Composite Elastic Modulus Aids Well Performance and Permeability Predictions

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Abstract— *A traditional analytical model for well performance predictions is based on Darcy's law and the slightly compressible fluid assumption. The latter assumption is not consistent with the principle of conservation of mass. Quadratic gradient terms are neglected. The elastic moduli of fluid and rock may give rise to quadratic terms. Many studies investigate the effect of one or two quadratic gradient terms. The objective of the present study is to generalize the traditional model to accommodate an arbitrary number of quadratic gradient terms and to facilitate well performance and permeability predictions. The methodology depends on the assumption that each pressure dependent variable may be approximated by an exponential function of pressure or equivalently of a constant value of the corresponding elastic modulus. This assumption may be reasonable for some deep reservoirs. Then, the non-linear Darcy equation may be linearized by use of a composite elastic modulus. We find that the effect of quadratic terms cannot be overlooked for large values of the composite elastic modulus and/or large pressure differences between the outer boundary and the wellbore. Hence, we expect the generalized model to work better for deep reservoirs than for shallow ones. The technique may be extended to time dependent flow, but with reduced accuracy. The diffusivity still depends on pressure. Then, perturbation techniques may be necessary.*

Keywords— *stress-sensitivity, well performance, permeability predictions*

I. Introduction

The slightly compressible flow assumption is not consistent with the principle of conservation of mass. Quadratic gradient terms are neglected. The elasticity of both fluid and rock properties gives rise to quadratic terms (Matthews and Russel [1]). The objective of the present study is to propose a generalized composite elastic modulus to extend a well-known method to accommodate an arbitrary number of quadratic terms. The maximum number is limited by the number of factors in the transport term of the diffusivity equation. The technique depends on the assumption that each pressure dependent variable may be approximated by an exponential function of pressure. The equivalent assumption is constant value elastic moduli. This study is limited to steady state flow. The technique has obvious extensions to time dependent flow, but with reduced accuracy. Perturbation techniques may be necessary (Kikani and Pedrosa [2]). Provided the elastic moduli are known, steady state flow analysis is useful for well performance and permeability predictions. Obtaining credible estimates for the moduli remains a challenge.

The majority of previous studies deal with pressure transient analysis. Chakrabarty et al. [3] investigated the effect of density changes by use of a logarithmic (Cole-Hopf) transformation. Jelmert and Vik [4] proposed a slightly different transformation to simplify the quadratic gradient term equation. They showed that the method of source and Green functions may be used to obtain approximate solutions. Kikani and Pedrosa [2] showed that the permeability modulus may be obtained by well testing. Jelmert and Selseng [5] analysed the same problem by use of a slightly different transformation. The transformed variable had the advantage of a clear physical interpretation, normalized permeability. It has physical integrity since the permeability function cannot be negative. They pointed out that use of the pseudo-potential method under the assumption of a constant permeability modulus lead to the same end result. More recently, Ai and Yao [6] presented a similar study. Their model included two quadratic gradient terms.

Formation evaluation of any sub-surface reservoir is problematic since there are more unknown variables than equations. Hence, information from many sources is required: well testing, logging, core and fluid analysis, etc. Fluid properties are usually obtained by correlations. The average variation of permeability and porosity with effective stress may be estimated from a group of cores (Jelmert and Selseng [7] and Jelmert et al. [8]).The basic idea in these studies is use of average permeability and porosity functions rather than relying on results from single cores.

Chen et al. [9] investigated the inflow performance of deep wells in stress sensitive reservoirs under the steady state flow assumption. Their study includes real data from the Qingxi field in China. Fang and Yang [10] presented a study based on the pseudo-pressure approach.

II. Theory

We limit this study to investigate the effect of three pressure dependent variables: permeability, $k\ \ p$, density, $\rho\ \ p$, and viscosity, $\mu\ \ p$. By assumption, each variable is characterized by a constant elastic modulus or equivalently an exponential function of pressure. If experimental or model data are available, these may be fit to an exponential pressure equation by linear regression.

The elastic moduli are denoted: γ, c, and υ, respectively. We define a combined modulus, τ, for the mobility, $M(p) = k(p) / \mu(p)B(p)$. The deviation of mobility from the reference condition is: $\Delta M\ p = M_{ref} - M\ p$, which is a function of distance and pressure. In the same way, the pressure difference is: $\Delta p = p_{ref} - p$. We chose the pressure at the external boundary as reference. The pressure at the external boundary depends on the difference between the fluid volume produced and injected into the drainage area. The reference pressure may change with time, but slowly to warrant the steady state assumption. Material balance calculation and formation evaluation may aid estimation of the reference pressure.

