

# Application of Queuing Theory in a Small Enterprise

Anindita Sharma<sup>1#</sup>, Parimal Bakul Barua<sup>2#</sup>

<sup>1</sup>M.E. student, <sup>2</sup>Professor and H.O.D.

<sup>#</sup>Department of Mechanical Engineering, Jorhat Engineering College, Jorhat, Assam, India

**Abstract**— Waiting lines are common occurrence in everyday life. All situations where there is involvement of customers such as restaurants, hospitals, departmental stores, theatres, banks, post offices etc. are likely to have queues or waiting lines. Queuing theory is the mathematical study of waiting lines. In queuing theory, a model is constructed which helps to predict the lengths of queue as well as the waiting times. The aim of this study is to review the queuing congestion in a grocery store and develop a suitable queuing model for the system. Also, an attempt is made to propose an alternate model and compare its efficiency levels with that of the existing model.

**Keywords**— Queue, queue length, waiting time, M/M/1 model

## I. INTRODUCTION

Queues help facilities or businesses to provide services in an orderly manner. Forming a queue can be helpful to the society if it can be managed so that both the waiting unit and the unit that serves get the most benefits. Generally, queue occurs when service rendered is low compared to the high level demand in a particular place and time. Waiting lines are likely to form in almost all situations where customers are involved [1]. A busy grocery store is such an example. Grocery is a daily need in all households and thus, such stores are visited for household commodities by customers everyday. At times, the heavy traffic becomes a problem for the serving unit, i.e., the grocery shop.

In the grocery store selected for the study, customers arrive at the check-out counter, wait to be served and leave once they get the service.

## II. LITERATURE REVIEW

A brief summary of the empirical literature survey done for the study is given below-

**Manuel, U.R., et al. (2014) [1]** states the importance of applying queuing theory to the ATM using Monte Carlo simulation in order to determine, control and manage the level of queuing congestion. **Patel, B., et al. (2012) [2]** used Little theorem and M/M/1 queuing model for the improvement of service time by the bank ATMs. They also developed a simulation model to confirm the results of their analytical model. **Marsudi, M., et al. (2014) [3]** carried out queuing analysis to examine multi-stage production line performance to facilitate more realistic resource planning. The comparison between the analytical results and the standard data of the company

showed an accuracy of 93.80%. **Mehandiratta, R. (2011) [4]** attempted to analyse the queuing theory and instances of using the theory in health care organisations around the world and benefits acquired for the same. **Nafees, A., (2007) [5]** carried out the analysis of queuing systems for the empirical data of supermarket checkout service unit. He also described a queuing simulation for a multiple server process as well as for single queue models. **Vasumathi, A., et al. (2010) [6]** adopted a simulation technique for identifying the pitfalls of existing 3 ATM services of 3 banks and also planning to propose a new ATM service from any one of these banks or other banks based upon the service required from the customers. **Sztrik, J. (2010) [7]** discussed about the queuing theory and some mathematical models of queuing systems. **Anokye, M., et al. (2013) [8]** sought to model the vehicle traffic flow and explored how vehicle traffic could be minimised using queuing theory in order to reduce the delay on roads in the Kumasi metropolis of Ghana. The results showed that traffic intensity  $\rho < 1$  for all sessions, a condition that suggested a perfect traffic system. **Yakubu, A.W.N., et al. (2014) [9]** applied queuing theory to determine optimal service level for a case ATM base on a customer defined criterion of wait time not exceeding eight minutes. **Afrane, S., et al. (2014) [10]** investigated the application of queuing theory and modelling to the queuing problem at the out-patient department at AngloGold Ashanti hospital in Obuasi, Ghana. **Bihani, A. (2014) [11]** proposed a new approach to Monte Carlo simulation of operations thereby optimising multi-server operations. Besides, analysis of the Monte Carlo methods was done against the deterministic methods.

## III. OBJECTIVES OF THE STUDY

The objectives of this research done in a grocery store are-

- To study the congestion in the check-out counter of the store.
- To validate the data collected in the investigation.
- To develop a queuing model for the system.
- To propose an alternate model and compare its efficiency levels with the existing model.

## IV. METHODOLOGY

### A. Data Collection:

The data was collected for the check-out counter in a grocery store for a period of 6 weeks. A stop-watch

was used in the process. The data collection was done for three different timings during the working hours of the store- morning, afternoon and evening. Two weeks were allotted to each timing.

