Sum Distance in Bipolar Fuzzy Graphs

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Abstract

In this paper, sum distance in bipolar fuzzy graph is defined and the properties of eccentricity, radius and sum distance of a bipolar fuzzy graph are studied. A characterization of self centered complete bipolar fuzzy graph is obtained. The sufficient condition for a cycle to be self centered based on sum distance is discussed.

Key words: *Bipolar fuzzy graph, sum distance, fuzzy cycle, eccentricity, central vertex*

AMS subject classification: 05C12, 03E72, 05C72.

1 Introduction

Euler first introduced the concept of graph theory in 1736. Fuzzy set theory was first introduced by Zadeh in 1965[15]. The first definition of fuzzy graph was introduced by Haufmann in 1973 based on Zadeh's fuzzy relations in 1971. In 1975, Zhang initiated the concept of bipolar fuzzy sets as a generalization of fuzzy sets in 1994. Bipolar fuzzy sets whose range of membership degree is [-1,1]. In bipolar fuzzy sets, membership degree 0 of an element means that the element is irrelevant to the corresponding property, the membership degree within (0,1] of an element indicates that the element somewhat satisfies the property and the membership degree within [-1,0) of an element indicates the element somewhat satisfies the implicit counter property. It is noted that positive information represents what is granted to be possible, while negative information represents what is considered to be impossible. For instance, when we assess the position of an object in a space, we may have positive information expressed as a set of possible places and negative information expressed as a set of impossible places.

1.1 Review of Literature

A.Rosenfeld introduced the concept of fuzzy graphs[12]. Now, fuzzy graphs have many applications in branches of engineering and technology. A. Nagoorgani and K. Radha introduced the concept of degree,total degree, regular fuzzy graphs in 2008 [9]. M.Akram and Wieslaw A.Dudek introduced regular and totally regular bipolar fuzzy graphs. Also they introduced the notion of bipolar fuzzy line graphs and present some of their properties[4]. Sovan Samanta and Madhumangal Pal introduced irregular bipolar fuzzy graphs[13]. K.Radha and N.Kumaravel introduced an edge regular bipolar fuzzy graphs and totally edge regular bipolar fuzzy graphs[11]. Mini Tom and M.S. Sunitha introduced the concept of sum distance in fuzzy graphs. These motivates us to introduce sum distance in bipolar fuzzy graphs. Throughout this paper, $m_1^+ ,m_2^+ \in [0, 1]$ and $m_1^- ,m_2^- \in [-1, 0]$.

2 Preliminaries

We present some known definitions related to fuzzy graphs for ready reference to go through the work presented in this paper.

Definition 2.1. [7] A fuzzy graph $G : (\sigma, \mu)$ is a pair of functions (σ, μ) , where $\sigma : V \rightarrow [0, 1]$ is a fuzzy subset of a non empty set V and $\mu : V X V \rightarrow [0, 1]$ is a symmetric fuzzy relation on σ such that for all u, v in V , the relation $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$ is satisfied. A fuzzy graph G is called complete fuzzy graph if the relation $\mu(u, v) = \sigma(u) \wedge \sigma(v)$ is satisfied.

Definition 2.2. [1] A bipolar fuzzy graph with an underlying set V is defined to be the pair (A, B), where $A = (m_1^+ ,m_1^-)$ is a bipolar fuzzy set on V and $B = (m_2^+,m_2^-)$ is a bipolar fuzzy set on E such that $m_2^+(x, y) \leq min\{(m_1^+(x), m_1^+ (y)\}$ and $m_2^-(x, y) \geq max\{m_1^-(x), m_1^-(y)\}$ for all $(x,y) \in E$. Here, A is called bipolar fuzzy vertex set on V and B is called bipolar fuzzy edge set on E.

Definition 2.3. [14] G is said to be a bipolar fuzzy cycle if there does not exists unique edge uv such that $m_2^+(uv) = \wedge\{m_2^+(xy) : xy \in E\}$ and $m_2^-(uv) = \vee\{m_2^-(xy) : xy \in E\}$.

Definition 2.4. [13] A bipolar fuzzy graph $G = (A,B)$ is connected if the underlying crisp graph is connected.

