Cordial Labeling of Cosplitting Graphs

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Abstract: The **cosplitting graph** CS(G) is obtained from G, by adding a new vertex w for each vertex v ϵ V and joining to those vertices of G which are not adjacent to v in G. In this paper, we proved that the cosplitting graph of path, cycle, complete bipartite graph, wheel and star graph are cordial.

Keywords: Cordial labeling, Cosplitting graph.

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1 INTRODUCTION

All graphs considered here are finite, simple and undirected. The origin of graph labelings can be attributed to Rosa [3]. Gallian [1] has given a dynamic survey of graph labeling. For graph theoretic terminologies and notations we follow Harary [2]. Cosplitting graph was introduced by Selvam Avadayappan [4].

2 PRELIMINARIES

Definition 2.1. A graph G is called a **complete bipartite graph** $K_{m,n}$ with bipartition V (G) = $V_1 \cup V_2$ where $V_1 = \{x_1, x_2, ..., x_m\}$ and $V_2 = \{y_1, y_2, ..., y_n\}$ and all vertices in V_1 are adjacent to all vertices in V_2 but no vertices in V_1 and V_2.

Definition 2.2. A **wheel graph** W_n is obtained from a cycle C_n by adding a new vertex and joining it to all the vertices of the path by an edge, the new edges are called the spokes of the wheel.

Definition 2.3. The graph $K_{1,n}$,s $n \geq 1$ is called a **star** at the vertex has degree n is called centre.

Definition 2.4. Let G = (V, E) be a graph. A mapping f : V(G) $\rightarrow \{0,1\}$ is called **binary vertex labeling** of G and f(v) is called the label of the vertex of G under f.

For an edge e = uv, the induced edge labeling $f^* : E(G) \rightarrow \{0,1\}$ is given by $f^*(e) = |f(u)-f(v)|$. Let v_f (0)and v_f (1) be the number of vertices of G having labels 0 and 1 respectively under f and let $e_f(0)$, $e_f(1)$ be the number of edges having labels 0 and 1 respectively under f*.

Definition 2.5. A binary vertex labeling of a graph G is called a **cordial** if $|v_f(0)-v_f(1)| \leq 1$ and $|e_f(0)-e_f(1)| \leq 1$. A graph G is cordial if it admits **cordial labeling.**

Definition 2.6. The **cosplitting graph** CS(G) is obtained from G, by adding a new vertex w for each vertex v ϵ V and joining w to those vertices of G which are not adjacent to v in G.

3 Main Results

Theorem 3.1. The graph $CS(P_n)$ is cordial.

Proof. Let $v_1, v_2, ..., v_n$ be the vertices of P_n and $v_1', v_2', ..., v_n'$ be the duplicate vertices of $CS(P_n)$.
Then $|V(CS(P_n))| = 2n$ and $|E(CS(P_n))| = n^2-n+1$.

The vertex labeling $f : V(CS(P_n)) \rightarrow \{0, 1\}$ is given by

Case (i) : n is odd

$$f(v_i) = \begin{cases} 1 \text{ if } i \equiv 0,1 \pmod 4 \\ 0 \text{ if } i \equiv 2,3 \pmod 4 \end{cases} \quad 1 \leq i \leq n$$

$$f(v_i') = \begin{cases} 1 \text{ if } i \equiv 0,1 \pmod 4 \\ 0 \text{ if } i \equiv 2,3 \pmod 4 \end{cases} \quad 1 \leq i \leq n$$

Case (ii) : n is even

$$f(v_i) = \begin{cases} 1 \text{ if } i \equiv 0,1 \pmod 4 \\ 0 \text{ if } i \equiv 2,3 \pmod 4 \end{cases} \quad 1 \leq i \leq n$$

$$f(v_i') = \begin{cases} 1 \text{ if } i \equiv 0,1 \pmod 4 \\ 0 \text{ if } i \equiv 2,3 \pmod 4 \end{cases} \quad 1 \leq i \leq n$$

The following table shows that the graph CS(Pn) satisfies the conditions $|v_f(0)-v_f(1)| \leq 1$ and $|e_f(0)-e_f(1)| \leq 1$.

n	Vertex conditions	Edge Conditions
odd	$v_f(0)=v_f(1)$	$e_f(1)=e_f(0)+1$
even	$v_f(0)=v_f(1)$	$e_f(0)=e_f(1)+1$

Hence, $CS(P_n)$ is cordial.

Illustration 1. The cordial labeling of $CS(P_4)$ and $CS(P_5)$ are shown in the Figure 1(a) and Figure 1(b).

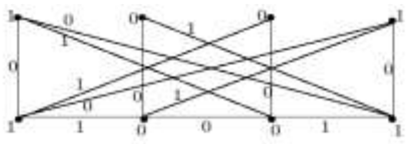

Figure 1(a)

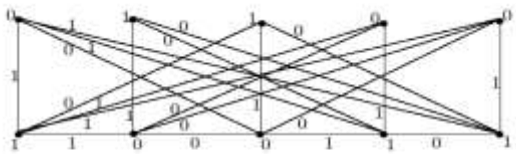

Figure 1(b)

Theorem 3.2. The graph $CS(C_n)$ is cordial for $n \not\equiv 1 (\text{mod } 4)$ and $n \not\equiv 2 (\text{mod } 4)$, $n \geq 3$.