**B. Data Validation:**

By running the Goodness of Fit test in the JMP Statistical software, the data were validated. The inter-arrival times were found to follow Poisson distribution and the service times followed Exponential distribution. The number of servers was one as there was only a single check-out counter in the store. Thus, it was found that the system was fit for the M/M/1 queuing model.

**C. Development of Queuing Model:**

Certain assumptions that were made during the research are-

- Customers’ behaviour (or queue discipline) was assumed to be First-Come-First-Serve (FCFS) type.
- Reneging, balking and jockeying of the customers were not taken into consideration in the study.
- The population source was taken to be infinity.
- Infinite number of customers are allowed in the system.

According to Kendall’s notation, the model for the system could be represented as M/M/1:FCFS/∞/∞ [12]. The following parameters were analysed for the model-

Mean customers arrival rate =  $\lambda$

Mean service rate =  $\mu$

Utilisation factor,  $\rho = \frac{\lambda}{\mu}$

Probability of zero customers in the system,  $P_0 = 1 - \rho$

Probability of having n customers,  $P_n = P_0 \rho^n = (1 - \rho) \rho^n$

Average number of customers in the system,  $L_s = \frac{\lambda}{\lambda - \mu}$

Average number of customers in the queue,  $L_q = L_s - \frac{\lambda}{\mu}$

Average time spent in the system,  $W_s = \frac{1}{\mu - \lambda}$

Average time spent in the queue,  $W_q = \frac{\rho}{\mu - \lambda}$

Moreover, the model parameters were analysed for each of three timings individually and a comparison was drawn amongst the three timings. Finally, an alternate model was proposed and the efficiency levels were compared to that of the existing model.

**V. RESULTS AND DISCUSSION**

The results found from the analysis are as follows-

**A. Analysis of the Parameters of the Queuing Model for the Whole System:**

Number of servers = 1

Mean customers arrival rate,  $\lambda = 13$  customers per hour  
Mean service rate,  $\mu = 19$  customers per hour

Utilization factor,  $\rho = \frac{\lambda}{\mu} = 0.6842$

Probability of zero customers in the system,  $P(0) = 1 - \rho = 0.3158$

Average number of customers in the system,  $L_s = \frac{\lambda}{\mu - \lambda} = 2.1667$

Average number of customers in the queue,  $L_q = L_s - \frac{\lambda}{\mu} = 1.4825$

Average time spent in the system,  $W_s = \frac{1}{\mu - \lambda} = 0.1667$  hours

Average time spent in the queue,  $W_q = \frac{\rho}{\mu - \lambda} = 0.114$  hours

TABLE 1: RESULTS OF THE M/M/1:FCFS/∞/∞ MODEL FOR THE WHOLE SYSTEM

| N  | Probability, P(n) | Cumulative probability, P(n) |
|----|-------------------|------------------------------|
| 0  | 0.3158            | 0.3158                       |
| 1  | 0.2161            | 0.5319                       |
| 2  | 0.1478            | 0.6797                       |
| 3  | 0.1012            | 0.7808                       |
| 4  | 0.0692            | 0.8500                       |
| 5  | 0.0474            | 0.8974                       |
| 6  | 0.0324            | 0.9298                       |
| 7  | 0.0222            | 0.9520                       |
| 8  | 0.0152            | 0.9671                       |
| 9  | 0.0104            | 0.9775                       |
| 10 | 0.0071            | 0.9846                       |
| 11 | 0.0049            | 0.9895                       |
| 12 | 0.0033            | 0.9928                       |

From Table-1, it is seen that the probability of customers in the queue decreases with increasing number of customers. The probability of having one customer in the queue is 0.2161 and the probability of having three customers is 0.1012. The cumulative probability is quickly approaching 1, for example for 12 customers it is 0.9928. This implies that it is rare to have more than 12 customers in the queue under normal conditions.