Definition 2.5. [3] Let G be a bipolar fuzzy graph. A vertex v is called central vertex if radius of G= eccentricity of v . The set of all central vertices in G is denoted by $C(G)$. G is said to be self-centered if each vertex is a central vertex.

Definition 2.6. [3] Let $G = (A,B)$ be a bipolar fuzzy graph. Then the fuzzy subgraph induced by $C(G)$ is known as center of G.

3 Sum Distance in Bipolar Fuzzy Graph

Definition 3.1. For any two vertices u, v in G let $P = \{p_i : p_i \text{ is a } u - v \text{ path}, i = 1, 2, 3 \ldots\}$. For any path P, the positive length of P is defined as the sum of positive weight of edges in P and the negative length of P is defined as the sum of negative weight of edges in P. i.e; $L^+(P) = \sum_{i=1}^{n} m_2^+(u_{i-1}, u_i)$ and $L^-(P) = \sum_{i=1}^{n} m_2^-(u_{i-1}, u_i)$. The positive sum distance between two vertices is defined as $d_s^+(u, v) = min\{L^+(P)\}$ and the negative sum distance between two vertices is defined as $d_s^-(u, v) = max\{L^-(P)\}$.

Definition 3.2. The positive eccentricity $e^+(u)$ is the positive sum distance to a vertex farthest from u i.e; $e^+(u) = max\{d_s^+(u, v) : v \in V\}$. The negative eccentricity $e^-(u)$ is the negative sum distance to a vertex farthest from u i.e; $e^-(u) = min\{d_s^-(u, v) : v \in V\}$. The eccentricity of a vertex u is defined as $e(u) = (e^+(u), e^-(u))$.

Definition 3.3. For a vertex u, each vertex at sum distance $e^+(u)$ from u is positive eccentric vertex of u denoted by $u^{(*,+)}$ and each vertex at sum distance $e^-(u)$ from u is negative eccentric vertex of u denoted by $u^{(*,-)}$. The eccentric vertex of u is denoted by $u^* = (u^{(*,+)}, u^{(*,-)})$.

Definition 3.4. The radius of G is defined as $r(G) = (r^+(G), r^-(G))$ where $r^+(G) = min\{e^+(u) : u \in G\}$, $r^-(G) = max\{e^-(u) : u \in G\}$.

Definition 3.5. The diameter of G is defined as $d(G) = (d^+(G), d^-(G))$ where $d^+(G) = max\{e^+(u) : u \in V\}$ and $d^-(G) = min\{e^-(u) : u \in V\}$.

Definition 3.6. A vertex u is called a central vertex if $e^+(u) = r^+(G)$ and $e^-(u) = r^-(G)$. A vertex u is called peripheral vertex if $e^+(u) = d^+(G)$ and $e^-(u) = d^-(G)$

Definition 3.7. A bipolar fuzzy graph G is self centered if each vertex in G is a central vertex

Example 3.8. Consider a bipolar fuzzy graph on $G^*(V,E)$

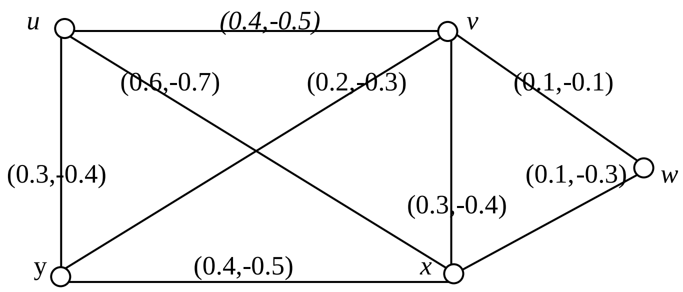


Figure.1

$d_s^+(u, v) = 0.4$, $d_s^+(v, x) = 0.1$, $d_s^-(u, v) = -0.5$, $d_s^-(v, x) = -0.3$

$d_s^+(u,w) = 0.5$, $d_s^+(v, y) = 0.2$, $d_s^-(u,w) = -0.6$, $d_s^-(v, y) = -0.3$

$d_s^+(u, x) = 0.6$, $d_s^+(w, x) = 0.3$, $d_s^-(u, x) = -0.7$, $d_s^-(w, x) = -0.4$

$d_s^+(u, y) = 0.3$, $d_s^+(w, y) = 0.3$, $d_s^-(u, y) = -0.4$, $d_s^-(w, y) = -0.4$

