Parametric Method Based PSD Estimation using Gaussian Window

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Abstract- *Non-parametric methods of Spectrum Estimation Such as Periodogram, Modified Periodogram, Welch, Bartlett and Blackman-Tukey are generally used but are not always efficient in finding out the power spectral densities. These methods lacks due to their limitations like windowing of the autocorrelation sequence, spectral leakage, poor resolution and incapability to include the available information about the process into the estimation procedure. In these cases parametric approach of spectrum estimation outperforms the nonparametric ones and helps in producing high resolution spectral estimates. Also, parametric methods are more accurate and computationally more efficient than the non-parametric methods. As Gaussian window is an important window in digital signal processing applications. So in the proposed work, Burg's, Yule-Walker, Covariance and Modified Covariance methods are used to estimate the PSD of the Gaussian window function. The simulation is done in MATLAB and compared to analyze the power in different spectral components using different methods.*

Keywords — *Parametric Methods, PSD, Burg's method, Yule Walker method, Covariance method, Modified Covariance method.*

I. INTRODUCTION

Fourier transform of finite energy signals can be calculated. Such signals may be characterized in spectral domain by its energy density spectrum. But, stationary random signals do not possess finite energy; hence, its Fourier Transform cannot be calculated[1, 2]. However, such signals have finite average power. Thus, they are characterized by power spectrum density (PSD). The estimation of PSD involves sampling of the signal over a finite time interval. The shorter the record length of the signal better will be the power spectral resolution [3, 4]. The finite record length of the signal is a major limitation on the quality of power spectrum estimate. The methods of estimating the PSD of a wide sense stationary random process can be broadly classified into two categories: Parametric and Non-Parametric methods. Non-Parametric methods include Periodogram, the Bartlett and Welch Modified Periodogram and Blackman-Tukey based on the Fourier Transformation of the auto-correlation of the data. Parametric methods of power spectral estimation include Burg's method,

Yule-Walker, Covariance and Modified Covariance methods [5]. The advantage of non-parametric methods is that they can be implemented using the fast Fourier Transform but have spectral leakage and limited frequency resolution. These problems can be overcome by using parametric methods which give least spectral leakage and better frequency resolution even with less computational complexity [6]-[7]. Parametric methods involve formation of a model of the process from which spectral density is estimated. It involves modeling of the data sequence x(n), as the output of linear system expressed by a rational system function of the form given by (1).

$$H(z) = \frac{\sum_{k=0}^{q} b_k z^{-k}}{1 + \sum_{k=1}^{p} a_k z^{-k}} \qquad \dots (1)$$

The corresponding difference equation is given by (2).

$$x(n) = -\sum_{k=1}^{p} a_k \, x(n-k) + \sum_{k=0}^{q} b_k w(n-k) \dots(2)$$

Where w(n) represents the input sequence and x(n) represents the output sequence of the system. The PSD of the data is given by (3).

$$\Gamma_{xx}(f) = |H(f)|^2 \Gamma_{ww}(f) \qquad \dots (3)$$

Where $\Gamma_{xx}(f)$ is the PSD of the observed data and $\Gamma_{ww}(f)$ is the PSD of the input data.

With a parametric approach, the first step is to select a suitable model for the process. This selection of suitable model may be based on a priori knowledge of about how the process is generated, or, perhaps, on experimental results indicating that a particular model "works well". The parametric models that are commonly used include autoregressive (AR), moving average (MA) and autoregressive moving average (ARMA) [8, 9]. Once a model has been selected, the next step is to estimate the model parameters from the given data. The final step is to estimate the power spectrum by incorporating the estimated parameters into the parametric form for the spectrum [10]-[12]. In this paper we are estimating the power spectrum of Gaussian Window function using parametric methods, explained in section II. The Gaussian window function is given by (4). This is one of the common window function used for FIR filter design.

$$g(t;\sigma) \equiv \frac{1}{\sqrt{2\pi}\sigma} e^{-\omega^2/2\sigma^2} \qquad \dots (4)$$

Where, σ is the standard deviation. The higher σ gets, the wider gets the Gaussian window and the more severe gets the truncation.
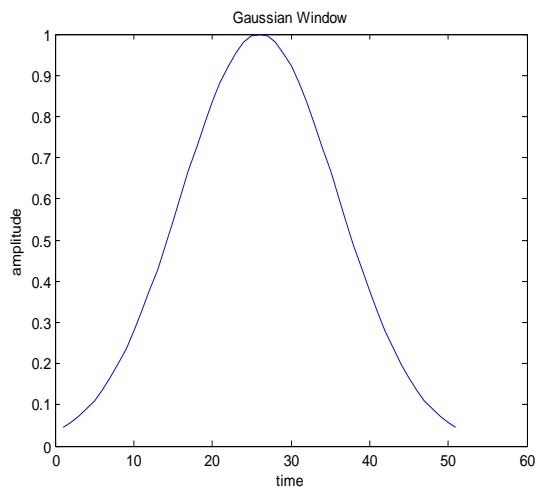