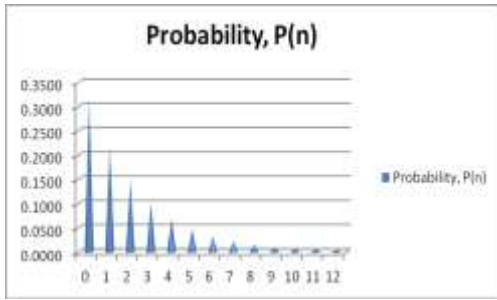


Fig.1: Plot of P(n) vs N for the whole system

**B. Analysis of the Parameters of the Queuing Model for each of the 3 timings individually:**

**1) In the evening-**

Number of servers = 1  
 Mean customers arrival rate,  $\lambda = 15$  customers per hour  
 Mean service rate,  $\mu = 20$  customers per hour  
 Utilisation factor,  $\rho = \frac{\lambda}{\mu} = 0.75$

Probability of zero customers in the system,  $P(0) = 1 - \rho = 0.25$

Average number of customers in the system,  $L_s = \frac{\lambda}{\mu - \lambda} = 3$

Average number of customers in the queue,  $L_q = L_s - \frac{\lambda}{\mu} = 2.25$

Average time spent in the system,  $W_s = \frac{1}{\mu - \lambda} = 0.2$  hours

Average time spent in the queue,  $W_q = \frac{\rho}{\mu - \lambda} = 0.15$  hours

TABLE 2: RESULTS OF THE M/M/1:FCFS/ $\infty/\infty$  MODEL FOR THE EVENING CONGESTION

| N  | Probability, P(n) | Cumulative probability, P(n) |
|----|-------------------|------------------------------|
| 0  | 0.2500            | 0.2500                       |
| 1  | 0.1875            | 0.4375                       |
| 2  | 0.1406            | 0.5781                       |
| 3  | 0.1055            | 0.6836                       |
| 4  | 0.0791            | 0.7627                       |
| 5  | 0.0593            | 0.8220                       |
| 6  | 0.0445            | 0.8665                       |
| 7  | 0.0334            | 0.8999                       |
| 8  | 0.0250            | 0.9249                       |
| 9  | 0.0188            | 0.9437                       |
| 10 | 0.0141            | 0.9578                       |
| 11 | 0.0106            | 0.9683                       |
| 12 | 0.0079            | 0.9762                       |
| 13 | 0.0059            | 0.9822                       |
| 14 | 0.0045            | 0.9866                       |
| 15 | 0.0033            | 0.9900                       |

From Table-2, it is seen that the probability of customers in the queue decreases with increasing number of customers. The probability of having one customer in the queue is 0.1875 and the probability of having three customers is 0.1055. The cumulative probability is quickly approaching 1, for example for 15 customers it is 0.9900. This implies that it is rare to have more than 15 customers in the queue under normal conditions.

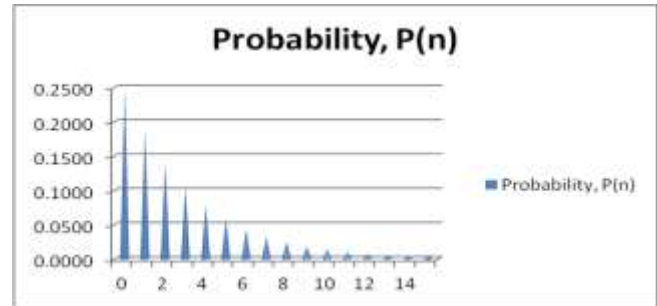


Fig.2: Plot of P(n) vs N for the evening time

**2) In the morning-**

Number of servers = 1  
 Mean customers arrival rate,  $\lambda = 12$  customers per hour  
 Mean service rate,  $\mu = 19$  customers per hour  
 Utilisation factor,  $\rho = \frac{\lambda}{\mu} = 0.6316$

Probability of zero customers in the system,  $P(0) = 1 - \rho = 0.3684$

Average number of customers in the system,  $L_s = \frac{\lambda}{\mu - \lambda} = 1.7143$

Average number of customers in the queue,  $L_q = L_s - \frac{\lambda}{\mu} = 1.0827$

Average time spent in the system,  $W_s = \frac{1}{\mu - \lambda} = 0.1429$  hours

Average time spent in the queue,  $W_q = \frac{\rho}{\mu - \lambda} = 0.0902$  hours

