A Guaranteed Stable Sliding Discrete Fourier Transform Algorithm to Reduced Computational Complexities

Naveen Kumar[1], Dr. Rajesh Mehra[2], Buddhi Prakash Sharma[3] M.E Schlorer[1,3] ,Associate Professor[2]
Department of Electronics and Communication Engineering
National Institute of Technical Teachers Training and Research Chandigarh

Abstract- *Discrete Fourier Transform (DFT) and Fast Fourier Transform is important in the field of Digital Signal Processing, communication and filtering. But now a new concept comes in Digital signal processing is Sliding Discrete Fourier Transform. In this process the Transform window is shifted one sample at a time and this transform process is repeated continuously. In this paper: Background and implementation issues, and the advantages and disadvantages of the Sliding Discrete Fourier Transform(SDFT) as compared with a more traditional Fast Fourier Transform (FFT). The Sliding Discrete Fourier Transform (SDFT) is computationally stable but it also have some errors and potential instabilities. A much more efficient Simple Sliding Inverse DFT that makes sliding a serious alternative to jumping between overlapping frames. In digital signal processing (DSP) by applying a Sliding Discrete Fourier Transform (SDFT) technique we are removing ripple in side lobes of a spectrum. In this method we receive an input signal which includes a number of discrete samples taken at regular time intervals, So as to remove the potential instabilities and errors. Finally we assess the quality of transformations based on the Sliding Discrete Fourier Transform (SDFT).*

Keywords-*Frequency Estimation, DFT, FFT, Sliding, Windowing.*

I. INTRODUCTION

The Discrete Fourier Transform (DFT) represents the Finite duration data inputs. The DFT plays a important role in formation of a variety of Digital signal and its applications. But there are several methods Comes into plays like Sliding Discrete Fourier Transform (SDFT) in which the transform window is shifted one sample at a time and this transform process is repeated. The received signal is corrupted by some noise [1].

The best frequency is estimated by the signal that is from the peak of the N-point Discrete Fourier Transform (DFT) of the received signal. A N-point DFT is calculated from an array of length of N. Samples that gives us a resolution of 2N.For some real time spectral analysis a well-known computationally efficient method is the sliding DFT especially in the cases when a new

DFT spectrum is needed every sample that has been taken into for Observation. The sliding DFT is computationally efficient than the radix-2 FFT. The sliding DFT performs a N point DFT within a sliding window of N samples. The window is then shifted by a sample for the next stage and a new N point DFT is calculated which uses the old N point DFT values. The SDFT algorithm is implemented as a filter, to which a filter can be added so as to calculate all N-DFT spectral components [1].

The sliding DFT though has a marginally stable transfer function because all its poles lie in the z-domain's unit circle. The rounding off the filter coefficient equation might force the poles outside of the unit circle which results in instability. A damping factor (d) may be used to force the pole inside the unit circle of radius(r) that's guarantees stability [2].

The Sliding Discrete Fourier Transform (SDFT) having a repeated algorithm that deals with a DFT on sample to sample Basis. The Sliding Discrete Fourier Transform (SDFT) is a computationally efficient but it also has some accumulated errors and some instability. In this paper we discuss, new SDFT algorithm that is gSDFT.
Here g stands for guaranteed, without compromising with accuracy[3].

In this guaranteed Sliding Discrete Fourier Transform (gSDFT) we directly put the sum in the Sliding Window of Length (N). By doing this we remove the twiddle factor from the feedback and avoid the Computation complexities and errors and some potential instability [4].

The standard Method in DSP is the Discrete Fourier Transform (DFT) mostly implemented with FFT. Now a new term known as Sliding DFT whose spectral output is equal to the input data rate with an advantage with less number of computations for real time spectral analysis. The Sliding DFT is computationally easier than the other methods. The Sliding Discrete Fourier Transform (SDFT) is the process of frequency domain convolution in time domain by windowing. The time window is advanced by one or two sample. The incremental advance of the time window for each output leads to a sliding window Discrete Fourier Transform (DFT)[5].

The principle used in Sliding Discrete Fourier Transform (SDFT) is the Discrete Fourier Transform (DFT) shifting or circular shift property. It states that the DFT of a time domain windowed sequence is X(k).So that the Discrete Fourier Transform (DFT) of the window is sequence circular shifted by one or two or more sample[5].

