The sub model finding Te and wr is shown in fig 13.

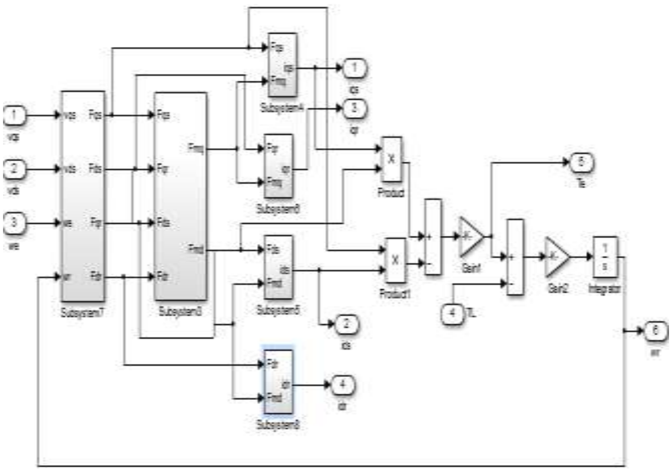

Fig.13. sub model for Te and wr.

III. PROPOSED MODEL

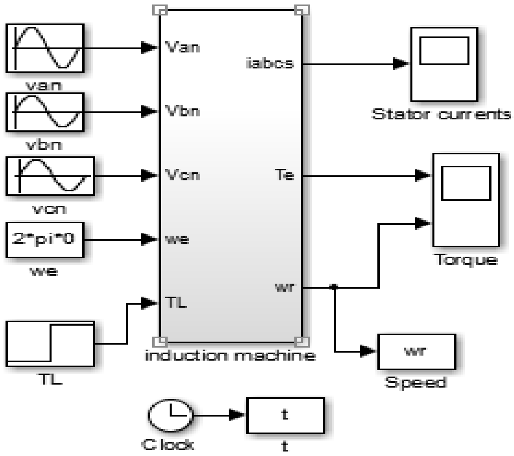

Fig.14.Proposed overall model of induction motor

IV.SIMULATION RESULTS

The induction motor has the following parameters
Type: three phase, 6-pole, $Rs = 0.855\Omega$,
$R_r = 1.15\Omega$, $L_{ls} = 0.00392H$, $L_{lr} = 0.00392H$
$J = 0.06\ kg\ m2$

To study the transient operation of the induction motor, a simulation study is demonstrated. At t=0, the motor is stand still. The moment supply is given, the motor reaches its nominal speed gradually. The load torque is initially kept zero and then at time t=0.3, the load torque is made 40 N.m.
Figure shows the results of computer simulation using the simulink model.

