Development of Universal Decline Curve Analysis Technique for Forecasting the Performance of Oil Wells

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ABSTRACT

Linear and Nonlinear regression analysis were carried out on Arps decline curve by using analytical Natural Cubic Spline Interpolation and Levenberg-Marquardt algorithm respectively. The linearised model was used to initialize the nonlinear model and this shows improvement in the result of nonlinear model by Levenberg-Marquardt algorithm.The results of these analyses were used for history-matching existing oil well production data from Niger Delta Field, Nigeria. The nonlinear regression by Levenberg-Maquardt algorithm shows high degree of accuracy in history matching the oil field production data and also for future production forecast.

Keywords: *Linear and Non-linear Regression, Levenberg-Marquardt algorithm, History Matching*

1. INTRODUCTION

The integral part of oil well development plan is essentially the requirement of forecasting the performance of oil wells in a reservoir or in the entire field. Different techniques have been developed in the past in an attempt to forecast future production performance from oil wells. Generally, only information about production history is available to initiate any evaluation of reservoir reserve from decline curve analysis. The varieties of methods developed and published in various literatures for predicting oil well performance range from the basic material balance analysis to decline curve analysis techniques. Among the techniques, the decline curves have been found quite accurate to forecast oil well performance in the absence of known reservoir parameters. It appears to be a very useful technique for performing future projections and evaluating original oil in place and hydrocarbon reserves. These estimates are needed to determine the economic viability for project development.

The common methods of forecasting production from oil wells vary in complexity and in the amount of detail required. In practice then, many engineers avoid the use of hyperbolic decline curve. Some use the French curve to approximate well production declines. Another common approach is to assume shapes composed of series of straight segments. For example, 50% decline for two years, and then 20% decline for three year, followed by 8% decline to an economic limit. While these methods may give satisfactory results for a group of similar wells, a pertinent question is that why do these wells follow a decline shape which is apparently arbitrary?

In addition, the several unaddressed limitations in predicting the performance of oil well from decline curve analysis include: The trial and error method of selecting the type of decline (exponential, harmonic and hyperbolic) to fit during decline curve analysis, the constraints to accurately estimate decline parameters from the Arps decline function using the conventional decline curve analysis and the difficulty of finding a proper nonlinear algorithm to tune the Arps decline equation to match historical oil production. For this reason, most applications opt for type-curve matching whose inefficiency to adequately predict future performance of oil wells has been widely criticized and the inability of the current methods, type-curve method inclusive, to track the behaviour of oil wells during the transient period.

The decline curve analysis approach by Arps equation(1945) was proposed more than sixty years ago. However, a large number of studies on production decline analysis are still based on this empirical method(2013 journal). The successive derivatives of the production rate with time were determined by Arps' methodolgy by using numerical forward difference technique. The production data presented by Arps offers a good illustration for demonstrating the utility of the proposed linear technique of decline curve analysis. In the example, Arps computed the loss ratio for 6 months interval to eliminate monthly fluctuations and to embrace the general trend of the curve without difficulty. He then defined the loss ratio as the production rate divided by the first derivative of the rate-time curve. In computing for the example, Arps introduced a correction factor of 1/6 to find the proper values of the loss ratio. This depicts one important weakness of the Arps loss-ratio method. That is, the values of the loss-ratio difference were enlarged in an effort to smooth the data, hence Arps introduction of the correction factor in his example. The factor is arbitrary and does not follow any defined mathematical formulation. Thus it cannot be generalized to other problems. The loss ratios thus obtained by Arps indicated a fairly uniform arithmetic series and consequently the differences between successive loss ratio values are reasonably constant. The average of the individual loss ratio is the decline exponent.

In practice however, production rate is so erratic to follow the Arps' loss ratio technique, thereby rendering the technique inaccurate. Obviously, the error of the Arps' loss ratio method is attributed mainly to the linear numerical method(forward difference technique) it employed to evaluate the derivatives of production rate with respect to time. The method works if the data points are linearly ordered, but fails when the data are erratically staggered as in field data.