By use of elementary derivation rules we obtain:

$$\tau = \gamma + c - \upsilon = \frac{1}{M_n}\frac{dM_n}{dp} = \frac{1}{M_n}\frac{d\Delta M_n}{d\Delta p} \tag{1}$$

Index n denotes normalized to the reference condition, i.e.

$M_n = M(p) / M_e$ and $M_{ne} = 1$.

Integration gives M_n as an exponential function of pressure:

$$M_n = 1 + \Delta M_n = e^{-\tau\ p_e - p} \tag{2}$$

and a logarithmic inverse function as:

$$p = p_e + 1/\tau\ \ln M_n \tag{3}$$

The mobility modulus, τ, involves three constants: γ, c and υ. These show up the same way in the model, hence it is impossible to distinguish between them when matching a model to observed behavior. If two of these may be estimated by other means, then the effect of the third one may be isolated. Traditionally, the fluid variables are estimated by correlations.

The permeability modulus may be estimated from well testing (Kikani and Pedrosa [2]) and/or core analysis (Jelmert and Selseng [7]).

Conservation of mass under steady state flow may be described by:

$$\frac{1}{r}\frac{d\ \rho\ r\ q\ r}{dr} = 0 \tag{4}$$

Division by the density at standard condition, ρ_{sc}, yields:

$$\frac{1}{r}\frac{dq_{sc}\ r}{dr} = 0 \tag{5}$$

Integration leads to a surface flow rate that is constant and independent of position. Then:

$$q_{sc} = q_{sc}\ r\ = q_{sc}\ r_w\ = q_{sc}\ r_e$$

The flow rate depends on Darcy's law, hence:

$$q_{sc} = \pm\frac{2\pi k_e h M_n\ r}{\mu_e B_e} r \frac{d\Delta p}{dr} \tag{7}$$

The upper sign is for production and the lower one for injection. Let the normalized radial distance be: $r_n = r / r_e$,

Substitution of equation 1 into equation 7 yields:

$$q_{sc} = \pm\frac{2\pi k_e h}{\mu_e B_e \tau} r_n \frac{d\Delta M_n}{dr_n} \tag{8}$$

Equation 8 is equivalent to the traditional Darcy equation, but with another dependent variable. As such, the equations share similar solutions. All traditional equations may be "rediscovered".

Equation 8 may be integrated to yield:

$$M_n\ r_n\ = 1 + \Delta M_n\ r_n\ = 1 \pm \frac{q_{sc}\mu_e B_e \tau}{2\pi k_e h}\ln r_n \tag{9a}$$

$$M_{nw} = 1 \pm \frac{q_{sc}\mu_e B_e \tau}{2\pi k_e h}\ \ln r_{nw} - S_\tau \tag{9b}$$

The volumetric average pressure may be estimated from material balance calculations. As a first approximation, one may assume that the volumetric average pressure and volumetric average mobility occur at the same position. From traditional analysis, we know the average mobility change is located half the distance out to the external boundary, $\bar{M}_n\ r_n\ = M_n\ 0.5$.

Equation 9a will show up as straight lines on a semi-log plot, see Fig. 1 and Fig. 2. The slope of the straight lines, τ_D, which is dimensionless, has been called the non-linear flow parameter. The non-linear flow parameter is proportional to the rate:

$$\tau_D = \frac{q_{sc}\mu_e B_e \tau}{2\pi k_e h} \tag{10}$$

Substituting the wellbore radius into equation 9b yields:

