Design and Analysis of 5 DOF Wheeled Mobile Robot

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Abstract
This dissertation work is aimed for research over the 5 DoF wheeled mobile robot application. A kinematics models which including direct kinematics, inverse kinematics, and differential kinematics for a wheeled mobile robot is established. For direct kinematics, the Euler angles are used to represent the posture of the end effector, which are more convenient for measurement and control than the posture vectors. During the analysis of differential kinematics, a direct differentiation method is used, which is more accurate than the traditional method. The critical criteria of the tip-over stability for the wheeled mobile robot has been established and validated using stress analysis of wheel over ground. In RoboAnalyzer the arm of WMR is simulated and path traced from initial to final end position. A total of two variants are analyzed for concluding the thesis work. On the extended arm position side the stress are greater and on opposite side it was found to be enough to have friction between wheel and ground to keep WMR stable.

Keywords — *Wheeled mobile robot, Nonholonomic, wheel ground interaction.*

I. INTRODUCTION

The thesis conveys an inventive background, for enabling the mobile robot to explore a congested and cluttered real world surrounding safely, especially, an impulsively fluctuating environment thereby avoiding structured or unstructured obstacles. The work described in this thesis has been carried out in the context of the navigation through various environments with mobile robots. This chapter specifies background information and the basic concept and an overview of the research areas concerning the work carried out as well as the motivation pertaining to the work carried out in this thesis. It then briefly enlightens the overview of major goals of this research i.e. what type of demanding problems have been undertaken and how, which are reaffirmed later in more depth in the successive thesis chapters. Finally, the thesis structure has been sketched.

A. Autonomous Wheeled Robot

Autonomous mobile robots are widely used in industry, defence and rescue operations, space exploration, transportation in warehouses, inspection in constrained spaces and services such as office, restaurants, fire and security, people with disabilities, etc. Some of the mobile robots are shown in Figure 1.1. Wheeled Mobile Robots (WMRs) are mechanical devices capable of moving in an environment with a certain degree of autonomy [8]. These robots are increasingly required to navigate and perform purposeful autonomous tasks in more complex domains, where the environment is uncertain and dynamic. A wheeled robot is an autonomous robot and it can autonomously plan and control its own motion in order to accomplish specified tasks. However, these mobile robots are quite restricted in their motion by nonholonomic constraints on their wheel mechanism. The control design of a mobile robot with nonholonomic constraints on a desired path is very difficult in trajectory tracking with determined velocity [9].

Figure 1.1: Timeline of mobile manipulator development (Photo courtesy of MTECH, Aalborg University, Denmark)

Trajectory tracking generates the control commands for the robot to follow the previously defined path by taking into account the actual position and orientation, linear and angular velocities, nonholonomic constraint and dynamic constraints imposed by the robot. The changes in the terrain topography, texture or in wheel properties due to wear, contamination or deformation play a major role in the robot motion. These variations can easily affect the traction properties and hence the robot movement may cause slippage [17].

B. Wheeled Mobile Robot Definition and Applications

A Wheeled Mobile Robot (WMR) is defined as a wheeled vehicle that can move autonomously without assistance from external human operator. The WMR is equipped with a set of motorized actuators and an array of sensors, which help it to carry out useful work. In order to govern its motion, usually, there is an on-board computer to command the motors to drive, based on reference inputs and the signals gathered by the sensors. Unlike the majority of industrial robots that can only move about a fixed frame in a specific workspace, the WMR has a distinct feature of moving around freely within its predefined workspace to fulfill a desired task [11].

Wheeled Mobile Robot - A robot capable of locomotion on a surface solely through the actuation of wheel assemblies mounted on the robot and in contact with the surface [4]. A wheel assembly is a device which provides or allows relative motion between its mount and a surface on which it is intended to have a single point of rolling contact.

Mobile robot control is an area of research that has attracted attention recently due to the need for autonomous systems. Although the field of robotics has achieved great success in the manufacturing sector, these industrial robots suffer from the major disadvantage of the lack of mobility [5]. Robot manipulators with a fixed base allow limited range of motion and a significantly small workspace compared to robots mounted on movable bases, i.e. a Wheeled Mobile Robots (WMRs).