This study therefore resolves these conflicts by utilizing numerical algorithm and hence initialization of instantaneous decline rate, instantaneous decline exponent and adjusted initial production rate follow. The initialization was obtained from linear least square regression performed on the linearly transformed Arps decline curve by using natural cubic spline interpolation. Also, the Arps decline function is regressed nonlinearly using the Levenberg-Marquadrt algorithm to obtain the values of instantaneous decline rate, instantaneous decline exponent and adjusted initial production rate that will provide a match of the regression model and the actual rate decline. The target of the tuning process is to find a Levenberg-Marquadrt perturbation that would update the initialization and minimize the objective function.

II. MATERIALS AND METHODS

The efficiency and performance of Levenberg-Marquardt algorithms in solving variety of problems is higher than simple gradient descent and other conjugate gradient methods. The Levenberg-Marquardt algorithm was first shown to be a blend of vanilla gradient descent and Gauss-Newton iteration. Vanilla gradient descent is the simplest and most intuitive method of determining minima in a function. Parameter update is performed by summing the negative of the scaled gradient at each step:

$$c_{i+1} = c_i - \nabla f \qquad (1)$$

Simple gradient descent is associated with various convergence problems. Logically, it would be likely that taken large steps down the gradient at locations where the gradient is small (the slope is gentle) and conversely, taken small steps when the gradient is large, will not rattle out of the minima. With the above update rule, do just the opposite of this. The efficiency of the above condition can be enhanced by using curvature as well as gradient information, namely second derivatives, which is the Newton's method.

$$c_{i+1} = c_i - H^{-1} \times \left[J^T \times D \right] \qquad (2)$$

Provided that the residuals *D* are small, the Hessian *H* need not be evaluated exactly but can be approximated essentially by the equation given as

$$H = J^T \times J \qquad (3)$$

Where the Jacobian is defined by

$$J = \frac{\partial \hat{q}}{\partial c} \qquad (4)$$

For *m* number of data points and *n* number of regression constants, the Jacobian is $n \times m$ matrix.

The main advantage of this method is rapid convergence. Therefore, convergence rate is sensitive to the starting location or more concisely, the linearity around the starting location. It can be deduced that simple gradient descent and Gauss-Newton iteration are complementary in the advantages they provide. Levenberg-Marquardt algorithm was proposed based on this observation. Also, another perspective on the algorithm is provided by considering it as a trust-region method. Whichever perspective the Levenberg-Marquardt algorithm is viewed, the tuning objective is to find the argument that minimizes the sum of the squares of the errors between the production rate history and the Arps decline function to achieve a reasonable match.

In fitting the production rate, Arps function $\hat{q}(t, c)$ of an independent time variable *t* and a vector of *n* regression constants *c* to a set of *m* production rate history (t, q), the objective is to find the update

$$c_{i+1} = \arg\min \{F(c)\} \qquad (5)$$

$F(c)$ is defined by the Euclidian norm

$$F(c) = \frac{1}{2} \left\| \frac{f(c)}{w} \right\|^2 \qquad (6)$$

Where

$$f(c) \equiv D = q(t) - \hat{q}(t) \qquad (7)$$

The weighting factor(w) is defined as

$$w = \sqrt{\frac{D^T D}{n - m + 1}} \tag{8}$$

Here, the vector of n regression constants c represents the vector with elements instantaneous decline rate, instantaneous decline exponent and adjusted initial production rate. In this case the Jacobian is the $m \times 3$ matrix of the derivatives

And

$$\frac{\partial \hat{q}}{\partial D_i} = -\frac{q_i t}{(1 + bD_i t)^{1+1/b}} = -\frac{qt}{1 + bD_i t} \tag{9}$$

$$\frac{\partial \hat{q}}{\partial q_i} = \frac{1}{(1 + bD_i t)^{1/b}} \tag{10}$$

$$\frac{\partial \hat{q}}{\partial b} = \frac{q_i}{b(1 + bD_i t)^{1/b}} \left[\frac{\ln(1 + bD_i t)}{b} - \frac{D_i t}{1 + bD_i t} \right] = \frac{q}{b} \left[\frac{\ln(1 + bD_i t)}{b} - \frac{D_i t}{1 + bD_i t} \right] \tag{11}$$