Fig. 1: Gaussian window function

The higher σ the better the frequency response approximates the ideal one below π but also the more distinctive are the bumps above π. The plot of the Gaussian window function in time domain is shown in fig.1. The proposed work in this paper is to estimate the PSD of Gaussian window function using the parametric methods discussed above. The PSD obtained with different methods are compared and the ability of every method to measure the spectral characteristics of the Gaussian window is discussed. Rest of the paper is organized as: Section II discusses the different parametric methods to estimate the PSD of the Gaussian window function. Section III explains the simulated results obtained through MATLAB. Section IV concludes the paper by discussing the pros and cons of each method.

II. PSD ESTIMATION METHODS

Yule-Walker Method

In this method we simply estimate the autocorrelation from the data to solve for AR model parameters. In Yule–Walker method or the autocorrelation method, by minimizing an estimate of prediction error power, the AR parameters can be estimated. Biased form of the autocorrelation estimate is desirable to use,

$$r_{xx}(m) = \frac{1}{N}\sum_{n=0}^{N-m-1} x^*(n)x(n+m), \qquad m \geq 0 \quad \dots(5)$$

To ensure semi definite autocorrelation matrix is positive semidefinite. A stable AR model is resulted. It is conjectured that a stable AR model best represents the data, though for power spectrum estimation, stability is not a critical issue. From Levinson-Durbin algorithm the AR parameters are obtained. The power spectrum estimate corresponding to this is

$$P_{xx}^{YW}(f) = \frac{\sigma_{\omega p}^2}{\left|1+\sum_{k=1}^{p}\hat{a}_p(k)e^{-j2\pi fk}\right|^2} \qquad \dots(6)$$

Where, $\hat{a}_p(k)$ are estimates of AR parameters obtained from the Levinson-Durbin recursions.

Burg Method

Based on autoregressive (AR) model for the PSD estimation, Burg Method comes under the class of parametric methods. This system is described by the following difference equation.

$$x(n) = -\sum_{k=1}^{p} a_k x[n-k] + e[n] \qquad \dots(7)$$

where $e[n]$ is the unobserved input data, $x[n]$ is the observed output of the system and the a_k are its coefficients. This method is an order recursive least-squares lattice method, based on the minimization of the forward and backward errors in linear predictors, having constraint of the AR parameters that satisfy the Levinson-Durbin recursion. From estimates of the AR parameters, the power spectrum estimate is expressed as

$$P_{xx}^{BU}(f) = \frac{\hat{E}_p}{\left|1+\sum_{k=1}^{p}\hat{a}_p(k)e^{-j2\pi fk}\right|^2} \qquad \dots(8)$$

Where $\hat{E}_p$ is the total least-square error having order p. The major advantages of the Burg Method are that it has high frequency resolution, the AR model is always stable and is computationally very efficient. It however exhibits, several limitations which are frequency shifting from the true frequency, occurs especially for short data record and for high signal to noise ratios, line splitting may appear in the PSD [13].

Covariance Method

The major and only difference between the Yule–Walker method and the covariance method is the range of summation present in the prediction error power estimate. To compute the prediction error power estimate, all the data points are needed in the covarianve method. Zeroing of the data is not necessary. The solution of the equations as AR parameter estimate is given as

$$\begin{bmatrix} c(1,0) \\ \vdots \\ c(p,0) \end{bmatrix} + \begin{bmatrix} c(1,1) & \dots & c(1,p) \\ \vdots & \ddots & \vdots \\ c(p,1) & \dots & c(p,p) \end{bmatrix}\begin{bmatrix} a(1) \\ \vdots \\ a(p) \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix} \dots(9)$$

Where,

$$c(j,k) = \frac{1}{N-p}\sum_{n=p}^{N-1} x^*(n-j)x(n-k) \qquad \dots(10)$$

From (10) the AR parameter estimates are found as

$$a = -C_p^{-1}c_p \qquad \dots(11)$$

An autoregressive (AR) model of order p is fitted to the input signal in the covariance method, which is assumed to be an AR system white noise driven output[12]. This method focuses on minimization of the forward prediction error in least-squares sense. Normalized estimate of the AR system parameters are contained in the output vector. The power spectral density is given as

$$P_{xx}^{COV}(f) = \frac{\varepsilon}{\left|1+\sum_{k=1}^{p} \hat{a}_p(k)e^{-j2\pi fk}\right|^2} \qquad …(12)$$