Wheeled Mobile Robotic Systems

A Wheeled Mobile Robot is an autonomous robot and the class of so called wheeled mobile robotic systems consists of several subclasses. The basic element of every mobile robot is the wheel, which can be simplified to the dynamics of a rolling disk. Many multi wheeled systems found in practice, can be simplified to a kinematic model equal to the model of a unicycle. Some systems have the kinematics of automobile, such as those in car like robots, modeled with wheels of both front and back axle which can be simplified to a bicycle model that is back driving wheels and a front steering wheel.

Rolling Disk

The simplest depiction of this wheeled robot model is the rolling disk, modeled by a vertical disk that rolls without slipping on a horizontal plane, as illustrated in Figure 1.2. Its configuration is completely described by four variables; the planar position coordinates (x, y) of its center point in a fixed frame, the disk orientation θ w with respect to the x-axis and the angle ϕ w between any chosen radial axis on the disk and the vertical axis, where, r is radius of the wheel.

The configuration space of this system has dimension, n = 4, state, q = (x, y, θ w , ϕ w). Due to the non slipping constraint, the generalized velocities of the system are restricted to satisfy the following constraints.

$$\dot{x} - \dot{r}\ \phi\ w \cos\theta\ w = 0 \qquad (1.1)$$

$$\dot{y} - \dot{r}\ \phi\ w \cos\theta\ w = 0 \qquad (1.2)$$

Unicycle Robots

The class of unicycle kinematics is descriptive for many wheeled mobile robots. Its kinematics are described by where x, y are the Cartesian coordinates of center of the rear axle, θ measures the orientation of the robot body with respect to the x axis and ϕ is the rolling angle of the wheel, as depicted in Figure 1.3. Depending on the speed of rotation of each wheel and its direction, the vehicle will maneuver in various directions with the center of rotation anywhere in the line joining the two wheels[4].

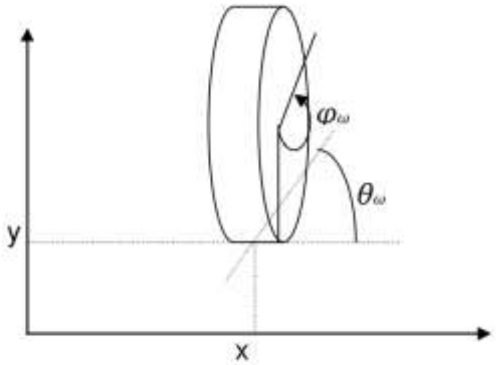

Figure 1.2: Rolling Disk Configuration

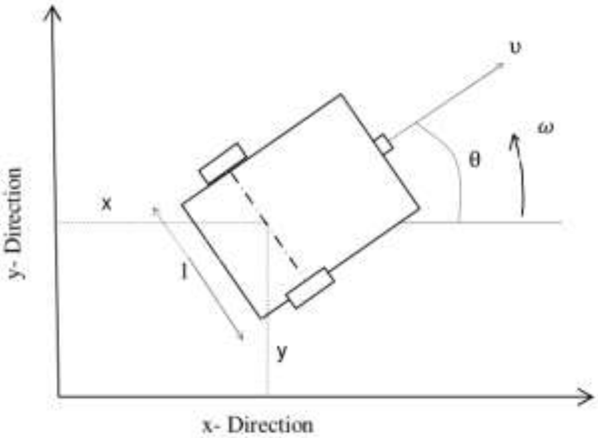

Figure 1.3: Schematic view of a unicycle

Bicycle Robots

The bicycle model consists of a fixed drive wheel at the back and a steerable front wheel. This is also the simplified model for a mobile robot that is similar to an automobile. The generalized coordinates for this kinematic model are q = (x, y, θ , α), where (x, y) are the Cartesian coordinates of the rear axle midpoint, θ describes the orientation of the car with respect to the x-axis and α is the steering angle, as depicted in

Figure 1.4. The robot's motion is restricted by two nonholonomic constraints for each wheel.

$$\dot{y}\cos\theta - \dot{x}\sin\theta = 0 \quad (1.3)$$

$$x_f = x + d \times \cos\theta \quad (1.4)$$

$$y_f = y + d \times \sin\theta \quad (1.5)$$

$$\dot{y}_f\cos(\theta + \alpha) - \dot{x}_f\sin(\theta + \alpha) = 0 \quad (1.6)$$

Mobility

Mobile robots can move from place to place across the ground. Mobility of the robot depends on the vehicle dimensions, locomotion principles and wheel characteristics. Mobility gives a robot a much greater flexibility to perform new, complex and exciting tasks [7]. The world does not have to be modified to bring all needed items within the reach of the robot. The robots can move where needed. Robots with mobility can perform more natural tasks in which the environment is not designed specially for them. These robots can work in a human centered space and cooperate with men by sharing a workspace together.