$$M_{nw} = 1 \mp \tau_D\ \ln r_{eD} + S_\tau \tag{11}$$

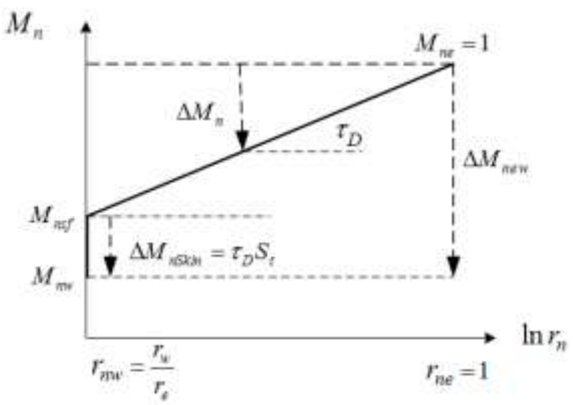

Fig. 1 Mobility function (M_n) vs. radial distance for production

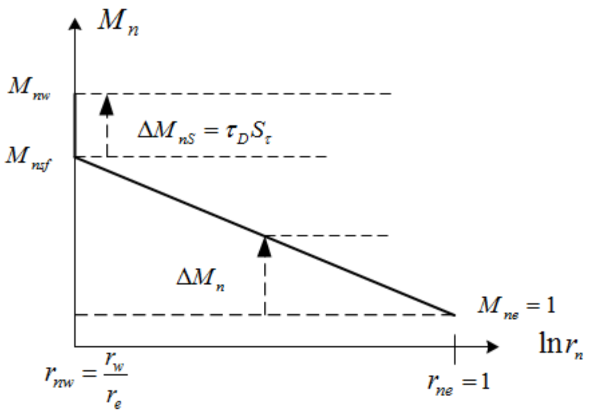

Fig. 2 Mobility function (M_n) vs. radial distance for injection

From Figs. 1 and 2 we see that there are limits on the M_{nw} function. For production, it is bounded between two theoretical limits, $0 < M_{nw} < 1$ which leads to: $0 < 1 - \tau_D \ln\ r_{eD} + S_\tau\ < 1$. For injection the limit is: $M_{nw} > 1$ or equivalently $1 + \tau_D \ln(r_{eD} + S_\tau) > 1$. These conditions spell out the admissible operating range of the model. Beyond these limits, the mathematical model leads to predictions that are physical impossible. The practical limits are stricter. For example, wellbore pressure, p_w, cannot be negative, equation 12. Also, the pressure at the external boundary, p_e, may decline so much that it is impossible to lift the fluid to the surface.

The traditional inflow performance plot, Fig. 5, shows that the effect of decreasing reservoir pressure may be predicted by downward shift of the well performance curves.

Combination of the equation of state, equation 2, and the flow equation, equation 9b leads to:

$$p_e - p_w = -\frac{1}{\tau}\ln\left\{1 \mp \frac{q_{sc}\mu_e B_e \tau}{2\pi k_e h}\ \ln r_{eD} + S_\tau\right\} \tag{12}$$

or

$$p_e - p_w = -\frac{1}{\tau}\ln\ 1 \mp \tau_D\ \ln r_{eD} + S_\tau \tag{13}$$

III. Core Analysis

The most direct way to identify stress-sensitive rocks is by special core analysis. This is a time-consuming and expensive process. Usually, a limited number of cores are selected for investigation. The choice may be biased for practical reasons. Still, results from a small group of cores may give more reliable results than single core results.

Routine core analysis, which is conducted at laboratory conditions, provides both results for each core and also the central tendency of the group. The traditional measures are: the median, arithmetic-, geometric-, hyperbolic- and power-law averages. The same measures, in terms of pressure functions, are available for stress-sensitive reservoirs (Jelmert and Selseng [7]). Obtaining these functions may be thought about as simple upscaling. In principle, an average function may be used directly in the deliverability model or as a check on the results obtained by well testing.

The exponential permeability-pressure function lends itself naturally to the traditional geometric average. Suppose we have group of cores, then the geometric average function becomes:

$$\bar{k}_g\ \ p\ = \sqrt[N]{k_1 k_2 \cdots k_n e^{\ \gamma_1+\gamma_2 ... +\gamma_N\ \ p-p_e}}$$
$$= \sqrt[N]{k_1 k_2 \cdots k_N}\, e^{\frac{1}{N}\ \gamma_1+\gamma_2 ... +\gamma_N\ \ p-p_e} \tag{14}$$

As a result, the arithmetic average permeability modulus, $\bar{\gamma}_a$, should be plugged into equation 1. The geometric average reference permeability, $\bar{k}_{eg}$, should be used in equation 8 (Jelmert and Selseng [5]):

$$\bar{\gamma}_a = \frac{1}{N}\ \ \gamma_1 + \gamma_2 + \cdots \gamma_N \tag{15a}$$

$$\bar{k}_{eg}\ \ p_e\ = \sqrt[N]{k_1\ \ p_e\ \ k_2\ \ p_e\ \cdots k_N\ \ p_e} \tag{15b}$$

The reference permeability depends on reference pressure. As such, $\bar{k}_{eg}\ \ p_e$, should be updated with changes in the reference pressure. The permeability modulus, $\bar{\gamma}_a$, shows up as an addend in the definition of the composite elastic modulus, equation 1.