$d_s^+(v,w) = 0.1$, $d_s^+(x, y) = 0.4$, $d_s^-(v,w) = -0.1$, $d_s^-(x, y) = -0.5$

Thus, $e^+(u) = 0.6$, $e^+(v) = 0.4$, $e^+(w) = 0.5$, $e^+(x) = 0.6$, $e^+(y) = 0.4$
$e^-(u) = -0.7$, $e^-(v) = -0.5$, $e^-(w) = -0.6$, $e^-(x) = -0.7$, $e^-(y) = -0.5$
$e(u) = (0.6,-0.7)$, $e(v) = (0.4,-0.5)$, $e(w) = (0.5,-0.6)$, $e(x) = (0.6,-0.7)$, $e(y) = (0.4,-0.5)$

Also, $r^+(G) = 0.4$, $r^-(G) = -0.5$, $d^+(G) = 0.6$, $d^-(G) = -0.7$.
So, $r(G) = (0.4,-0.6)$ and $d(G) = (0.6,-0.7)$

Theorem 3.9. For any connected bipolar fuzzy graph G, $r^+(G) \le d^+(G) \le 2r^+(G)$ and $r^-(G) \ge d^-(G) \ge 2r^-(G)$.
Proof. $r^+(G) \le d^+(G)$ and $r^-(G) \ge d^-(G)$ follows from the definition. Let w be the central vertex of G. Then, $e^+(w) = r^+(G)$ and $e^-(w) = r^-(G)$. Let u and v be the peripheral vertices of G. Then, $e^+(u) = e^+(v) = d^+(G)$ and $e^-(u) = e^-(v) = d^-(G)$. By triangle inequality, $d_s^+(u, v) \le d_s^+(u,w) + d_s^+(w, v)$ and $d_s^-(u, v) \ge d_s^-(u,w) + d_s^-(w, v) \Rightarrow d^+(G) \le r^+(G) + r^+(G)$ and $d^-(G) \ge r^-(G) + r^-(G) \Rightarrow d^+(G)$

$\leq 2r^{+}(G)$ and $d^{-}(G) \geq 2r^{-}(G)$. Hence the result follows.

Theorem 3.10. For every two adjacent vertices u and v in a connected bipolar fuzzy graph G, $|e^{+}(u) - e^{+}(v)| \leq 1$ and $|e^{-}(u) - e^{-}(v)| \geq -1$

Proof. Assume that $e^{+}(u) \geq e^{+}(v)$ and $e^{-}(u) \leq e^{-}(v)$. Let x be a vertex farthest from u . Then, $e^{+}(u) = d_s^{+}(u, x) \leq d_s^{+}(u, v) + d_s^{+} v, x)$ and $e^{-}(u) = d_s^{-}(u, x) \geq d_s^{-}(u, v) + d_s^{-}(v, x)$. Since $e^{+}(v) \geq d_s^{+}(v, x)$ and $e^{-}(v) \leq d_s^{-}(v, x)$, $e^{+}(u) \leq d_s^{+}(u, v) + e^{+}(v)$ and $e^{-}(u) \geq d_s^{-}(u, v) + e^{-}(v)$. Since u and v are adjacent vertices, $d_s^{+}(u, v) \leq 1$ and $d_s^{-}(u, v) \geq -1$. So, $e^{+}(u) \leq 1 + e^{+}(v)$ and
$e^{-}(u) \geq -1 + e^{-}(v) \Rightarrow e^{+}(u) - e^{+}(v) \leq 1$ and $e^{-}(u) - e^{-}(v) \geq -1$. Hence $|e^{+}(u) - e^{+}(v)| \leq 1$ and $|e^{-}(u) - e^{-}(v)| \geq -1$.

Theorem 3.11. Let u and v be the adjacent vertices in G. Then $| d_s^{+}(u, x) - d_s^{+}(v, x)| \leq 1$ and $| d_s^{-}(u, x) - d_s^{-}(v, x)| \geq -1$, for every vertex x of G.