Proof. Let $v_1, v_2, ..., v_n$ be the vertices of cycle C_n and $v_1', v_2', ..., v_n'$ be the duplicate vertices of $CS(C_n)$.
Then $|V (CS(C_n))| = 2n$ and $|E(CS(C_n))| = n(n - 1)$.

The vertex labeling $f : V (CS(Cn)) \rightarrow \{0, 1\}$ is given by

Case(i): $n \equiv 0 \ (\text{mod } 4)$

$$f(v_i) = \begin{cases} 1 \ if \ i \equiv 0,1 \ (mod \ 4) \\ 0 \ if \ i \equiv 2,3 \ (mod \ 4) \end{cases} \quad 1 \leq i \leq n$$

$$f(v_i') = \begin{cases} 1 \ if \ i \equiv 0,3 \ (mod \ 4) \\ 0 \ if \ i \equiv 1,2 \ (mod \ 4) \end{cases} \quad 1 \leq i \leq n$$

Case(ii): $n \equiv 3 \ (\text{mod } 4)$

$$f(v_i) = \begin{cases} 1 \ if \ i \equiv 0,1 \ (mod \ 4) \\ 0 \ if \ i \equiv 2,3 \ (mod \ 4) \end{cases} \quad 1 \leq i \leq n$$

$$f(v_i') = \begin{cases} 1 \ if \ i \equiv 1,3 \ (mod \ 4) \\ 0 \ if \ i \equiv 0,2 \ (mod \ 4) \end{cases} \quad 1 \leq i \leq n$$

Here, $v_f(0)=v_f(1)$ for all n and $e_f(0)=e_f(1)$ for all n. Therefore, the graph $CS(C_n)$ satisfies the conditions $|v_f(0)-v_f(1)| \leq 1$ and $|e_f(0)-e_f(1)| \leq 1$.

Hence, $CS(C_n)$ is cordial.

Illustration 2. The cordial labeling of $CS(C_3)$ and $CS(C_4)$ are shown in the Figure 2(a) and Figure 2(b).

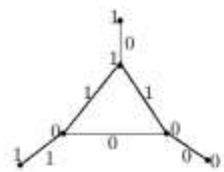

Figure 2(a)

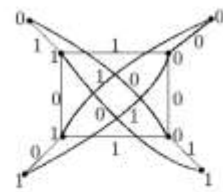

Figure 2(b)

Theorem 3.3. The graph $CS(W_n)$ is cordial.

Proof: let $u, v_1, v_2, ..., v_n$ be the vertices of W_n and $u', v_1', v_2', ..., v_n'$ be the duplicate vertices of $CS(W_n)$.
Then $|V(CS(W_n))| = 2n+1$ and $|E(CS(W_n))| = n^2 + 1$.
The vertex labeling $f : V (CS(W_n)) \rightarrow \{0, 1\}$ is given by

$f(u) = 0$, $f(u0) = 1$.

Case(i): $n \equiv 0, 2, 3 (\text{mod } 4)$

$$f(v_i) = \begin{cases} 1 \ if \ i \equiv 0,1 \ (mod \ 4) \\ 0 \ if \ i \equiv 2,3 \ (mod \ 4) \end{cases} \quad 1 \leq i \leq n$$

$$f(v_i') = \begin{cases} 1 \ if \ i \equiv 1,3 \ (mod \ 4) \\ 0 \ if \ i \equiv 0,2 \ (mod \ 4) \end{cases} \quad 1 \leq i \leq n$$

Case(ii): $n \equiv 1 (\text{mod } 4)$

$$f(v_i) = \begin{cases} 1 \ if \ i \equiv 0,1 \ (mod \ 4) \\ 0 \ if \ i \equiv 2,3 \ (mod \ 4) \end{cases} \quad 1 \leq i \leq n$$

$$f(v_i') = \begin{cases} 1 \ if \ i \equiv 0,2 \ (mod \ 4) \\ 0 \ if \ i \equiv 1,3 \ (mod \ 4) \end{cases} \quad 1 \leq i \leq n$$

The following table shows that the graph $CS(W_n)$ satisfies the conditions $|v_f(0)-v_f(1)| \leq 1$ and $|e_f(0)-e_f(1)| \leq 1$.

n	Vertex Conditions	Edge Conditions
$n \equiv 0 \ mod \ 4$	$V_f(0)=v_f(1)$	$e_f(1)=e_f(0)+1$
$n \equiv 1 \ mod \ 4$	$V_f(0)=v_f(1)$	$e_f(0)=e_f(1)$
$n \equiv 2 \ mod \ 4$	$V_f(0)=v_f(1)$	$e_f(0)=e_f(1)+1$
$n \equiv 3 \ mod \ 4$	$V_f(0)=v_f(1)$	$e_f(0)=e_f(1)$

Hence, $CS(W_n)$ is cordial.