Modified Covariance Method

Suppose $x(n)$, $n=0,1,.,$ N-1, be a given data used to derive the estimator, consider the estimates of forward and backward linear prediction of order m , as

$$\hat{x}(n) = -\sum_{k=1}^{m} a_m(k)x(n-k) \qquad …(13)$$
$$\hat{x}(n-m) = -\sum_{k=1}^{m} a_m^*(k)x(n+k-m) …(14)$$

and the corresponding forward and backward errors as

$$g_m(n) = x(n-m) - \hat{x}(n-m) \qquad \text{and}$$
$$f_m(n) = x(n) - \hat{x}(n)$$

The least square error is

$$\varepsilon_m = \sum_{n=m}^{N-1}\left[|f_m(n)|^2 + |g_m(n)|^2\right] \qquad …(15)$$

ε_m should have a minimized value. As considered the case of covariance method the summations are over prediction errors only that involve observed data samples. An alternative way of seeing this estimator is to recognize that the prediction error power estimate is obtained by flipping the data record around and complex conjugating it and applying a forward predictor to this new data set. For any set of $\hat{a}_p(k)$ the forward and backward prediction of error estimates will be slightly different because of the range of summations[14, 15] .

III. PSD ESTIMATION SIMULATION

The MATLAB stimulated results of PSD estimate using Yule Walker, Burg's, Covariance and Modified Covariance method is shown in fig. 2, 3, 4 & 5 respectively. Fig. 2 shows the Power Spectral Density estimate of Gaussian window function using Yule Walker method. It can be clearly observed from fig. 2 that power spectral density estimated by Yule Walker method is giving a flat estimate of power. The power spectral density estimate of Gaussian window function

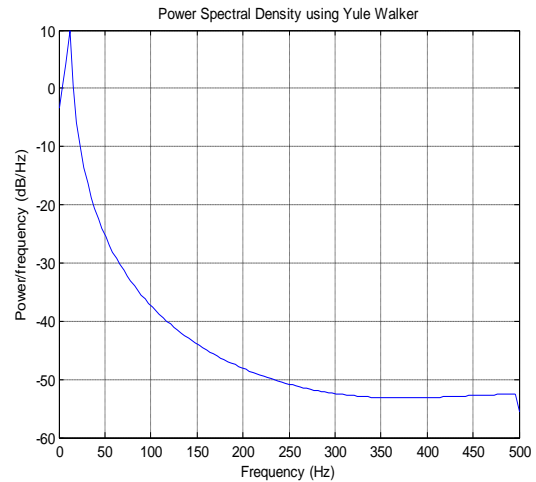using Burg's method is clearly shown in fig. 3.

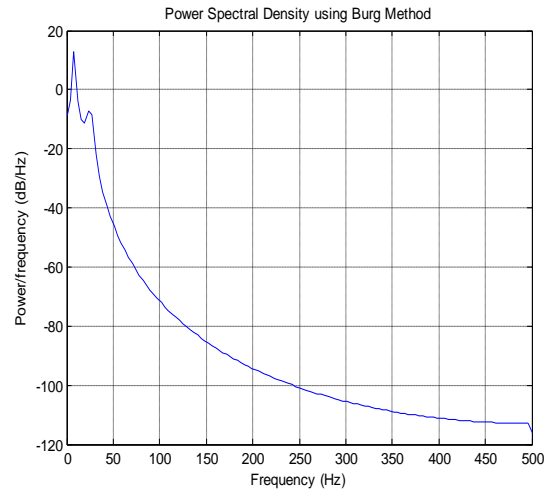Fig. 2: Yule Walker method for PSD.

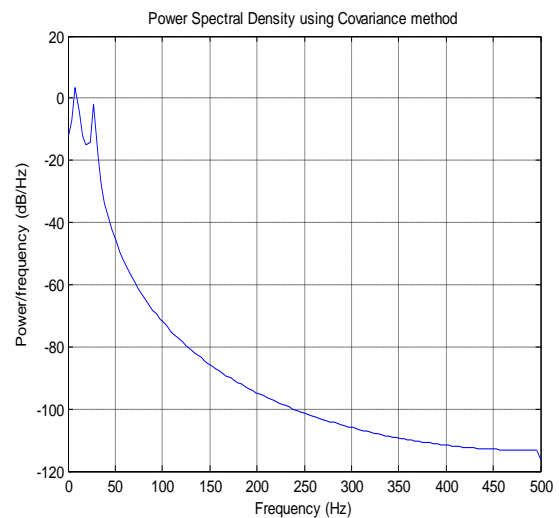Fig. 3: Burg's method for PSD.