II. A SLIDING DFT

The Sliding DFT performs an N point Discrete Fourier Transform on Time Samples within a Sliding Window then the Time window is incremented by one Sample. The Incremental advanced of the time window for every signal, then the output is named as Sliding DFT.
The principle used in Sliding DFT is Circular Shift property and Time shifting property and DFT.
We have a function f (n), with discrete samples f_0, f_1, f2…..F_n.
Let the Window Size is N and suppose that the signal repeats at a certain period over that period of time continuously.

The DFT at a Time period t:

$$F_t(n) = \sum_{j=0}^{N-1} f_{j+t} \cdot e^{-2\pi ijn/N} \quad (1)$$

Where F_t (n) is the value in the n^{th} bucket in the frequency Domain.

The idea behind the Sliding Discrete Fourier Transform (SDFT) is to make use of the known values of F_t (n) to Calculate the value for the next window. Suppose we are moving it by 1 sample:

$$F_{t+1}(n) = \sum_{K=0}^{N-1} f_{k+t+1}\, e^{-2\pi ikn/N} \quad (2)$$

$$F_{t+1}(n) = \sum_{K=1}^{N} f_{k+t}\, e^{-2\pi i(k-1)n/N} \quad (3)$$

$$= \sum_{K=0}^{N-1} \left(f_{k+t}\, e^{-2\pi i(k)n/N} - f_t + f_{t+N} \right) e^{-2\pi in/N} \quad (4)$$

$$= \left(F_t(n) - f_t + f_{t+N}\right) e^{-2\pi in/N} \quad (5)$$

This means to get a new sample you have to forget the previous one signal. The samples are real this is a little simpler than might appear [6].

The SDFT can reduce the Computational Complexities of the DFT in a unpleasant way. The Transform is computed on a fixed length window of the signal.
Let there is a complex input signal X (n), where n is 0, 1, 2, and 3… and so on .And which is divided into overlapped windows of size M.
Let k be the frequency domain index. The value of 'k' lies in between 0 to M. At any time index n, the M-point DFT is Computed as:

$$X_n(k) = \sum_{m=0}^{M-1} x(\hat{n} + m)\, W_M^{-km} \quad (6)$$

Where $\hat{n} = n-M+1$ and $W_M = e^{j2\pi/M}$

The Sliding discrete Fourier Transform (SDFT) is marginally Stable transfer Function because its pole lies within a unit
circle in z-domain. The Numerical rounding off of the complex twiddle factor may be move some poles outside the unit circle and this leads to a unstable system [7].
Another stable SDFT named as r – SDFT. In Transform forces the pole to be within the radius 'r' inside the unit circle by utilizing the damping factor 'r'.
The DFT approximation using a damping factor 'r' is expressed as:

$$X_n(k) = \sum_{K=0}^{M-1} x(n+m) \quad r^{M-m} \quad W_M^{-km} \quad (7)$$

Where r lies in between 0 and 1.

The Sliding DiscreteFourierTransform (SDFT) algorithm performs an N-point DFT with time sample within a sliding window. Then the time window is advanced one or two sample and a new sample of DFT comes out. The new DFT results are directly depends on the previous DFT. The incremental advance of the time window and a new DFT comes out, that DFT is known as Sliding Discrete Fourier Transform (SDFT).

The principle used for Sliding Discrete Fourier Transform (SDFT) is shifting or circular shift property[8].

III. SDFT STABILITY

The Sliding Discrete Fourier Transform (SDFT) is marginally stable because their pole lies within the z-domain unit circle. The Sliding Discrete Fourier Transform (SDFT) is Bounded input Bounded output(BIBO) stable. In SDFT stability Filter instability is the main issue. On the other hand while rounding off the values some of the values goes outside from the unit circle.
So, we can use damping factor (d) to force the pole to be in the radius of r that is inside the unit circle. That means guaranteed stability that is given by:

$$H_{sdft,gs}(z) = (1-r^{N} z^{-N}) / 1- re^{j2\pi k/N} z^{-1} \quad (8)$$

Where gs mean guaranteed stability.