Illustration 3. The cordial labeling of $CS(W_4)$ and $CS(W_5)$ are shown in the Figure 3(a) and Figure 3(b).

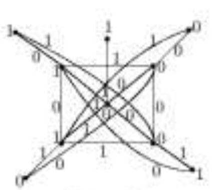

Figure 3(a)

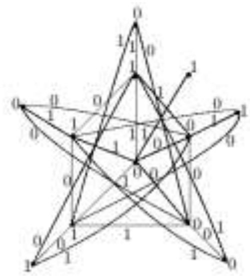

Figure 3(b)

Theorem 3.4. The graph $CS(K_{m,n})$ is cordial.

Proof. Let $u_1, u_2, ..., u_n$ and $v_1, v_2, ..., v_n$ be the vertices of $K_{m,n}$ and $u_1', u_2', ..., u_n', v_1', v_2', ..., v_n'$ be the duplicate vertices of $CS(K_{m,n})$.
Then $|V(CS(K_{m,n}))| = 2(m + n)$ and $|E(CS(K_{m,n}))| = m^2+n^2+mn$.
The vertex labeling $f : V(CS(K_{m,n}))\to\{0, 1\}$ is given by

$f(u_i) = f(v_j) = 1$ and $f(u_i') = f(v_j') = 0$, if i and j is odd $1 \le i \le m$ and $1 \le j \le n$

$f(u_i) = f(v_j) = 0$ and $f(u_i') = f(v_j') = 1$, if i and j is even $1 \le i \le m$ and $1 \le j \le n$

The following table shows that the graph $CS(K_{m,n})$ satisfies the conditions
$|v_f(0)-v_f(1)| \le 1$ and $|e_f(0)- e_f(1)| \le 1$.

m	n	Vertex Conditions	Edge Conditions
even	Even	$v_f(0)=v_f(1)$	$e_f(0)=e_f(1)$
even	Odd	$v_f(0)=v_f(1)$	$e_f(1)=e_f(0)+1$
odd	Even	$v_f(0)=v_f(1)$	$e_f(1)=e_f(0)+1$
odd	Odd	$v_f(0)=v_f(1)$	$e_f(1)=e_f(0)+1$

Hence, $CS(K_{m,n})$ is cordial.

Illustration 4. The cordial labeling of $CS(K_{2,3})$ and $CS(K_{3,3})$ are shown in the Figure 4(a) and Figure 4(b).

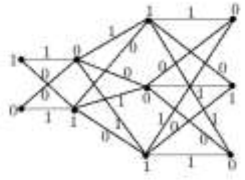

Figure 4(a)

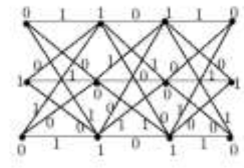

Figure 4(b)

Theorem 3.5. The graph $CS(K_{1,n})$ is cordial.

Proof. let $u, v_1, v_2, ..., v_n$ be the vertices of $K_{1,n}$ and $u', v_1', v_2', ..., v_n'$ be the duplicate vertices of $CS(K_{1,n})$.
Then $|V(CS(K_{1,n}))| = 2(n+1)$ and $|E(CS(K_{1,n}))| = n^2+n+1$.
The vertex labeling $f : V(CS(K_{1,n})) \to\{0, 1\}$ is given by
$f(u) = 1$ and $f(u') = 0$

$$f(v_i) = \begin{cases} 1 \; if \; i \; is \; odd \\ 0 \; if \; i \; is \; even \end{cases} \quad 1 \le i \le n$$

$$f(v_i') = \begin{cases} 0 \; if \; i \; is \; odd \\ 1 \; if \; i \; is \; even \end{cases} \quad 1 \le i \le n$$

or

$f(u) = 0$ and $f(u') = 1$

$$f(v_i) = \begin{cases} 0 \; if \; i \; is \; odd \\ 1 \; if \; i \; is \; even \end{cases} \quad 1 \le i \le n$$

$$f(v_i') = \begin{cases} 0 \; if \; i \; is \; odd \\ 1 \; if \; i \; is \; even \end{cases} \quad 1 \le i \le n$$

Here, $v_f(0)=v_f(1)$ for all n and $e_f(1)= e_f(0)+1$ for all n.
Therefore, the graph $CS(K_{1,n})$ satisfies the conditions $|v_f(0)-v_f(1)| \le 1$ and $|e_f(0)- e_f(1)| \le 1$.

Hence, $CS(K_{1,n})$ is cordial.

Illustration 5. The cordial labeling of $CS(K_{1,3})$ and $CS(K_{1,4})$ are shown in the Figure 5(a) and Figure 5(b).

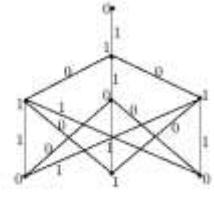

Figure 5(a)

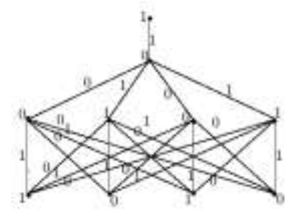

Figure 5(b)

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