TABLE 3: RESULTS OF THE M/M/1:FCFS/∞/∞ MODEL FOR THE MORNING CONGESTION

| N  | Probability, P(n) | Cumulative probability, P(n) |
|----|-------------------|------------------------------|
| 0  | 0.3684            | 0.3684                       |
| 1  | 0.2327            | 0.6011                       |
| 2  | 0.1470            | 0.7481                       |
| 3  | 0.0928            | 0.8409                       |
| 4  | 0.0586            | 0.8995                       |
| 5  | 0.0370            | 0.9365                       |
| 6  | 0.0234            | 0.9599                       |
| 7  | 0.0148            | 0.9747                       |
| 8  | 0.0093            | 0.9840                       |
| 9  | 0.0059            | 0.9899                       |
| 10 | 0.0037            | 0.9936                       |

From Table-3, it is seen that the probability of customers in the queue decreases with increasing number of customers. The probability of having one customer in the queue is 0.2327 and that of having three customers is 0.0928. The cumulative probability is quickly approaching 1, for example for 10 customers it is 0.0037. This implies that it is rare to have more than 10 customers in the queue under normal conditions.

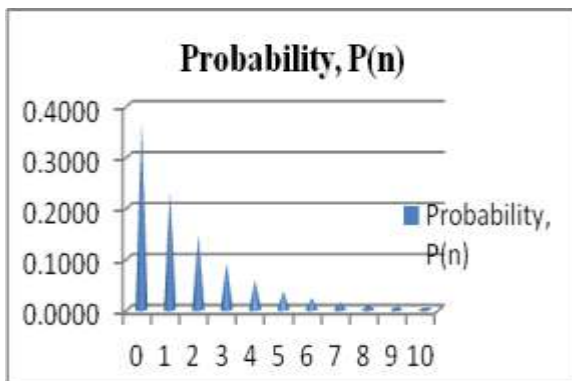


Fig.3: Plot of P(n) vs N for the morning time

3) In the afternoon-

Number of servers = 1  
 Mean customers arrival rate,  $\lambda = 11$  customers per hour

Mean service rate,  $\mu = 18$  customers per hour

Utilisation factor,  $\rho = \frac{\lambda}{\mu} = 0.6111$

Probability of zero customers in the system,  $P(0) = 1 - \rho = 0.3889$

Average number of customers in the system,  $L_s = \frac{\lambda}{\mu - \lambda} = 1.5714$

Average number of customers in the queue,  $L_q = L_s - \frac{\lambda}{\mu} = 0.9603$

Average time spent in the system,  $W_s = \frac{1}{\mu - \lambda} =$

0.1429hours

Average time spent in the queue,  $W_q = \frac{\rho}{\mu - \lambda} =$

0.0873 hours

TABLE 4: RESULTS OF THE M/M/1:FCFS/∞/∞ MODEL FOR THE AFTERNOON CONGESTION

| N  | Probability, P(n) | Cumulative probability, P(n) |
|----|-------------------|------------------------------|
| 0  | 0.3889            | 0.3889                       |
| 1  | 0.2377            | 0.6265                       |
| 2  | 0.1452            | 0.7718                       |
| 3  | 0.0888            | 0.8605                       |
| 4  | 0.0542            | 0.9148                       |
| 5  | 0.0331            | 0.9479                       |
| 6  | 0.0203            | 0.9682                       |
| 7  | 0.0124            | 0.9805                       |
| 8  | 0.0076            | 0.9881                       |
| 9  | 0.0046            | 0.9927                       |
| 10 | 0.0028            | 0.9956                       |

From Table-4, it is again seen that the probability of customers in the queue decreases with increasing number of customers. The probability of having one customer in the queue is 0.2377 and the probability of having three customers is 0.0888. The cumulative probability is quickly approaching 1, for example for 10 customers it is 0.0028, implying that it is rare to have more than 10 customers in the queue under normal conditions.