Another method for stable output is decrease the largest component value of the filter by $e^{j2\pi k/N}$ and feedback coefficients by at least one significant bit [9]. Due to rounding off the value rounding errors also comes which gives precise results in finite precision $e^{j2\pi k/N}$. Here coefficients are greater than unity .
Finally, SDFT and DFT output values are completely dependent on N.

Time domain windowing in frequency domain: The spectral leakage is reduced by windowing the function x(n) that is input time samples [9]. Thus, by doing windowing in time domain multiplication may leads to simplicity in the Sliding discrete Fourier Transform by computationally. Thus, we implement a time domain window by means of frequency domain convolution [10].

IV. SDFT ALGORITHM

The frequency of the $(k+1)^{th}$ transform is $X_{f,k+1}$ is calculated with k^{th} transform is $X_{f,k}$. For each frequency with index f, the difference of the newest time-domain input sample, x_{k+N}, and the oldest sample in the DFT window x_k .The result is multiplied by the frequency-dependent exponential term, $e^{-j2pf/N}$, to produce the updated output form.
In SDFT one complex addition, $x_{k+N}-x_k$, and one complex multiplication and addition for each output[11].

Deriving the Sliding DFT:

Suppose that a transform is taken with every new time-domain sample, so that the length-N transform window moves along the time domain stream a sample at a time. The input stream with samples x_k, where k runs over an index with a range larger than N, can then yield a length-N transform at every k^{th} sample, where f is the frequency index and n the time index within the length-N transform window[11].

Suppose a given sequence is x[n] of size M. where the Size of window is N. And Y_1 is the DFT of first N samples of sequence, which can be compute by the FFT if N is a power of two.

For n = 2 to M − N + 1

Following equations are:

$$= X_{f,k} = \sum_{n=0}^{N-1} x_{n+k} \,.\, e^{-j2\pi fn/N} \quad (9)$$

Sliding Sequence will be $(K+1)^{th}$ term

Equation for it is:

$$X_{f,k+1} = \sum_{n=0}^{N-1} x_{n+k+1} \,.e^{-j2\pi f.n/N} \quad (10)$$

Suppose p = n+1 in eq. (2)

$$X_{f,k+1} = \sum_{p=1}^{N} x_{p+k} \,.e^{-j2\pi.f(p-1)/N} \quad (11)$$

The Nth term can be taken out of the overall Equation and added separately. Now, we introduce a zero term in the Overall Equation, and we subtract it again outside of the Equation. The resulting equation is:

$$x_{f,k+1} = \left[\sum_{p=0}^{N-1} x_{p+k} \,.e^{-j.2.\pi.f.(p-1)/N} \right] + x_{k+N} \,.\, e^{-j.2.\pi.f.(N-1)/N}$$

$$- x_k \,.e^{j.2.\pi.f/N} \quad (12)$$

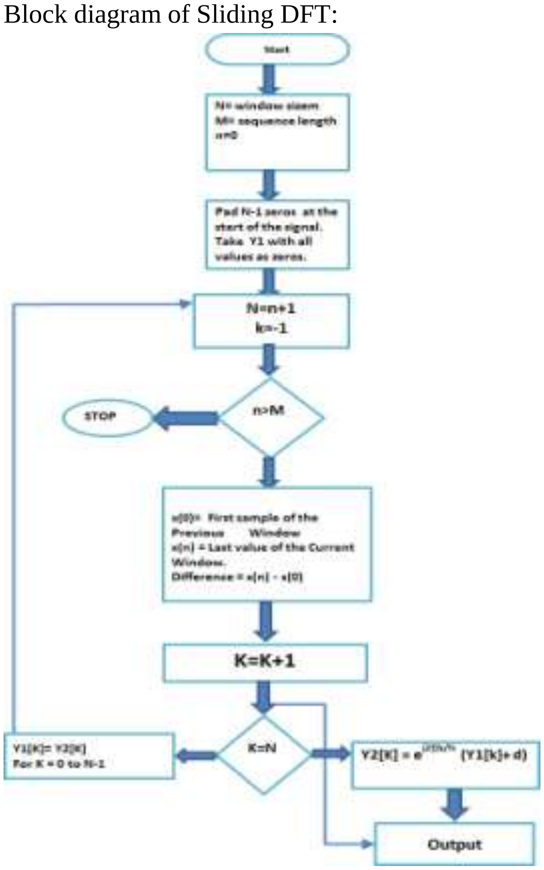
$$x_{f,k+1} = e^{j.2.\pi.f/N} \left[\sum_{p=0}^{N-1} x_{p+k} . e^{-j.2.\pi.f.(p)/N} + x_{k+N} . e^{-j.2.\pi.f.(N)/N} - x_k \right] \quad (13)$$