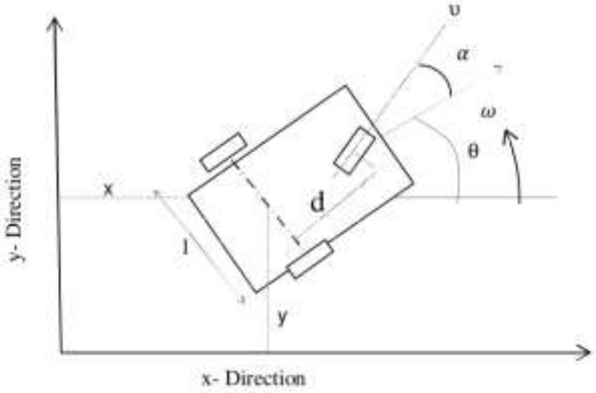

Figure 1.4: Schematic view of a car-like mobile robot

Wheel-Ground Interaction

The function of the wheel is to carry the load and to produce the traction force. The traction force can be used to overcome the rolling resistance and to generate pulling force. Pulling force is the difference between traction and motion resistance and is the force that is available to pull or push an additional payload until the maximum available traction is reached.

Low speed robot model are valid for relatively fast motions, provided either that the turns are not too tight or that the friction between the wheels and the surface is sufficiently large [10]. The actual wheel ground interaction needs to be considered in order to improve the robot motion control. Mobile robots having wheels whose slip rate, rolling, inertia force, moments and mass distribution contribute to the forces exerted on the structure of the vehicle thus affecting the tip over stability [2], accuracy and maneuverability of the robot. Wheeled robots are almost always designed so that all wheels are in ground contact at all times. To avoid tip over of robot, when arm is extended for performing any task the wheels should the exerting at all times some positive force on surface on which it rolls.

Thus three wheels are sufficient to guarantee stable balance, although two wheeled robots can also be stable. When more than three wheels are used, a suspension system is required to allow all wheels to maintain ground contact when the robot encounters uneven terrain. Instead of worrying about balance, wheeled robot research tends to focus on the problems of traction and stability, maneuverability and control which can provide sufficient traction and stability for the robot to cover all of the desired parameters.

This thesis focuses on the possibility of instability of robot when robot is acting and at the end effector robot arm in extended position from robot base.

C. Holonomic and Nonholonomic

In most cases, the system motion is subject to constraints that may arise from the structure itself or from the way it is actuated or controlled [6]. One of the possible classifications of this type of constraints is the separation into bilateral and unilateral constraints, based on whether they can be expressed as equalities or inequalities respectively and as explicitly depending on time or not.

Holonomic constraints are typically introduced by joints in mechanical systems. This class of constraints limits the admissible motions of the system by restricting the set of generalized velocities. In mechanics, this class usually encountered as a Pfafian constraint that is linear in the generalized velocities. It may occur that the kinematic constraints are not integrable [14]. In this case, the constraint and the system is referred to as nonholonomic. This class of nonholonomic constraints influences the system's behaviour in a different way from the holonomic type.

Nonholonomic robots are more prevalent because of their simple design and ease of control. By their nature, nonholonomic mobile robots have fewer degrees of freedom than holonomic mobile robots. These few actuated degrees of freedom in nonholonomic mobile robots often, either independently controllable or mechanically decoupled; further simplify the low level control of the robot. Since they have fewer degrees of freedom, there are certain motions they cannot perform. This creates difficult problems for motion planning and implementation of motion control. In general, motion restrictions that can be written in the form holonomic constraints. Holonomic constraints are typically introduced by joints in Mechanical systems.