Proof. Let u and v be the adjacent vertices in G and let x be any vertex. Assume that $d_s^{+}(u, x) \geq d_s^{+}(v, x)$ and $d_s^{-}(u, x) \leq d_s^{-}(v, x)$. Since u and v are adjacent vertices, $d_s^{+}(u, x) \leq 1 + d_s^{+}(v, x)$ and $d_s^{-}(u, x) \geq -1 + d_s^{-}(v, x) \Rightarrow 0 \leq d_s^{+}(u, x) - d_s^{+}(v, x) \leq 1$ and $0 \geq d_s^{-}(u, x) - d_s^{-}(v, x) \geq -1$. Hence $| d_s^{+}(u, x) - d_s^{+}(v, x)| \leq 1$ and $| d_s^{-}(u, x) - d_s^{-}(v, x)| \geq -1$.

Remark 3.12. For any real numbers $a, b \in [0, 1]$ and $c, d \in [-1, 0]$ such that $0 \leq a \leq b \leq 2a$ and $0 \geq c \geq d \geq 2c$, then there exist bipolar fuzzy graph G such that $r^{+}(G) = a$, $r^{-}(G) = d$, $d^{+}(G) = b$, $d^{-}(G) = c$.

Example 3.13. Consider a bipolar fuzzy graph on $G^{*}(V,E)$

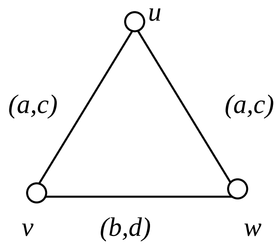


Figure.2

$d^{+}(u, v) = a$, $d^{-}(u, v) = c$, $d^{+}(u,w) = a$, $d^{-}(u,w) = c$, $d^{+}(v,w) = b$, $d^{-}(v,w) = d$.
So, $e^{+}(u) = a$, $e^{+}(w) = b$, $e^{+}(v) = b$, $e^{-}(u) = c$, $e^{-}(w) = d$, $e^{-}(v) = d$.
Hence $r^{+}(G) = a$, $r^{-}(G) = d$, $d^{+}(G) = b$,$d^{-}(G) = c$.

Theorem 3.14. If G is self centered bipolar fuzzy graph, then each vertex of G is eccentric.

Proof. Assume that G is self centered bipolar fuzzy graph and let u be any vertex of G. Let $v = (v^{+}, v^{-})$ be any eccentric vertex of u . Then, $u^{(*,+)} = v^{+}$ and $u^{(*,-)} = v^{-}$. Then $e^{+}(u) = d_s^{+}(u, v)$ and $e^{-}(u) = d_s^{-}(u, v)$. Since G is self centered, we have $e^{+}(u) = e^{+}(v)$ and $e^{-}(u) = e^{-}(v)$. So, $e^{+}(u) = d_s^{+}(u, v) = e^{+}(v)$ and $e^{-}(u) = d_s^{-}(u, v) = e^{-}(v) \Rightarrow u$ is an eccentric vertex of v . Hence each vertex of G is eccentric.

Example 3.15. Consider a bipolar fuzzy graph on $G^{*}(V,E)$.

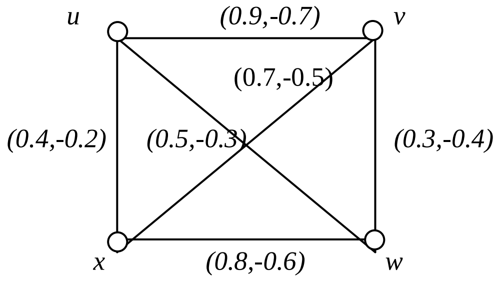


Figure.3

Remark 3.16. The condition given in Theorem 3.14 is not sufficient. In Figure.3, each vertex is eccentric but G is not self centered. Here, $r(G) =(0.8,-0.6)$ and $d(G) = (0.9,-0.7)$ and w, x are central vertices.

Theorem 3.17. If G is self centered bipolar fuzzy graph, then for every pair of vertices $u, v \in G$, $u^{+} \in V^{(*,+)} \Rightarrow v^{+} \in U^{(*,+)}$ and $u^{-} \in V^{(*,-)} \Rightarrow v^{-} \in U^{(*,-)}$ where $U^{(*,+)}$, $V^{(*,+)}$ are the positive eccentric vertices of u and v and $U^{(*,-)}$, $V^{(*,-)}$ are the negative eccentric vertices of u and v .