Starting with initial guess of the regression constants calculated from linear least square regression, the target is to find a perturbation

$$pert = [H + \lambda \cdot \text{diag}(H)]^{-1} \times [J^T \times (w \cdot D)] \tag{12}$$

to the regression constants that would give a new and hopefully a better match of the objective function. Provided that the residuals D are small, the Hessian H need not be evaluated exactly but can be approximated essentially by the equation given as

$$H = J^T \times w \cdot J \tag{13}$$

With λ being the blending factor, the Levenberg-Marquardt update is

$$c_{i+1} = c_i + pert \tag{14}$$

If the error declines following an update, λ is reduced and the algorithm degenerates to Gauss-Newton update. On the contrary, if the error amplifies following an update, λ is increased and the update becomes gradient descent iteration.

With the values of decline curve parameters determined, cumulative oil production is evaluated from the following equation:

$$N_p = \frac{q_i *}{(1 - b)D_i} \left[1 - \left(\frac{q}{q_i *} \right)^{1-b} \right] \tag{15}$$

Once the optimal curve-fit parameters are determined, the prediction error for the converged solution is computed from

$$\sigma = \frac{1}{m - n + 1} \sum_{j=1}^{m} (q_j - \hat{q}_j)^2 \tag{16}$$

III. RESULT AND DISCUSSION

A. LINEARIZED MODEL

The initialization of decline parameters from linear regression by using analytical natural cubic spline interpolation method is shown below

Table 1: Initialization of Decline Parameters from Linear Regression for Niger Delta Field Data

Decline Exponent	0.95482
Decline Rate	0.175941
Initial Oil Rate	2580
Regression MSE	349.2297
Forecast MSE	287.736

The study computed the decline parameters thus: b = 0.9548, $D_i = 0.1759$ and $q_i{*} = q(t=1) = 2580$. With the decline parameters defined, the Arps decline function becomes

$$q = \frac{2580}{(1 + 0.9548 \times 0.1759 \times t)^{1/0.9548}} \quad (17)$$

The oil rate computed from this equation is compared with the actual production data from a Niger Delta Field as tabulated in Table 2 and plotted in Figure 1.

Table 2: Production history and predicted rate data from linearised a algorithm

Time (Month)	Rate History (/Month)	Rate Predicted (/Month)
1	2580	2192.74727
2	2100	1904.87087
3	2090	1682.64223
4	1780	1506.01274
5	1860	1362.3248
6	1470	1243.19768
7	1510	1142.8642
8	1250	1057.22707
9	1330	983.296193
10	1220	918.838865
11	1090	862.15465
12	1050	811.925933

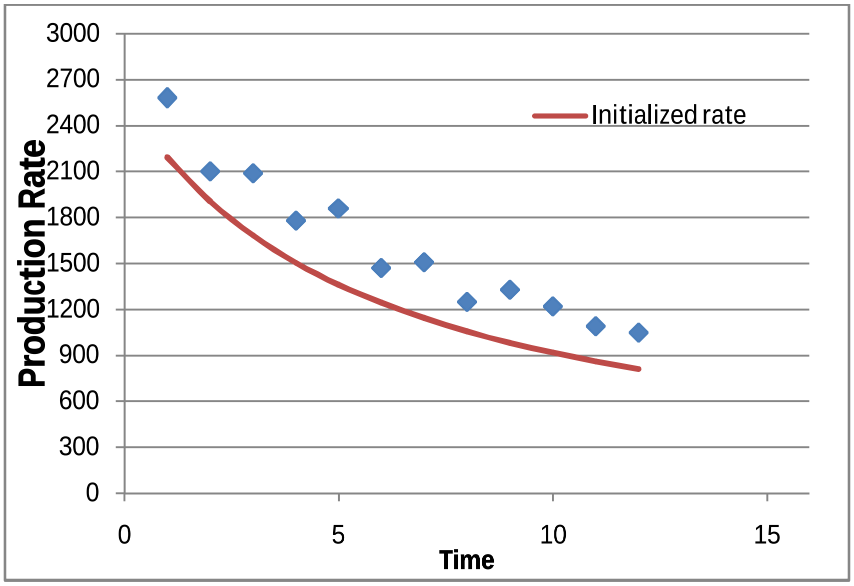

Figure 1: Field data initialized rate compared with rate history

The correlation confidence of the linear regression on the first data set (1st month to 12th month) was computed as mean square error of 349.23. To corroborate the fitness of the linear correlation, the second data set (13th month to 16th month) was used to forecast future production rate using the developed model.