$$g_i(q) = 0; i = 1,....,m < n \quad (1.7)$$

This motion of the mobile platform can be described by generalized coordinates $q_m \in R^n$ and generalized velocities $\dot{q}_m \in R^n$. The wheeled mobile platform should move without the slippage of its wheels. This is equivalent to the assumption that momentary velocity at the contact point between each wheel and the motion plane is equal to zero. A holonomic constraint can be expressed purely as a function of the configuration variables and is of the form,

$$g(q,t) = 0 \qquad (1.8)$$

A holonomic constraint reduces the dimension of the configuration space by one. A nonholonomic constraint is a constraint involving velocities and is of the form,

$$g(q',q,t) = 0 \qquad (1.9)$$

A nonholonomic system is one in which the kinematic constraints are non integrable and cannot be reduced to geometric constraints. In other words, a nonholonomic system is one in which the movement is restricted locally, but not globally. Usually, non integrable kinematic constraints are present when a system has fewer control inputs than states. In particular, it is necessary that all of the kinematic constraints are non integrable. A constraint may be non integrable on its own, but when taken together with the others, it may become integrable. Controllability and nonholonomy are closely related. If a system with kinematic constraints is controllable, then it is nonholonomic. This is because controllability implies that the system can attain any position in its state space. In general, it is easier to determine whether a system is controllable than it is to determine whether its kinematic constraints are non integrable.

Example of this system with nonholonomic constraints is a disk rolling on plane (Figure 1.2). The configuration of the disk is given by $q=(x, y, \theta)$ and external inputs in the form a linear velocity υ and angular velocity ω can be applied to it. Thus $u = [\upsilon\omega]^T$. The kinematic model of the rolling disk is where υ and ω are the linear and angular velocity inputs respectively. This disk is subject to the nonholonomic constraint,

$$\dot{y} \cos\theta - \dot{x} \sin\theta = 0 \qquad (1.10)$$

This constraint means that the disk can roll forward and backward but cannot move sideways. Many wheeled mobile robots, including most wheelchairs are governed by this kinematic model and are called differential drive robots. For such robots, the angular velocity can be set independently of the linear velocity and the robot can turn in place.

II. LITERATURE SURVEY

Not much literature exists on the subject of wheeled mobile robotics motion and its control. It is a well researched area but not the case of wheeled mobile robot due to the challenging theoretical nature of the problem and its practical importance.

A. Background

Wheeled mobile robots can have different wheel and axle configurations depending upon the Degrees Of Freedom (DOF). WMR are divided into two groups such as a 2-DOF and a 3-DOF robot. A 2-DOF mobile robot is a three wheeled vehicle with two drive wheels and one caster wheel (differential drive mechanism) and a 3-DOF mobile robot is a three wheeled vehicle with two drive wheels and one steering wheel.

Wheeled Mobile Robot can be defined as a kind of robot integrating a modular manipulator together with a mobile platform. Intelligent mobile manipulators have been paid extensive attention in recent years since they have many applications such as in modern factories for transporting materials, and in dangerous fields for dismantling bombs or moving nuclear infected objects. Traditionally, modular manipulators are mounted on a fixed base whose mobility is constrained. To extend the moving space of the manipulator, when a mobile platform is attached to the modular manipulator to increase the workspace of the modular manipulator greatly. However, building up the kinematics model for a mobile modular manipulator is a challenging task due to the interactive motion between the modular manipulator and the mobile platform. The way in which a given task is separated into motions to be achieved by either the mobile platform or the modular manipulator or by both of them is a key issue caused by the integration of the mobile platform with the modular manipulator. Stability is another concerning issue since the probability of tip-over increases as a result of the mechanical structure.

B. Related Work

Much research work has been published about the problem of wheeled mobile robot motion planning under nonholonomic constraints using only a kinematic model of a mobile robot , because they possess nonholonomic properties caused by non integrable differential constraints.

In many similar works on mobile manipulators, which this research work is based on, a mobile manipulator named mobipulator was proposed by [18] , which used a mobile platform for manipulation of different tasks. Motion planning methods were studied for different types of mobile manipulator to execute multiple missions. The effect of dynamic characteristics between the mobile base and manipulator on the coordinated control were also presented. A dynamic model automatically generated via the NewtonEuler method was presented. Dynamics analysis for the mobile manipulator with Kanes method has been explored. Parameter identification for modular manipulators was investigated. However, few research papers have focused on studying the kinematics, dynamics, and control of mobile modular manipulators. Based on this the kinematics and stability for the mobile modular manipulator are approached. Regarding the direct kinematics analysis, the mobile robot has been treated as a special module added to the bottom of the modular manipulator, which can both move on the horizontal plane and rotate about the Z axis. For convenience of measurement and control, ZYZ Euler angles are used to describe the posture of the mobile modular manipulator. A direct differentiation method which was used to analyse the differential kinematics, than the traditional geometric Jacobian method by the author, same method will be adopted in this research work. As for inverse kinematics analysis, velocity is a precondition for determining the solutions. By combining the mobile platform with the modular manipulator, the problem of stability will be introduced since in some positions and postures the wheeled mobile robot may easily tip over. Hence, a stability analysis for the mobile modular manipulator cannot be ignored.