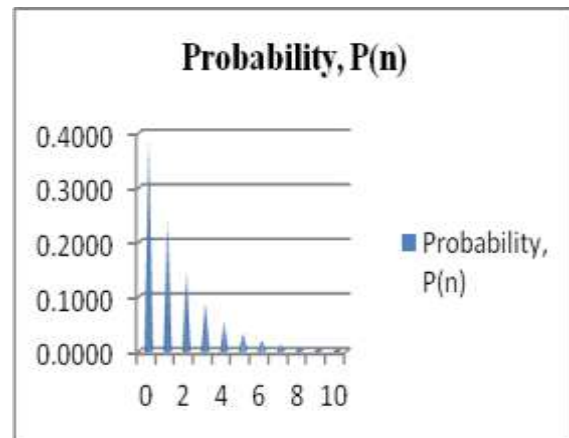


Fig.4: Plot of P(n) vs N for the afternoon time

TABLE 5: COMPARISON OF THE PARAMETERS OF THE M/M/1 MODEL FOR THE THREE TIMINGS

| Parameters | Evening | Morning | Afternoon |
|------------|---------|---------|-----------|
| $\lambda$  | 15      | 12      | 11        |
| $\mu$      | 20      | 19      | 18        |
| $\rho$     | 0.75    | 0.6316  | 0.6111    |
| $P(0)$     | 0.25    | 0.3684  | 0.3889    |
| $L_s$      | 3       | 1.7143  | 1.5714    |
| $L_q$      | 2.25    | 1.0827  | 0.9603    |
| $W_s$      | 0.2     | 0.1429  | 0.1429    |
| $W_q$      | 0.15    | 0.0902  | 0.0873    |

From Table-5, it is seen that the server remains the busiest during the evening with a traffic intensity of 75%. In its comparison, the utilisation of the server is quite low in the morning and afternoon with an intensity of 63.16% and 61.11% respectively.

**C. Comparison of the Single Server Model with a Proposed Multi-Server Model:**

The grocery store selected for this study used the single server model M/M/1:FCFS/ $\infty/\infty$ . But for efficiency purposes, it is worth to assume a multi-server model being used by the same store so as to compare the efficiency levels. To compare, the study has proposed an M/M/2 model. The efficiency parameters for the two models are shown in Table-6 below-

TABLE 6: COMPARISON OF THE QUEUING MODELS

| Parameters | M/M/1  | M/M/2  |
|------------|--------|--------|
| $\lambda$  | 13     | 13     |
| $\mu$      | 19     | 19     |
| $\rho$     | 0.6842 | 0.3421 |
| $P(0)$     | 0.3158 | 0.4902 |
| $L_s$      | 2.1667 | 0.7749 |
| $L_q$      | 1.4825 | 0.0907 |
| $W_s$      | 0.1667 | 0.0596 |
| $W_q$      | 0.1140 | 0.007  |

Table-6 above shows the efficiency parameters under two different queuing models. The traffic intensity has changed significantly from 68.42% to 34.21%. This shows that as more servers (check-out

counters) are introduced, then the store becomes less busy. This may attract more customers.

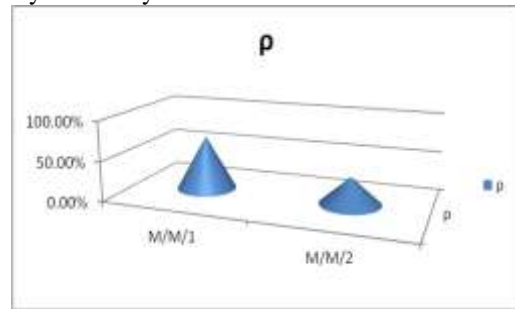


Fig.5: Plot of Server Utilisation vs Queuing Models

**VI. CONCLUSION**

In case of the grocery store selected, the results throw light on some important issues about the operation mode of the waiting line. The customers have to wait an average of 0.1667 hours in the system. The average number of customers who have to wait is 1.4825 and 68.42% of arriving customers have to wait to be served. These results show that there is a need to improve the operations that occur within the waiting line. If the store continues to use the single server waiting line, the number of waiting customers will increase. In general, there is need to improve the service rate. It is recommended that the store introduces another check-out counter in order to lower the congestion and also to attract more customers. However, the decision for introduction of another check-out counter must also focus on cost considerations (a break-even analysis).

**VII. FUTURE SCOPE**

For a more generalised result, the study can be done for a longer period over a number of grocery stores in the area. The cost estimation which was not included in the present study, can be incorporated.