Table 3: Forecast from Initialization

Time (Month)	Rate History (/Month)	Rate Forecast (/Month)
13	982	767.116105
14	940	726.89857
15	883	690.606211
16	850	657.694768

The forecast is compared with the actual production as shown in Figure 2. The mean square error of the forecast from the linear regression was computed as 287.74.

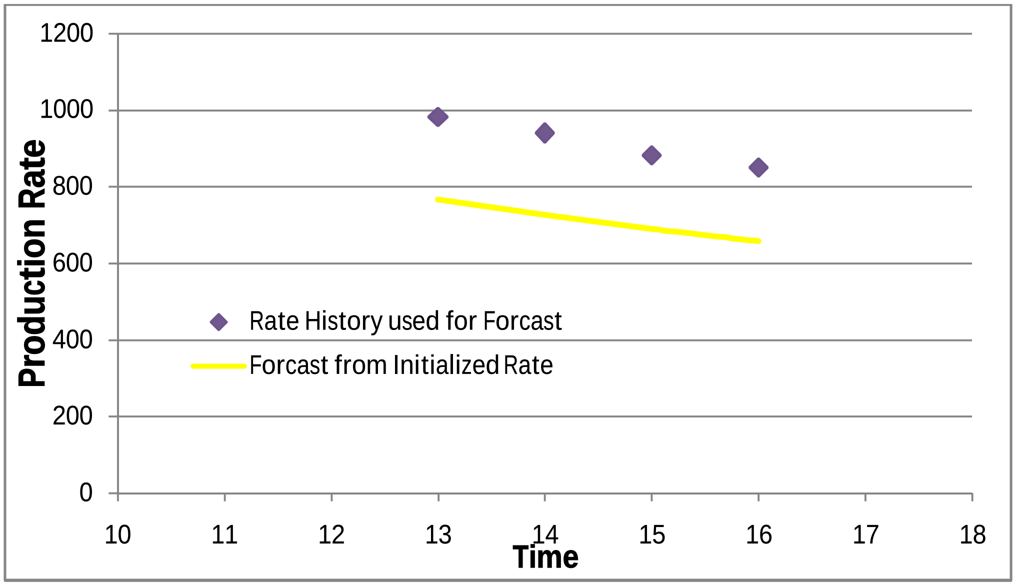

Figure 2: Field data forecast from initialized rate compared with rate history

The forecast mean square error although less than the regression mean square error still did not adequately minimize the Arps decline function to match production rate history. Hence, there is the need for refinement by applying the nonlinear optimization algorithm.

B. NONLINEAR ALGORITHM

The requirement of nonlinear optimization of the Arps decline curve function is the initialization of decline parameters and this is a major problem of nonlinear optimization. The problem of initialization in nonlinear optimization could give rise to non-uniqueness of the solution because nonlinear regression involves numerical iterations. The determination of an initial estimate that is realistically close to the desired local minimum will guide the Arps function to a suitable local point and hopefully turn out a unique combination of the decline parameters. Future rate-time prediction then can be made more confidently and as early as possible, even in the transient production period.

The principal objective of the nonlinear algorithm is to refine the decline parameters obtained from the linear regression. The advantage of this refinement is that there is an assurance that the initialized decline parameters closely approximate the actual values.

Therefore, with the initialized decline parameters b = 0.9548, $D_i = 0.1759$ and $q_i{}^* = q(t=1) = 2580$ from linear regression, a nonlinear regression algorithm was performed on the first set of the Arps data to produced a refined values of the decline parameters as $b = 1.1015$, $D_i = 0.1452$ and $q_i{}^* = 2863.59$.