In determining the tip over stability margin of a ground vehicle system, one is necessarily concerned with the stability of the central body which generally provides mobility , i.e. the vehicle body or base. It is assumed [13] work that the vehicle body is nominally in contact with the ground, as would be the case if mobility is provided via wheels, tracks, alternating (statically stable) legged support, or a combination thereof. A tip over or rollover instability occurs when a nominally upright vehicle body undergoes a rotation which results in a reduction of the number of ground contact points such that all remaining points lie on a single line (the tip over axis). Mobility control is then lost, and finally , if the situation is not reversed, the vehicle is overturned. A low c.g. height is always desirable from a stability point-of-view, heaviness on the other hand is stabilizing at low velocities and destabilizing at high velocities. There work concerns with low velocity systems exerting large forces on the environment, hence, heaviness was considered a stabilizing influence.

In their paper [15] present a solution for the trajectory tracking problem in a Newt mobile robot. They exploit the differential flatness property of the robot kinematic model to propose an input-output linearization controller which allows both the position and the orientation to track a desired trajectory. They make an important assumption that robot has to be initially placed at a point on such a desired trajectory. This controller provides the velocity profiles that the robot wheels have to track and a second controller has to be designed in order to ensure the orientation to track desired trajectory. This was accomplished by means of another differential flatness based control scheme which does not require measurements of any mechanical variables, i.e. velocities, to control the DC motors used as actuators at the wheels.

In the work presented by Chin Pei Tang et al., in [3], presented the differential flatness based integrated planning and control framework for a class of nonholonomic wheeled mobile manipulator (WMM) to achieve full-state controllability. They first showed that the kinematic model under consideration is differentially flat by establishing the one-to-one mapping between the states of the system and the corresponding flat output space. Thus the planning problem has been reduced to a curve fitting problem, i.e., using polynomials of appropriate order to satisfy the specified terminal conditions in the flat out put space.The corresponding control problem is simplified to a linear system pole-placement problem with guaranteed stability. The applicability of the framework on a custom-made electromechanical WMM platform is demonstrated.

In the work done by Nan Hu et al., [12], a new controller is proposed by using back stepping method for the trajectory tracking problem of nonholonomic dynamic mobile robots with nonholonomic constraints under the condition that there is a distance between the mass center and the geometrical center and the distance is unknown. In there approach Adaptive Controller Design is used to control the robot and keep it stable, the difference lies here in there approach. In this work a approach considering mechanical stability is taken.

III. REPRESENTATION OF 5 DEGREE OF FREEDOM WHEELED MOBILE ROBOT

To model the robot with D - H representation, the platform is assigned a local reference frame for each joint. For each joint assigning z-axis and x-axis, as all joints in the manipulator are revolute assigning z-axis about which the joint has to rotate and keeping x-axis in direction of common normal between two joint rotation axis.

A. Denavit-Hartenberg Representation of Forward Kinematics equation of Robot

According to the DenavitHartenberg notation, the simplified model of the robot can be drawn in Figure 3.1.

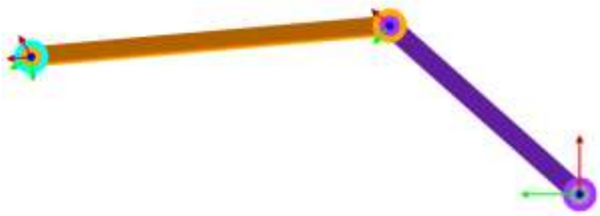

Figure 3.1: A simplified model of 3R Robot

Table 3.1: The DenavitHartenberg parameters

i	**α**	**l_i**	**q_i**	**d_i**
1	0	0	q_1	0
2	0	l_1	q_2	0
3	-90	l_2	q_3	0

Direct Kinematics Analysis

Considering the actual structure of WMR, the DenavitHartenberg parameters are listed in Table 3.1. The total transformation between the base to the end effector is given by equations 3.1 and 3.2 the equations is given by ${}^{B}T_{E}$ where,

$$
{}^{B}T_{E} =
\begin{array}{cccc}
c_{123} & 0 & -s_{123} & l_2\, c_{123} + l_1\, c_{12} \\
s_{123} & 0 & c_{123} & l_2\, c_{123} + l_1\, c_{12} \\
0 & -1 & 0 & 0 \\
0 & 0 & 0 & 1
\end{array}
\qquad (3.1)
$$