Proof. Assume that G is self centered and let u, v be any two vertices of G. Let u be the eccentric vertex of v . i.e; $d_s^+(v, u) = e^+(v)$ and $d_s^-(v, u) = e^-(v)$. Since G is self centered, we have $e^+(u) = e^+(v)$ and $e^-(u) = e^-(v)$. Also, $d_s^+(v, u) = d_s^+(u, v) = e^+(u)$ and $d_s^-(v, u) = d_s^-(u, v) = e^-(u) \Longrightarrow d_s^+(u, v) = e^+(u)$ and $d_s^-(u, v) = e^-(u)$. So v is an eccentric vertex of u.

Theorem 3.18. In a bipolar fuzzy graph G, all the peripheral vertices are eccentric vertices.
Proof. Let u be the peripheral vertex of G. Then $e^+(u) = d^+(G)$ and $e^-(u) = d^-(G)$. Then there exists at least one vertex v in G such that $e^+(u) = d_s^+(u, v) = d^+(G)$ and $e^-(u) = d_s^-(u, v) = d^-(G)$. So, $v^{(*,+)} = u^+$ and $v^{(*,-)} = u^-$. Thus u is an eccentric vertex of v . Hence all the peripheral vertices are eccentric vertices.

Remark 3.19. The converse of above theorem need not be true. In Figure.3, u, v, w, x are eccentric vertices but only u and v are peripheral vertices.

Remark 3.20. A bipolar fuzzy cycle need not be self centered.

Example 3.21. Consider a bipolar fuzzy graph on $G^*(V,E)$.

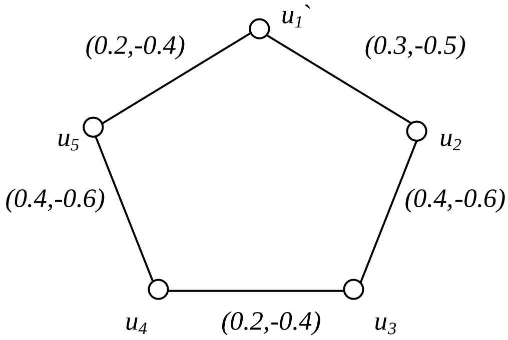


Figure. 4

Here, $r(G) = (0.6,-1.0)$ and $e(u_1) \neq r(G)$. So, u_1 is not a central vertex. Hence this graph is not self centered.

4 Self Centered Bipolar Fuzzy Cycle

Theorem 4.1. Let $G = (A,B)$ be a bipolar fuzzy graph on $G^*(V,E)$, a cycle of length n .Then G is self centered if

$$(i)\ B(e_i) = \begin{cases} (t_1, t_2) & for\ i = 1,3,5 \dots, n-1 \\ (s_1, s_2) & for\ i = 2,4,6,\dots, n \end{cases}$$

$$(ii)\ B(e_i) = \begin{cases} (s_1, s_2) & for\ i = 1,3,5 \dots, n-2 \\ (t_1, t_2) & for\ i = 2,4,6,\dots, n-1 \end{cases}$$

$$B(e_n) = (s_1, s_2),\ n = 4k-1,\ k = 1,2,3\dots$$

$$(iii) B(e_i) = \begin{cases} (t_1, t_2) & for\ i = 1,3,5 \dots, n-2 \\ (s, s_2) & for\ i = 2,4,6,\dots, n-1 \end{cases}$$

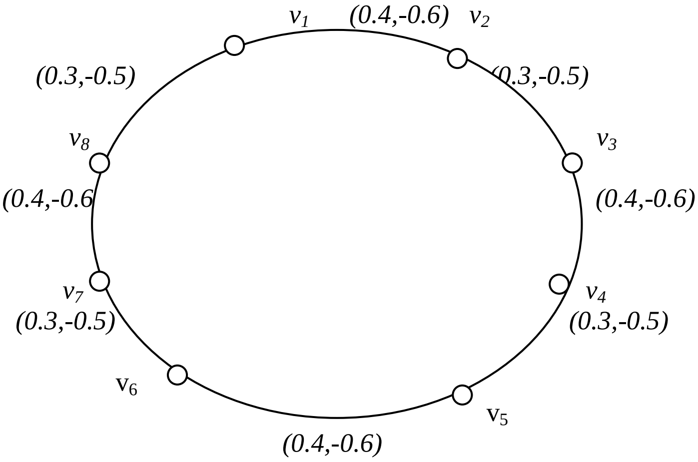
$$B(e_n) = (t_1, t_2),\ n = 4k+1,\ k = 1,2,3\dots$$