IV. Skin and Apparent Radius

The skin factor in terms of normalized mobility change, S_τ, is skin due to the traditional skin, but derives from a different concept. In the limit, they are equivalent, see equation 19, i.e. when $\tau \cdot \Delta p_{ew} \to 0$. Then, the traditional well performance indices are obtained as shown by equation 22.

From equation 9b we find that:

$$M_{nw} = 1 \pm \frac{q_{sc}\mu_e B_e \tau}{2\pi k_e h}\left(\ln\left(\frac{r_w}{r_e}\right) - S_\tau\right) \tag{16}$$

The terms inside the parenthesis appear in the same way in the above equation. They may be lumped into a single factor as follows:

$$M_{nw} = 1 \pm \frac{q_{sc}\mu_e B_e \tau}{2\pi k_e h}\ln\left(\frac{r_w e^{-S\tau}}{r_e}\right) \tag{17}$$

Hence, one may define an apparent wellbore radius, $r_{aw} = r_w e^{-S_\tau}$. The apparent wellbore radius may be thought of as a fictitious radius which makes the wellbore mobility without skin equal to the wellbore mobility with skin. The concept is illustrated in Fig. 3.

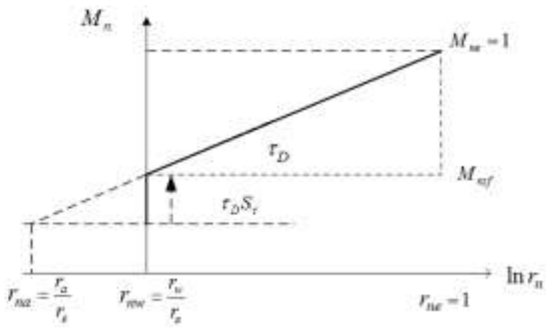

Fig. 3 Equivalent radius

Analytical well test models cannot handle negative skin, but use of the apparent wellbore radius is unproblematic.

V. Well performance predictions

Solving equation 12 for the rate yields:

$$q_{sc} = \pm \frac{2\pi k_e h}{\mu_e B_e \ \ln r_{eD} + S_\tau} \frac{1 - e^{-\tau \Delta p_e}}{\tau} \quad (18)$$

Division by $\Delta p_{ew} = p_e - p_w$ yields:

$$J = \frac{q_{sc}}{\Delta p_{ew}} = \pm \frac{2\pi k_e h}{\mu_e B_e \ \ln r_{eD} + S_\tau} \cdot \frac{1 - e^{-\tau \Delta p_{ew}}}{\tau \Delta p_{ew}} \quad (19)$$

The traditional way to investigate well performance is to prepare a plot of wellbore pressure vs. flowrate, p_w vs. q_{sc}. Then, from equation 19, we have:

$$p_w = p_e \mp \frac{q_{sc}}{J} \quad (20)$$

The equivalent equation for homogeneous reservoirs, without stress-sensitivity, is included in the above equation as limiting behaviour. Let $u = \tau \Delta p_{ew}$, then:

$$\lim_{u \to 0} \left\{ \frac{1 - e^{-u}}{u} \right\} = 1 \quad (21)$$

The above condition may occur for small values of Δp_{ew} and/or τ, (i.e. for the $\tau \cdot \Delta p_{ew}$ -product). Then:

$$\tilde{J} = \frac{\tilde{q}_{sc}}{\Delta p_{ew}} = \pm \frac{2\pi k_e h}{\mu_e B_e \ \ln r_{eD} + S_\tau} \quad (22)$$

which is the equation for the traditional well performance index for reservoirs without stress-sensitivity.

Except for opposite signs, the productivity and injectivity index are characterized by similar equations, see equation 19. The absolute values, however, are different, since the wellbore pressures, p_w, are at opposite sides of the reference pressure, p_e, see Fig. 4. The reservoir pressure difference is related to the mobility function by: $\Delta p_{ew} = -(1/\tau)\ln M_n$.

Figs. 5 and 6 (lower curves), which are based on equation 20, show the effect of the composite modulus, τ, on production and injection. The curves of Fig. 5 show that stress-sensitive mobility has unfavourable effect on the production rate. This is because of reduced fluid pressure in the near wellbore region. Also shown is the detrimental effect of reducing the pressure at the external boundary, p_e. The pressure at the external boundary can be estimated by material balance calculations. For an injection well, stress-sensitivity will improve the injectivity. This is a consequence of increased fluid pressure. These observations are confirmed by Figs 7 and 8. An intuitive explanation is that fracture apertures tend to decrease with decreasing fluid pressure and increase with increasing pressure.