$B_3n_x = c_{123}$
$B_3n_y = s_{123}$
$B_3n_z = 0$
$B_3o_x = 0$
$B_3o_y = 0$
$B_3o_z = -1$
$B_3a_x = -s_{123}$
$B_3a_y = c_{123}$
$B_3a_z = 0$
$B_3p_e = l_1c_{123} + l_2\, c_{12}$
$B_3p_e = l_2\, c_{123} + l_1\, c_{12}$
$B_3p_e = 0$
(3.2)

Wheel-ground interaction of the WMR

The actual wheel-ground interaction needs to be considered in order to improve the WMR motion control [16]. The ground is assumed to be rigid and the wheel deformable. Two wheel that roll on a plane while keeping its body vertical as shown in Figure 1.3. The configuration of the WMR can be described by a vector $q = (x, y, \theta, \varphi)$ of generalized coordinates, where x, y are the cartesian coordinates of center of the rear wheels, θ measures the orientation of the robot body with respect to the X axis, and ϕ is the rolling angle of the wheel. At the wheel ground contact point, the holonomic constraint is $\nu_{xc} = 0$, which ensures wheel ground contact is always maintained. Also at each instant, nonholonomic constraints which prevents instantaneous sliding and these are $\nu_{xc} = 0$, and $\nu_{yc} = 0$.

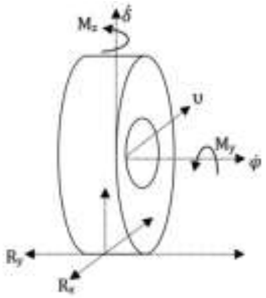

Figure 4.2: Wheel and ground interactions

For simplicity, dynamic model of the wheel is considered, as a thick cylinder that represents the cross section of the wheel and the linear velocity of the wheel center lies in the body plane of the wheel.

The general dynamic equation of the WMR is given by equation 4.3,

$$M(q)\ddot{q} + C(q\dot{q}) + G(q) + \tau_d = B(q)\tau + A^T(q)\lambda \qquad (3.3)$$

where M(q) is the inertia matrix, $C(q,\dot{q})$ is a matrix containing the centrifugal and coriolis terms, G(q) is the gravity force matrix matrix, B(q) is the input transformation matrix, τ is the input torque, A T (q) is the Jacobian matrix associated with the constraints, λ is the constraint force vector and q is the state vector representing the generalized coordinates. τ d denotes the bounded unknown external disturbance. For the continuous nature of the deformation and contact, the non-linear finite element method is selected for the best model and the contact force is measured from the built in geometric model of a wheel and a terrain. When considering the motion resistances, the dynamics of a single wheel as shown in Figure 3.2 and the equation given by

$$Ir\ \phi_w = \tau_r - M_y - F_x\, r_e \qquad (3.4)$$

$$Is\ \delta = \tau_s - M_z \qquad (3.5)$$

where, Ir is moment of inertia of the wheel about rolling, I s is moment of inertia of the wheel about turning, ϕ_w rolling velocity of the wheel, δ turning velocity of the wheel, τ_r rolling torque, τ_s steering torque, M_y moment of rolling resistance, Mz moment of turning resistance and r e effective radius of the wheel.

IV. RESULTS AND DISCUSSION

The Forward and Inverse Kinematic analysis experimentations are performed on a model of the WMR and simulated in RoboAnalyzer. The ground WMR interaction has been evaluated with a approach to find stress on the wheels.