$$x_{f,k+1} = e^{j.2.\pi.f/N} - x_{f,k} + x_{f,k+N} - x_k \quad (14)$$

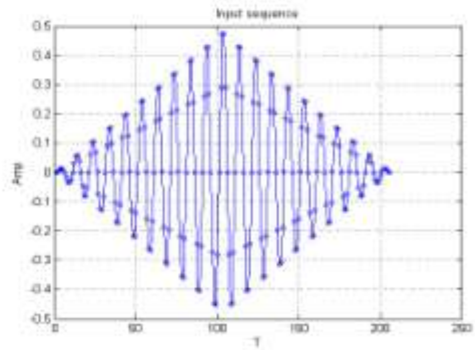
In SDFT, we have a windowed segment of the signal y_1 and its DFT is Y_1 containing information about the frequencies present in this time interval (small value of N gives better time resolution but worse frequency resolution and vice versa). Analysis is carried out and then the window is moved by one sample. Now the windowed segment would be y_2 and corresponding DFT would be Y_2 and N = 4.

Sequence y_2 is of the same length as that of y_1.The updated spectrum Y_2 is thus influenced by inclusion of x_n and exclusion of x_0, with the other time elements remaining unchanged. Each update is a complex multiplication and an addition. The removal of $x_0$will cause similar computation. Therefore, to compute Y_2 from Y_1, 2N calculations will be needed [12].

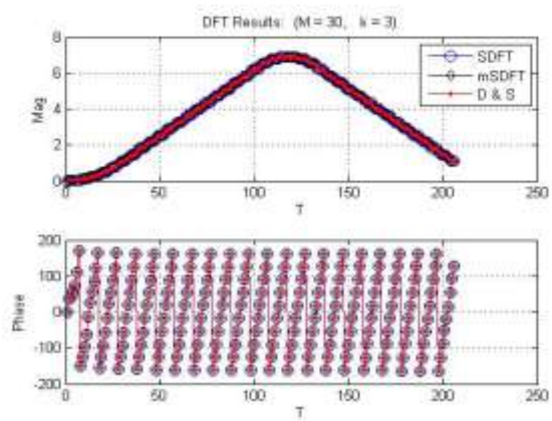
Block diagram of Sliding DFT:



V. RESULTS



In this paper, the computational efficiency of the DFT signal with number of test input signal more than 200. And the length of the DFT is M taken as 30. The Simulation is taken in Matlab environment. In this, SDFT is marginally stable by decrementing the largest component and by doing rounding off the errors which gives a better result and entire's pole lies within the unit circle in the z-domain.



The Sliding DFT reduce the processing time as compared to normal DFT. In these fig. the numerical errors of SDFT and the DFT are compared using a test signal. Efficiency of Sliding DFT and Guarantee 'Gs or m SDFT' and the DFT by taking a complex signal and with M equal to 30. In this, fig. graph errors in Sliding DFT and guarantee Stable DFT and DFT are respectively. Here, errors in Guarantee Stable DFT are less than in normal DFT. All algorithms are implemented using ANCI C code and performance is evaluated on an Intel core2 duo 1.97 GHZ CPU with 4GB RAM.

VI. Conclusion

In this paper we are computing the SDFT to remove potential instabilities and errors. We remove twiddle factor from the feedback in SDFT to get a finite and precise output.

We compared the algorithm of DFT, SDFT and gSDFT. In these algorithms gSDFT is stable and more accurate than DFT and SDFT in the same environment. In this experiment the result shows clearly that in gSDFT numerical error is very less than those of DFT and SDFT. In gSDFT the numbers of operation while doing calculation like addition and multiplication are less than DFT and SDFT.