$$\text{Also, } r^+(G = \begin{cases} k(t_1 + s_1) & n = 4k\ or\ 4k+1, k = 1,2,,3 \dots. \\ k(t_1 + s_1) - t_1 & n = 4k-1, k = 1,2,3,\dots \\ k(t_1 + s_1) + t_1 & n = 4k+2, k = 1,2,3,\dots \end{cases}$$

$$r^-(G = \begin{cases} k(t_2 + s_2) & n = 4k\ or\ 4k+1, k = 1,2,,3 \dots. \\ k(t_2 + s_2) + t_2 & n = 4k-1, k = 1,2,3,\dots \\ k(t_2 + s_2) + t_2 & n = 4k+2, k = 1,2,3,\dots \end{cases}$$

Illustration 1: Take $t_1 = 0.3$, $t_2 = -0.5$, $s_1 = 0.4$, $s_2 = -0.6$

Case 1 : n is even and $n = 4k, k = 1, 2$



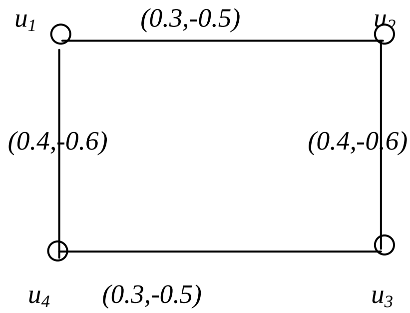
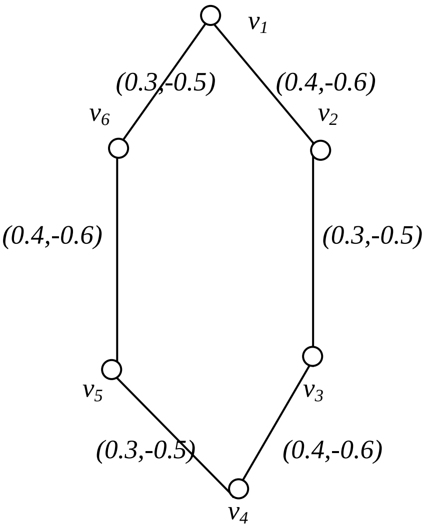