Fig. 4: Covariance method for PSD.

As seen in fig. 4, Power Spectral Density estimate of Gaussian window function using Covariance method is estimated. If the previous two methods are observed for estimating power spectrum then it can be clearly

seen that the power spectrum obtained using covariance method is not as smooth as Yule Walker method and Burg's method. Also, fig. 5 shows the Power Spectral Density estimate of Gaussian window function using Modified Covariance method.
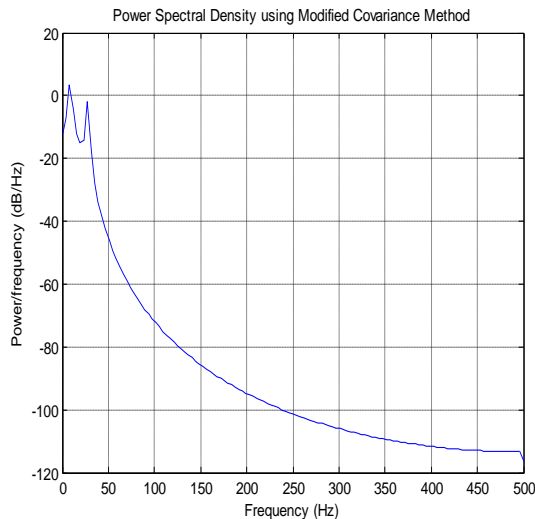

Fig. 5: Modified Covariance method for PSD.

IV. RESULT ANALYSIS

The comparison of PSD estimate using all the discussed parametric methods is shown in figure 6. The PSD estimate using Yule Walker method gives the relatively flat estimate which results in loss of some information about the variance in the spectral components. Covariance and Modified Covariance methods performs almost same. Methods other than Yule Walker give spikes in the PSD estimate. Taking a close view on the comparative analysis of the PSD estimates using different methods, one can clearly understand the difference in the estimation of power of different spectral components. Burg, Covariance and Modified Covariance methods give nearly same PSD estimates. However, Yule walker method does not give information about the spikes in the spectrum near the frequency range 25-30 Hz.
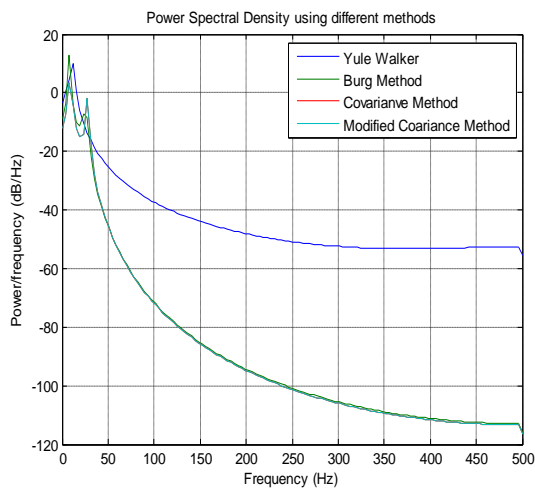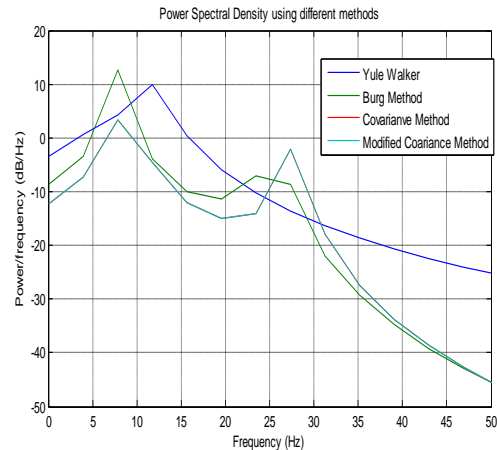

Fig. 6: PSD comparison.

Fig. 7: PSD comparison zoomed view.

Such analysis is shown in fig. 7.

V. CONCLUSION

The paper focuses on the power spectral density estimation of the Gaussian window function used in FIR filter designing. The spectrum estimation is done using Yule Walker, Burg's, Covariance and Modified Covariance methods of parametric power spectrum estimation. The simulated results are compared which clearly shows that the Yule Walker method gives relatively flat PSD estimate but this method is not as accurate as compared to the other methods discussed. Because, it results in less frequency resolution. Burg's method performs equally well as Covariance and Modified Covariance methods. However, Covariance and Modified Covariance exhibits a sharp spike in the range 25-30 Hz which is less prominent in Burg's method and almost flat in Yule Walker method. From the observations it is clear that Covariance and Modified Covariance methods outperforms the other parametric methods in estimating the PSD of the Gaussian Window function.

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