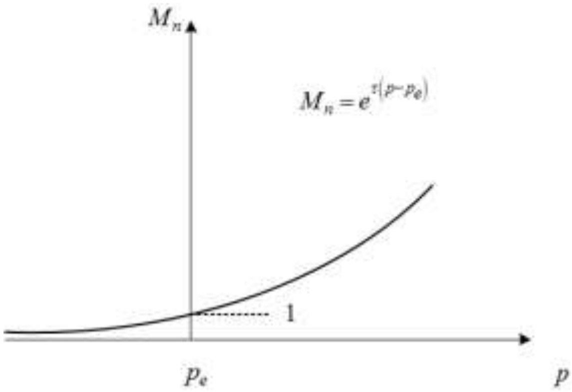

Fig. 4 Mobility function vs. pressure

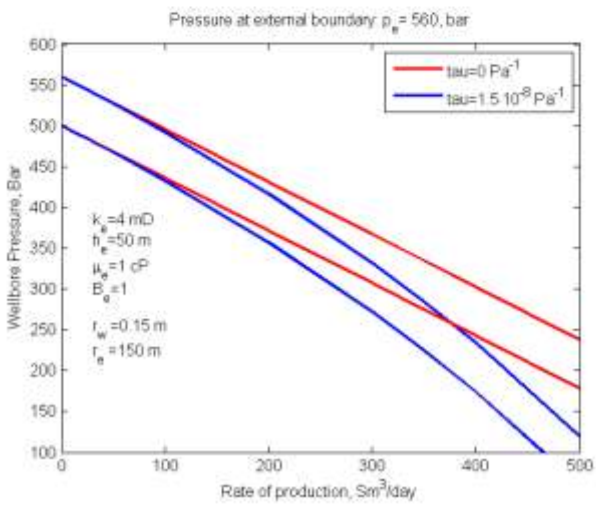

Fig. 5 Inflow performance

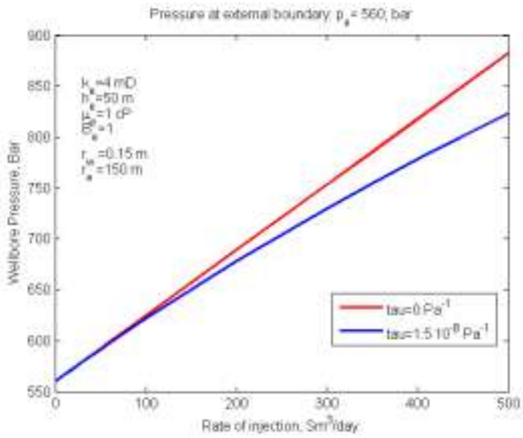

Fig. 6 Outflow performance

VI. VARIATION OF PERMEABILITY WITH DISTANCE

The variation of permeability with distance may be estimated once pressure as a function of distance is known. Combination of equations 3 and 9a yields the pressure difference at any radial distance:

$$p - p_e = +\frac{1}{\tau}\ln\left(1 \pm \tau_D \ln\frac{r}{r_e}\right) \qquad (23)$$

The permeability modulus is given by:

$$\gamma = \frac{1}{k_n}\frac{dk_n}{dp} \qquad (24)$$

The above equation may be integrated to give:

$$p_e - p = -\frac{1}{\gamma}\ln k_n \qquad (25)$$

Inversion yields:

$$k_n = e^{\gamma \; p - p_e} \qquad (26)$$

Substitution of equation 23 into equation 26 yields:

$$k_n \;\; r \;\; = e^{\frac{\gamma}{\tau}\left(\ln\left(1 \pm \tau_D \ln\frac{r}{r_e}\right)\right)} \qquad (27)$$

We define the modulus ratio:

$$\gamma^* = \frac{\gamma}{\tau}, \qquad \gamma^* = \in \;\; 0,1 \qquad (28)$$

Then:

$$k \;\; r \;\; = k_e\left(1 \mp \tau_D \ln\frac{r_e}{r}\right)^{\gamma^*} \qquad (29)$$

Fig. 7 and Fig. 8 show the variation of permeability as a function of distance for a production- and injection well respectively.