Contact relation of ground with WMR wheel

The resultant frictional forces can be defined by integration of all forces acting on the contact surface. The pressure distribution resulting from the normal contact can be calculated in the local reference. As a consequence, the tangential and the normal forces in the global reference can be calculated by integrating the contact pressures on contact of the X and Y axis for the tangential forces and for the normal force on the Z axis.

$$F_x = \int\!\!\int p_x\, dxdy \qquad (4.1)$$

$$F_y = \int\!\!\int p_y\, dxdy \qquad (4.2)$$

$$M_z = \int\!\!\int (xp_y - yp_x)dxdy \qquad (4.3)$$

At a point of the contact surface the projected force F_y on the Y axis is zero due to the symmetry of the vehicle structure. As a result, contact friction leads not only to a resultant force applied to the center of the area but also to a non-vanishing moment about the normal axis through the center of that area. This moment, M_z is a function of the size of the contact area A, wheel material, type of wheel ground contact, weight of the vehicle, etc. At the contact point, the contact force can be decomposed into normal and tangential components. Let F_x be the horizontal component of contact force and F_z be the normal component of contact force. Assuming that the coordinate frame and center of gravity are lying in symmetry axis of the wheels. So that the contact force $F_y = 0$ and F_z is expressed as the function of contact pressure.

Stress analysis of WMR with ground
An approach to check if the wheel and ground contact is broken is taken to conclude if the WMR tips over.

CAD model of WMR
A simplified model of wheeled mobile robot is shown in Figure **4**.1.

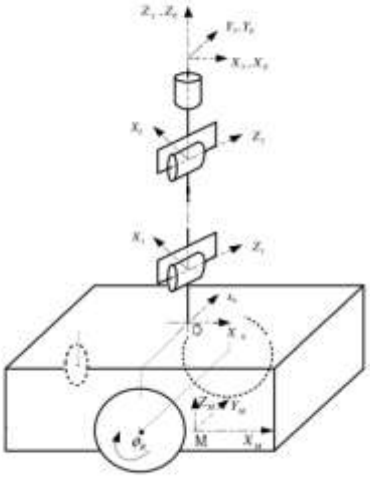

Figure 4.1: Schematic of a wheeled mobile robot

A CAD model of the WMR is shown in Figure 4.2 and the parameters of WMR are given in the Table 4.1.

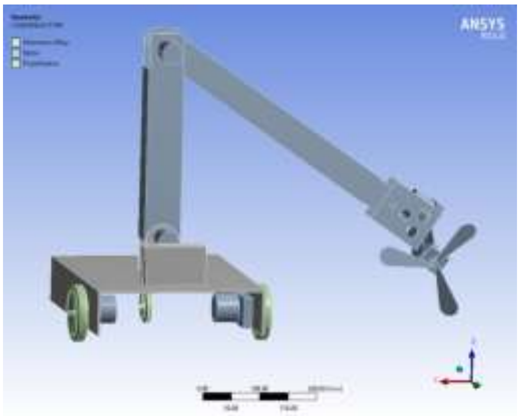

Figure 4.2: CAD model of wheeled mobile robot

Table 4.1: Parameters of Wheeled Mobile Robot

Part No	Material	Quantity	Density g/cm^3	Mass gm
1	Fan	1	1.4	19
2	Fan Motor	1	1	49
3	Motor Mount	1	7.86	33
4	End Effector Arm	1	1.16	13
5	End Effector Motor	1	1	155
6	End Effector mount	1	2.71	75
7	Pin shaft	3	7.86	42
10	Arm 01(Link)	1	1.16	186
8	Arm 02(Link)	1	1.16	233
9	Wheel Motor	2	1	51
11	Platform	1	2.71	2065
12	Wheel	2	0.91	122

Stress due to gravity alone on WMR with Arm Retracted

The WMR to be stable in static condition the stress acting on the two wheels are found in ANSYS R15.0. While the aim is to find the tip over possibility when no external load is acting on WMR, two iterations of analysis are performed.

1. In first iteration of analysis the assembly is considered having its Link 01 90 ◦ with the base and the Link 02 is as closest to the base.
2. In the second iteration of analysis the extended position of both the Links are considered.

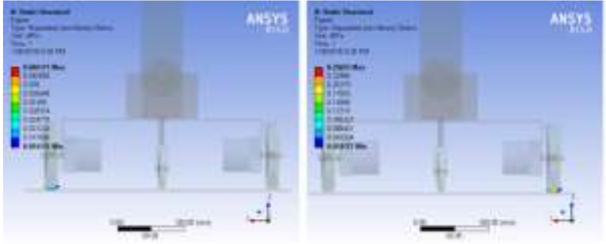

Figure 4.3: Stress on Left - Right Hand Wheel of WMR Arm Retracted

From first iteration of analysis its is clear that values of stress at the contact edge of wheels, Link 02 when not extended produce equivalent von-mises stress of magnitude 0.046111 M Pa on the left wheel of WMR. While in same position of the Links the non extended side wheel or the right hand side wheel bares stress of magnitude of 0.25653 M Pa. When stress of both the wheels are compared, it is seen from the Figure 4.3 that the stress on both wheels significantly different. From this we can conclude that the robot will tend to tilt if the stress values become negative on any one side.