Figure.5: C_8

Figure.6: C_4

Case 2: n is even and $n=4k+2, k = 1, 2$



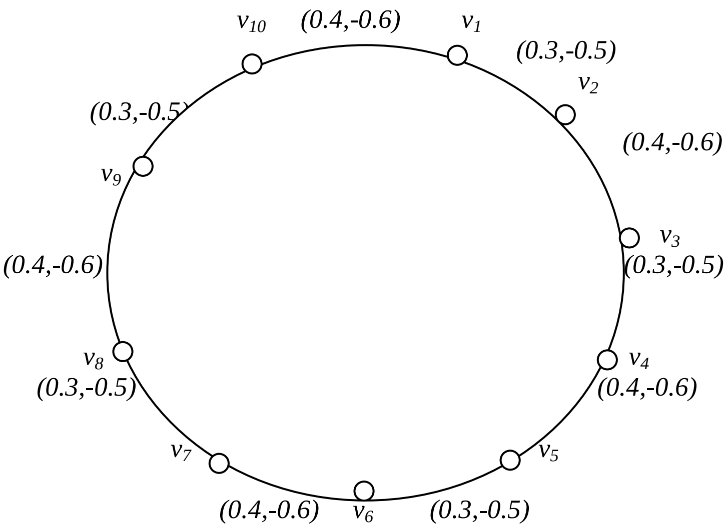


Figure.7: C_6

Figure.8: C_{10}

Case 3: n is odd and $n = 4k-1, k = 1, 2$

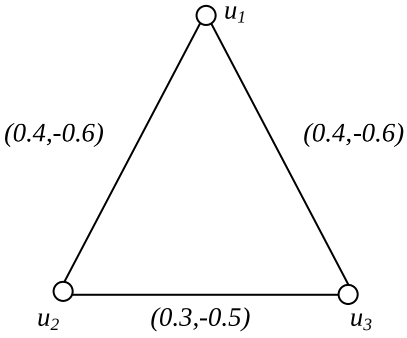


Figure.9: C_3

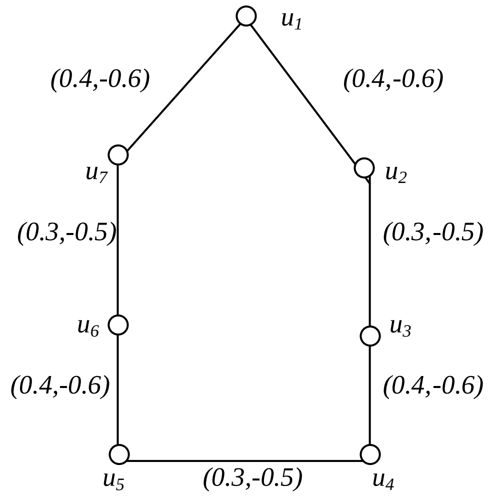


Figure.10: C_7

Case 4: n is odd and $n = 4k+1, k = 1, 2$

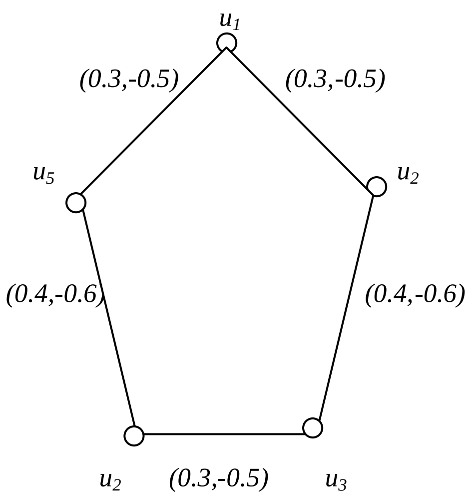


Figure.11: C_5

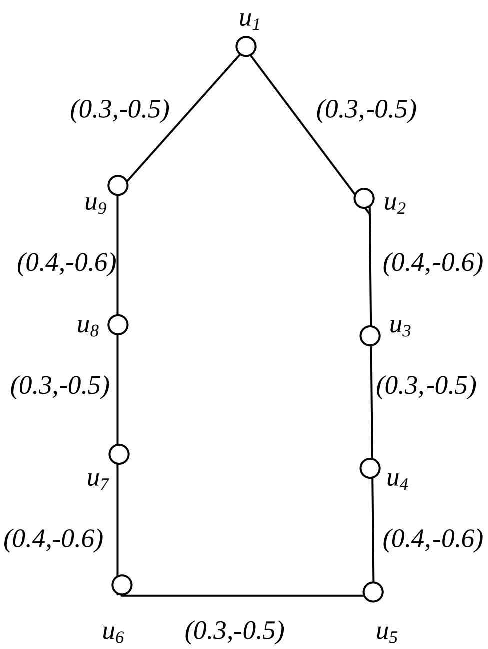


Figure.12: C_9

5 Sum distance in complete bipolar fuzzy graph

Remark 5.1. A complete bipolar fuzzy graph need not be self centered.

Example 5.2. Consider a bipolar fuzzy graph on $G^*(V,E)$.

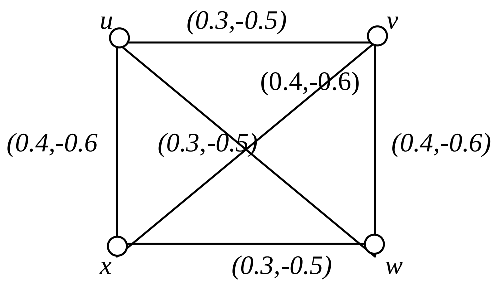


Figure.13

Theorem 5.3. Let G be a complete bipolar fuzzy graph on $G^*(V,E)$. Let the vertices takes the membership values $(c_1, k_1), (c_2, k_2),\ldots, (c_n, k_n)$ such that $c_1 \leq c_2 \leq \cdots \leq c_n$ and $k_1 \geq k_2 \geq \cdots \geq k_n$. Then the positive sum distance between any two vertices u_i, u_j is either $m_2^+ (u_i, u_j)$ or $2m_1^+ (u_1)$ and the negative sum distance between any two vertices u_i, u_j is either $m_2^- (u_i, u_j)$ or $2m_1^-(u_1)$