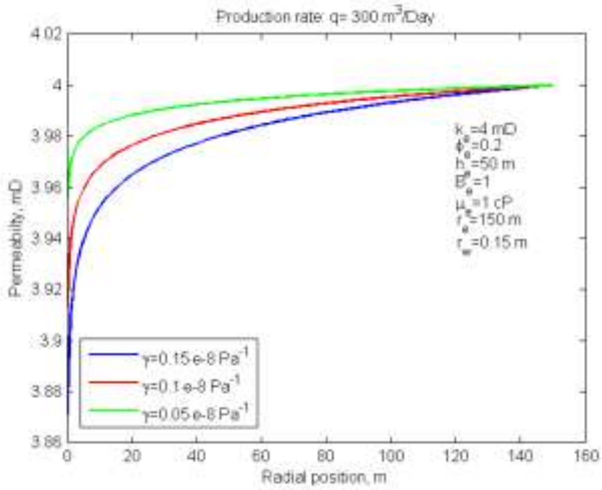

Fig. 7 Effect of permeability modulus on permeability

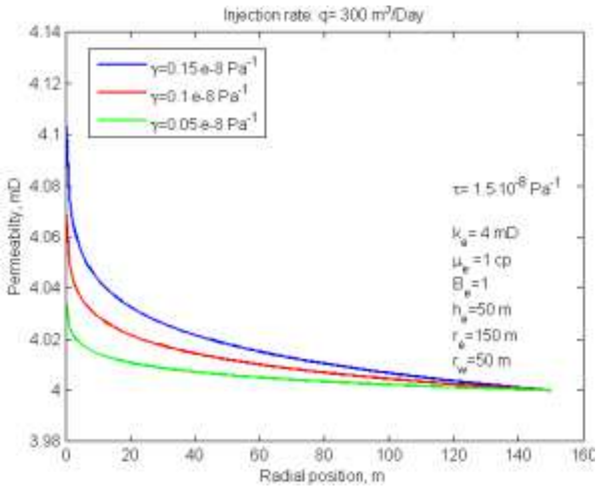

Fig. 8 Effect of permeability modulus on permeability

VII. CONCLUSIONS

A composite elastic modulus, obtained by simple addition, has been proposed. The new modulus may be useful in well performance and permeability predictions. The proposed methodology may be extended to an arbitrary number of quadratic terms. The maximum number is limited by the number of factors in the transport term of the diffusivity equation.

The generalized model simplifies to previous models by assigning zero-value to one or more elastic moduli. The proposed model will revert to the conventional model without stress-sensitivity, when all addends in the composite modulus are zero.

The effect of stress-sensitivity on rock and fluid properties should not be overlooked in cases characterized by moduli of high values and/or large pressure changes in the reservoir.

The traditional, homogenous reservoir model without stress-sensitivity is included in the proposed model as limiting behaviour. The conventional model may be used with negligible errors for small values of the composite elastic moduli, τ and/or small pressure changes in the reservoir, Δp_{ew}.

For production wells, stress-sensitive permeability has detrimental effect on well performance. For injection wells, it is the other way around.

The conventional relationship between the skin factor and apparent wellbore radius applies.

The theoretical operating range of the model is limited by the value the dimensionless distance to the external boundary and the skin factor.

Core analysis may be useful to identify possible stress-sensitivity.

NOMENCLATURE

B Formation volume factor

c Total fluid compressibility, Pa^{-1}

M_n Normalized mobility function, given by equation 2

ΔM_n Change in normalized mobility from the reference value, $\Delta M_n = 1 - M_n$

h Thickness, m

J Productivity/Injectivity index or rate per unit pressure change, equation 23, $Sm^3 s^{-1} Pa^{-1}$

k_e Permeability at the external boundary, m^2

$k\ p$ Permeability as a function of pressure

$\bar{k}_g\ p$ Geometric average of permeability function

p Fluid pore pressure, Pa

Δp_{ew} Pressure decrease/increase between external boundary and the well

Δp_{th} Threshold pressure change between the external boundary and the well

q_{sc} Flow rate, Sm^3 / s

$\tilde{q}_{sc}$ Flow rate for a reservoir without stress-sensitivity

r Radial distance, m

r_D Dimensionless distance, $r_D = r / r_w$

r_n Normalized radial distance, $r_n = r / r_e$

r_a Apparent wellbore radius

S_τ Skin factor to normalized mobility

Greek letters

γ Permeability modulus, Pa^{-1}

$\bar{\gamma}_a$ Arithmetic average of the permeability modulus

γ_S Permeability modulus of altered zone,

τ Composite modulus, Pa^{-1}

υ Viscosity modulus, Pa^{-1}

Δ Change from reference condition

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