Table 4: Initialization of decline parameters from nonlinear algorithm for Niger Delta field data

Decline Exponent	1.101473
Decline Rate	0.145179
Initial Oil Rate	2863.593
Regression MSE	96.24883
Forecast MSE	287.736

Also with the refined decline parameters defined, the Arps decline function finally becomes

$$q = \frac{2863.59}{(1 + 1.1015 \times 0.1452 \times t)^{1/1.1015}} \tag{18}$$

The oil rate computed from this equation is compared with the actual production as shown in Table 5 and plotted in Figure 3.

Table 5: Production history and predicted rate data from nonlinear regression

Time (Month)	Rate History (/Month)	Rate Predicted (/Month)
1	2580	2502.773589
2	2100	2225.861515
3	2090	2006.345609
4	1780	1827.86936
5	1860	1679.781662
6	1470	1554.84212
7	1510	1447.953211
8	1250	1355.419839
9	1330	1274.496622
10	1220	1203.100246
11	1090	1139.620593
12	1050	1082.793171

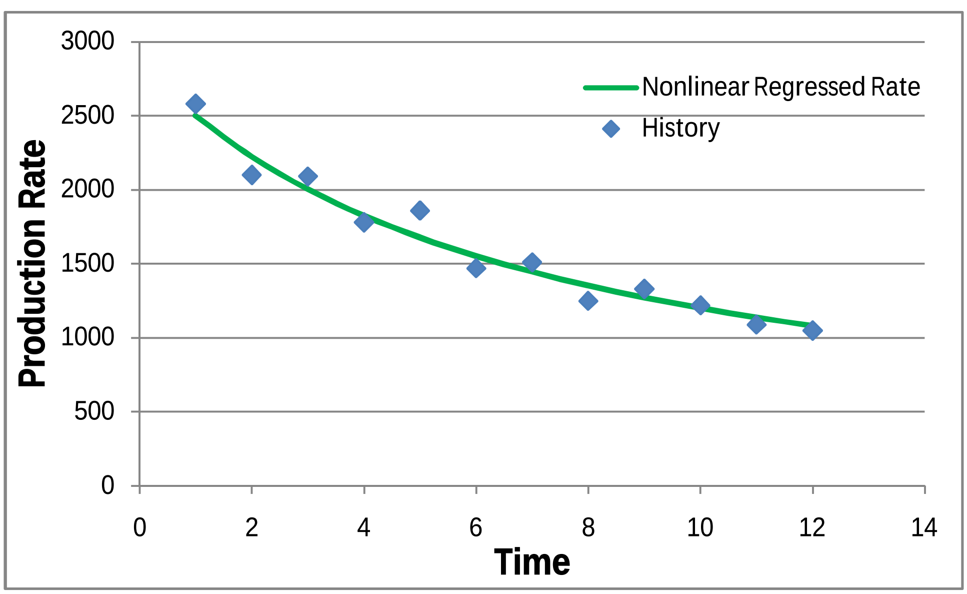

Figure 3: Field data nonlinear rate compared with rate history

The match indicated a mean square error of 96.25, which on the basis of the initialized rate is 98.57% reduction in error. Thus, the nonlinear regression refinement was able to improve the linear regression more than the Arps' data.

The confidence of the developed model is tested by using the second data set in the developed model equation for non-linear algorithm to forecast future oil production rate. The forecast is shown in Table 3 and plotted in Figure 4.8

Table 3: Forecast from Initialization

Time (Month)	Rate History (/Month)	Rate Forecast (/Month)
13	982	1031.6108
14	940	985.26105
15	883	943.08127
16	850	904.5254