Proof. Let u_i, u_j be any two vertices in G. We have

$d_s^+(u_i, u_j)=min\ \{m_2^+(u_i, u_j) , m_2^+(u_1, u_k) + m_2^+(u_k, u_j)\}$ and

$d_s^- (u_i, u_j)=max\{m_2^-(u_i, u_j) , m_2^-(u_1, u_k) + m_2^-(u_k, u_j)\}$

Also, since, $c_1 \leq c_n$ and $k_1 \geq k_n$, for all $i = 2, 3, . . . , n$

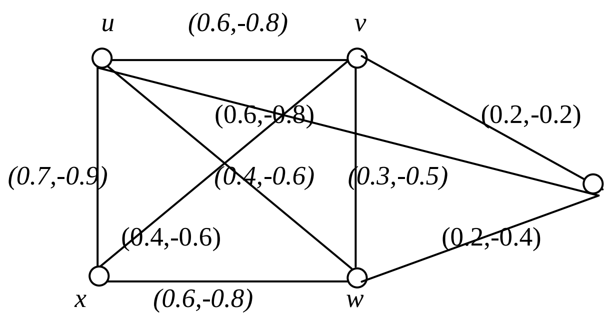
when $k = 1, m_2^+(u_i, u_1) = m_1^+ (u_1)$ and $m_2^+(u_i, u_j) = m_1^+(u_1)$ and

$m_2^- (u_i, u_1) = m_1^- (u_1)$ and $m_2^- (u_i, u_j) = m_1^- (u_1)$

So, $d_s^+(u_i, u_j)=min\ \{m_2^+(u_i, u_j), 2m_1^+(u_1)\}$, $d_s^- (u_i, u_j)=max\{m_2^- (u_i, u_j), 2m_1^- (u_1)\}$

Remark 5.4. A center of a bipolar fuzzy tree need not be either k_1 or k_2 .
Also, Center of a bipolar fuzzy tree need not be a fuzzy tree.

Example 5.5. Consider a bipolar fuzzy graph on $G^*(V,E)$



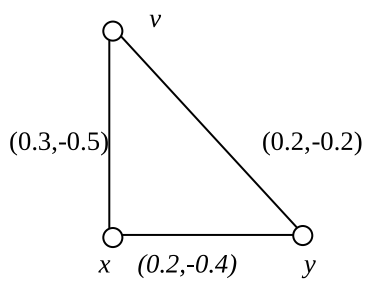


Figure.14: Bipolar Fuzzy tree G ***Center of G***

In Figure.14, the center of G is not a bipolar fuzzy tree.

Remark 5.6. A Center of a connected bipolar fuzzy graph need not be connected.

Example 5.7. Consider a bipolar fuzzy graph on $G^*(V,E)$

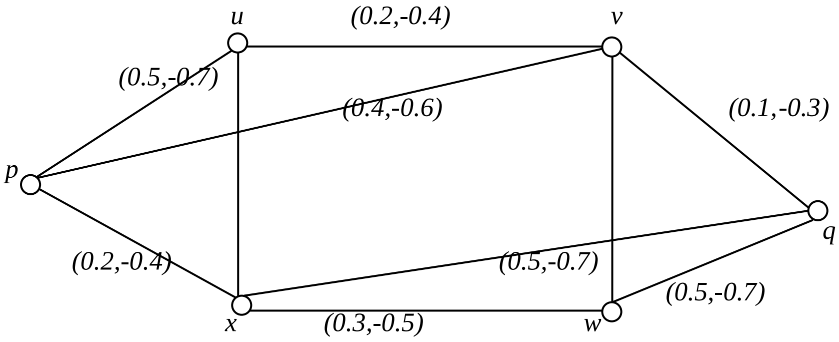
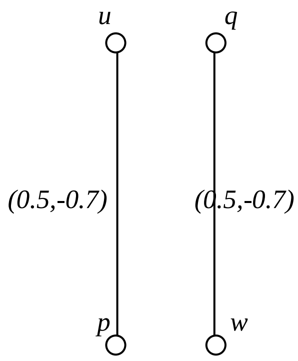


Figure.15:G



Center of G

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