References

[1] E. Jacobsen and R. Lyons, "The Sliding DFT", IEEE Signal Processing Magazine", pp. 74-80, March 2003.

[2] E.Jacobsen and P.Kootsookos, "Fast Accurate Freq- uency Estimators", IEEE Trans.Signal Process, Vol. 24, pp. 123-125, May 2007.

[3] E. Jacobsen and R. Lyons, "An update to the Sliding DFT", IEEE Signal Processing Magazine, pp.110-111 Jan.2004.

[4] T. Springer, "Sliding FFT computes frequency spectra in real time", *EDN Mag.*, pp. 161-170, Sept. 29, 1988.

[5] J. Proakis and D. Manolakis, "Digital Signal Proces- sing Principles, Algorithms and Applications", pp.480 -481, Prentice Hall.

[6] Pooja Kataria , Rajesh Mehra, "Comparative Analysis of FFT Algorithm for different windowing Techniques", International Journal of Science, Engineering and Technology Research (IJSETR), Vol. 2, Issue 9, September 2013.

[7] A. Nuttall, "Some windows with very good sidelobe behavior", IEEE Trans. Acoust., Speech, Signal Process- ing, Vol. 29, pp.84 -91, Feb. 1981.

[8] S. Douglas and J. Soh, "A numerically stable sliding-window estimator and its application to adaptive filters", Proc. 31st Annual Asilomar Conf. on Signals, Systems, and Computers, Vol. 1, pp.111 -115, 1997.

[9] B. Farhang-Boroujeny and Y. Lim, "A comment on the computational complexity of sliding FFT", IEEE Trans. Circuits Syst. II, Vol. 39, no. 12, pp.875 -876, 1992.

[10] Jacobsen, Eric and Richard Lyons, "Sliding Spectrum Analysis", streamlining digital signal processing a tricks of the trade guidebook, 2012.

[11] Duda, Krzysztof, " Accurate Guaranteed Stable, sliding Discrete Fourier Transform, DSP Tips and Tricks", IEEE Signal Processing Magazine, 2010.

[12] Duda , Krzysztof, " Accurate, Guaranteed-Stable, Sliding DFT, Streamlining Digital Signal Processing A Tricks of the Trade Guidebook", 2012.

Authors:

Naveen Kumar: Naveen is currently associated with IT Company ECLERX ltd., Chandigarh, India since Dec. 2013. He is currently pursuing M.E from National Institute of Technical Teachers Training and Research, Chandigarh India. He has completed his B. Tech from Chitkara University, Solan, India in 2013. He is having two years of Industrial experience.His research area Advanced Digital Signal Processing, Sensors and Embedded, Wireless communication.

Dr. Rajesh Mehra: Dr. Mehra is currently associated with Electronics and Communication Engineering Department of National Institute of Technical Teachers' Training & Research, Chandigarh, India since 1996. He has received his Doctor of Philosophy in Engineering and Technology from Panjab University, Chandigarh, India in 2015. Dr. Mehra received his Master of Engineering from Panjab Univeristy, Chandigarh, India in 2008 and Bachelor of Technology from NIT, Jalandhar, India in 1994. Dr. Mehra has 20 years of academic and industry experience. He has more than 250 papers in his credit which are published in refereed International Journals and Conferences. Dr. Mehra has 55 ME thesis in his credit. He has also authored one book on PLC & SCADA. His research areas are Advanced Digital Signal Processing, VLSI Design, FPGA System Design, Embedded System Design, and Wireless & Mobile Communication. Dr. Mehra is member of IEEE and ISTE.

Mr. Buddhi Prakash Sharma: Mr. Sharma is currently associated with Electronics & Communication Engineering Department of B. K. Birla Institute of Engineering & Technology, Pilani, Rajasthan, India since 2014. He is currently pursuing M.E from National Institute of Technical Teachers Training and Research, Chandigarh India. He has completed his B. Tech from ICFAI University, Dehradoon, India. Mr. Sharma is having six years of teaching experience. He has more than eight papers in his credit which are published in refereed International Journals and Conferences. His research area Advanced Digital Signal Processing, VLSI Design and Image Processing. Mr. Sharma is a member of IE.