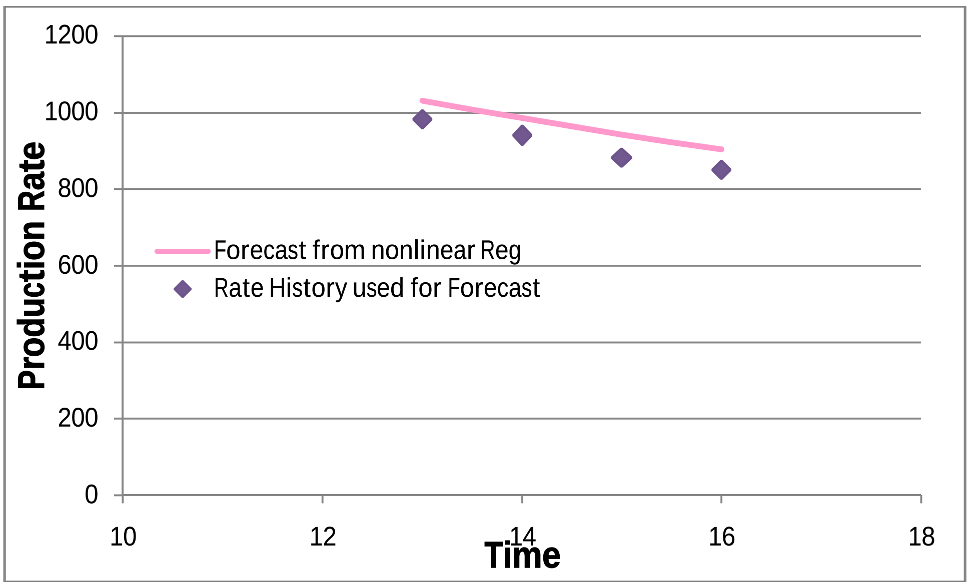

Figure 4: Field data Forecast from nonlinear regression compared with production history

CONCLUSION

This research study used linear regression to initialize the nonlinear optimization of the hyperbolic decline. The method is an advancement of existing methods, as no initial guesses are required to determine any of the decline curve parameters. The result of the initialization was refined using nonlinear regression algorithm to improve the accuracy of the decline curve analysis. Thus, the second stage of the method is computer-automated curve fitting algorithm based on Levenberg-Marquardt (LM) optimization.

The study concluded that a method that combined linear and nonlinear optimization to determine the hyperbolic decline curve parameters was developed, the technique was initialized from production history using linear regression thereby eliminating the efforts of trying out each type of the decline equations on the production rate history before decline curve analysis and the procedure is not only accurate but solved the problem of non-uniqueness as common in oil field data.

REFERENCES

[1]. Makinde, F.A., Orodu, O.D., Ladipo, A. O. and Anawe, P.A.L. (2012): Cumulative Production Forecast of an oil well using Simplified Hyperbolic-Exponential Decline models, Global Journal of Researches in Engineering, 2(12)

[2]. Ahmed, T. (2006): Reservoir Engineering Handbook. 2nd edition. Gulf Publishing Company, Texas. 850 – 853.

[3]. Khulud, M.R, Mohammed, H., Hissein, N. and Giuma,S. (2013): Prediction of Reservoir Performance Applying Decline Curve Analysis, International Journal of Chemical Engineering and Application, 4(2),74.

[4]. Manolis, I.A.L. (2005) A Brief Description of the Levenberg-Marquardt Algorithm implemented by Lemver, Technical Report, Institute of Computer Science, Foundation for Research and Technology, Hellas.

[5]. Henri, P.G. (2015): The Levenberg-Marquardt Method for Nonlinear Least Squares Curve Fitting Problems, Department of Civil and Environmental Engineering, Duke University.

[6]. Madsen, K., Nielsen, H.B. and Tingleff, O. (2004): Methods for Nonlinear Square Problems, Technical Report, Informatics and Mathematical Modeling, Technical University of Denmark, Lecture Note, Available at http://www.imm.dtu.dk/courses/02611/n11sq.pdf

[7]. Kewen, L and Roland, N.H. (2003): A Decline Curve Analysis Model based on Fluid Flow Mechanism, 83470-MS, SPE. A Paper Presented at the SPE Western Regional/AAPG Pacific Section Joint Meeting held at Long Beach, California, USA.

[8]. Marks, K.T and James, P.S. (2012) Improvements to the Levenberg-Marquardt Algorithm for Nonlinear Least Squares Minimization, Preprint Submitted to Journal of Computational Physics.

[9]. Chithra, C. N. C., Ki-Young, S., Deoki, N.S. and Madan, M.G. (2013): Production Forecasting of Petroleum ReservoirAapplying Higher-Order Neural Networks with limited Reservoir Data, International Journal of Computer Application,0975-8887,72(2)