**REFERENCES**

- [1] Udoanya Raymond Manuel and Aniekano Offiong, "Application of Queuing Theory to Automated Teller Machine (ATM) facilities using Monte-Carlo Simulation", *International Journal of Engineering Science and Technology (IJEST)*, vol.6(4), April 2014.
- [2] Bhavin Patel and Pravin Bhathawala, "Case Study for Bank ATM Queuing Model", *International Journal of Engineering Research and Applications (IJERA)*, vol.2(5), September-October 2012.
- [3] Muhammad Marsudi and Hani Shafeek, "The Application of Queuing Theory in Multi-Stage Production Line", in *Proceedings of the 2014 International Conference on Industrial Engineering and Operations Management Bali, Indonesia*, January 7-9, 2014.
- [4] Reetu Mehandiratta, "Applications of Queuing Theory in Health Care", *International Journal of Computing and Business Research*, vol.2(2), May 2011.
- [5] Azmat Nafees, "Queuing Theory and Its Application: Analysis of the Sales Checkout Operation in ICA Supermarket", M.Sc. thesis, University of Dalarna, June 2007.
- [6] Vasumathi A and Dhanavanthan P, "Application of Simulation Technique in Queuing Model for ATM Facility", *International Journal of Applied Engineering Research, Dindigul*, vol.1(3), 2010.
- [7] Janos Sztrik, "Queuing Theory and its Applications, A Personal View", in *Proceedings of the 8<sup>th</sup> International Conference on Applied Informatics, Eger Hungary*, January 27-30, 2010, vol.1 pp. 9-30.

- [8] Martin Anokye, A.R. Abdul-Aziz, Kwame Annin and Francis T. Oduro, “Application of Queuing Theory to Vehicular Traffic at Signalized Intersection in Kumasi-Ashanti Region, Ghana”, *American International Journal of Contemporary Research*, vol.3(7), July 2013.
- [9] Abdul-Wahab Nawusu Yakubu and Dr. Ussiph Najim, “An Application of Queuing Theory to ATM Service Optimization: A Case Study”, *Mathematical Theory and Modelling, IISTE*, vol.4(6), 2014.
- [10] Sam Afrane and Alex Appah, “Queuing Theory and the Management of Waiting Time in Hospitals: The Case of Anglo Gold Ashanti Hospital in Ghana”, *International Journal of Academic Research in Business and Social Sciences*, vol.4(2), February 2014.
- [11] Ankita Bihani, “A New Approach to Monte Carlo Simulation of Operations”, *International Journal of Engineering Trends and Technology*, vol.8(4), February 2014.
- [12] D.S. Hira and Prem Kumar Gupta, *Operations Research*, Edition 2007, S.Chand & Company Ltd.

**ANNEXURE**

**TABLE 7: A TYPICAL DATASHEET USED FOR THE RESEARCH**

| <b>Sl. No.</b> | <b>Arrival times</b> | <b>Inter-arrival times</b> | <b>Departure times</b> | <b>Service times</b> |
|----------------|----------------------|----------------------------|------------------------|----------------------|
| 1              | 3.30                 | 0                          | 3.32                   | 2                    |
| 2              | 3.34                 | 4                          | 3.36                   | 2                    |
| 3              | 3.40                 | 6                          | 3.43                   | 3                    |
| 4              | 3.42                 | 2                          | 3.44                   | 2                    |
| 5              | 3.45                 | 3                          | 3.47                   | 2                    |
| 6              | 3.51                 | 6                          | 3.53                   | 2                    |
| 7              | 3.53                 | 2                          | 3.56                   | 3                    |
| 8              | 3.55                 | 2                          | 4.00                   | 5                    |
| 9              | 4.01                 | 6                          | 4.04                   | 3                    |
| 10             | 4.03                 | 2                          | 4.05                   | 2                    |
| 11             | 4.04                 | 1                          | 4.07                   | 3                    |
| 12             | 4.05                 | 1                          | 4.09                   | 4                    |
| 13             | 4.07                 | 2                          | 4.11                   | 4                    |
| 14             | 4.10                 | 3                          | 4.13                   | 3                    |
| 15             | 4.11                 | 1                          | 4.12                   | 1                    |
| 16             | 4.11                 | 0                          | 4.14                   | 3                    |
| 17             | 4.13                 | 2                          | 4.16                   | 3                    |
| 18             | 4.15                 | 2                          | 4.18                   | 3                    |
| 19             | 4.18                 | 3                          | 4.21                   | 3                    |
| 20             | 4.20                 | 2                          | 4.24                   | 4                    |