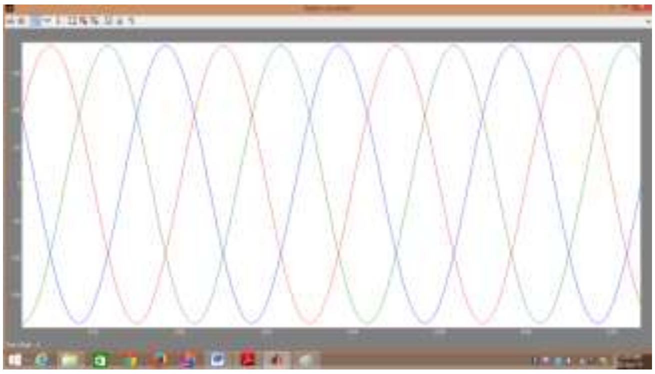

Fig.15. Three phase stator voltages

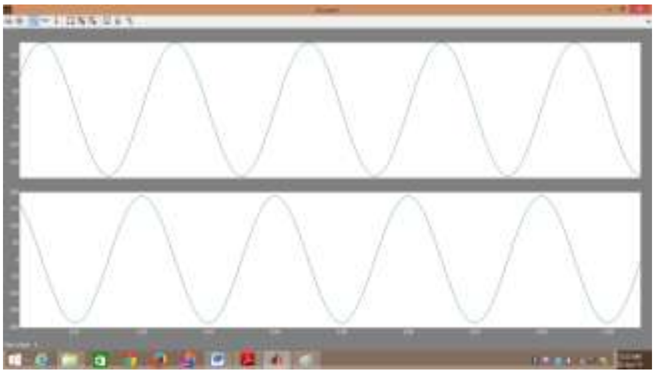

Fig.16.voltages Vqs and Vds

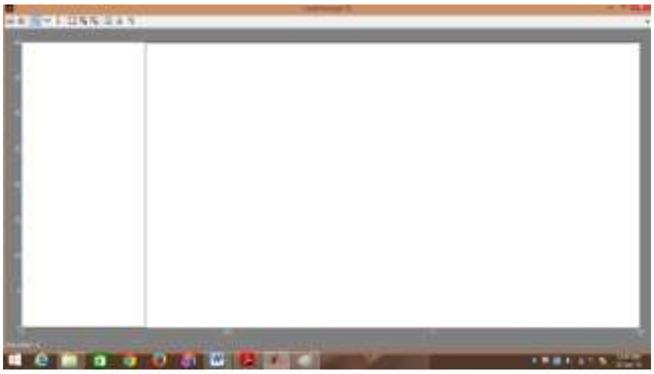

Fig.17. Load torque T_L, a step signal which is initially zero and has value 40Nm at T=0.3

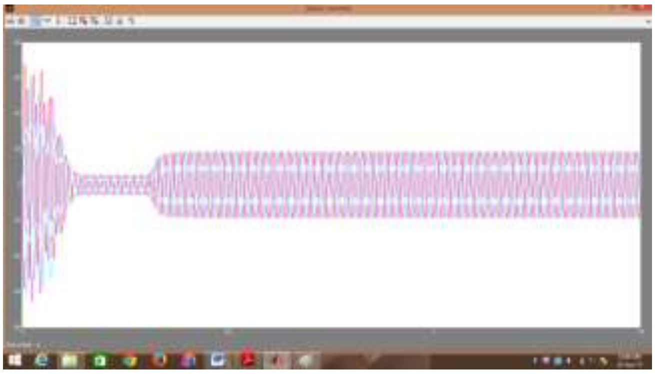

Fig.18.Three phase stator currents. Notice that current increases when T_L increases from 0 to 40 at T=0.3

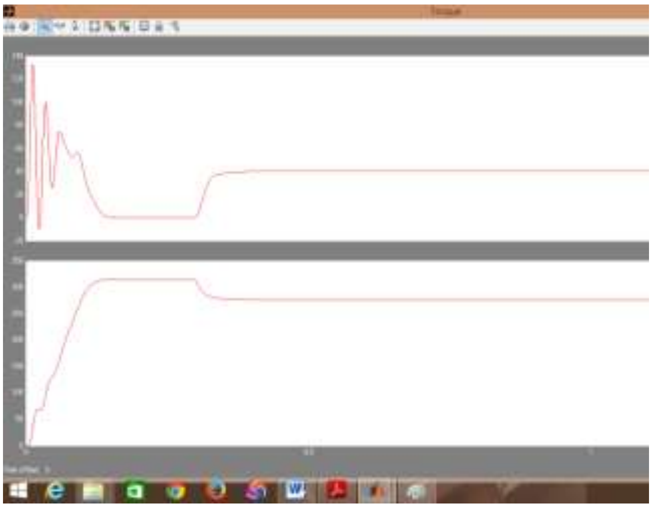

Fig.19. Waveforms for motor torque Te (upper) and Speed ω_r (lower). ω_r is in rad/sec. Note that when load torque increases to 40Nm, the speed slightly reduces from 314 rad/sec to 270 rad/sec.

V. CONCLUSION

Thus simulation of three phase induction motor has been carried out. Two phase stationary axis and rotating frame currents are obtained. This induction motor model not only can be used alone but also can be used in high performance motor drive systems such as field oriented control of induction motor, sensorless speed control of induction motor.

VI. REFERENCES

1] K.L.SHI, T.F.CHAN, Y. K. WONG and S.L.HO, "Modeling and simulation of the three-phase induction motor using simulink", Int. J. Elect. Enging. Educ., Vol. 36, pp. 163–172, Manchester U.P., 1999

2] P.M. Palmpankar, R U. Ghanmare, "Generalised dynamic model of induction motor using Simulink", ITSI TEEE, Vol.-1, Issue-5, 2013

3] Krause, P. C., Wasynczuk, O. and Sudhoff, S. D., Analysis of Electric Machinery, IEEE (1995)

4] Trzynadlowski, Andrzej,‟ Control of Induction Motors‟ United State of America (1994)

5] B. Ozpineci, L. M. Tolbert, "Simulink implementation of induction machine model – A Modular approach", IEEE, 2003, pp 728-734.

6] Muhammad Rashid," Power electronics handbook".

7] Electrical Machines by Nagrath and Kothari, Tata Mc Graw Hill, New Delhi

8] B.K.Bose,‟ Modern power electronics and AC drives. ‟ New Delhi, PHI Learning Private Limited, 2011.pp.413-408