Stress due to gravity alone on WMR with Arm Extended
From second iteration of analysis its is clear that the values of stress at the contact edge of wheels, Link 02 when extended produce equivalent von-mises stress of magnitude 0.054505 M Pa on the left wheel of WMR. While in same position of the Links the non extended side wheel or the right hand side wheel bares stress of magnitude of 0.60949 M Pa. When stress of both the wheels are compared, it is seen

from the Figure 4.4 that the stress on both wheels significantly different. From this we can conclude that the robot will tend to tilt if the stress values become negative on any one side.

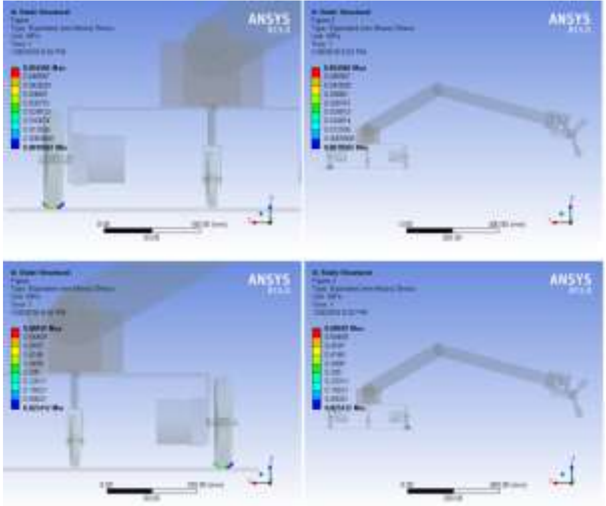

Figure 4.4: Stress on Left - Right Hand Wheel of WMR Arm Extended

At the current position the wheel has not yet left the contact of the ground. Thus the results of the two iteration helps to bring to a firm conclusion. This point of extended arm can be considered the limiting value for maximum position the end effector can trace.

A. *Simulation of WMR in RoboAnalyzer*

The WMR as a whole has been simulated in the open source software RoboAnalyzer. The plots of Force - Time curve are plotted for all the three joints as shown in Figure 4.5 and the Velocity - Acceleration curve for three Joints is shown in Figure 4.6.

This simulation results are helpful in motors selection for joints as the required torques should be generated by selected motors.

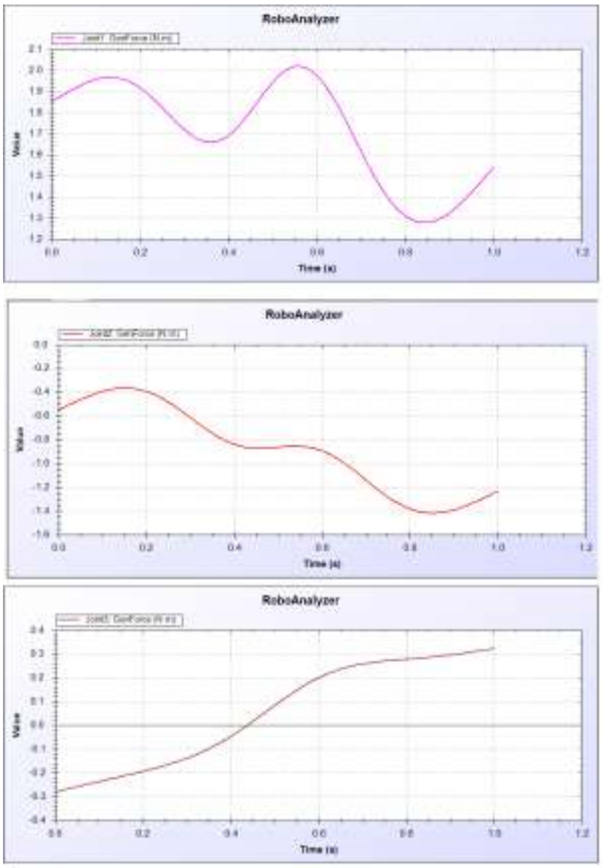

Figure 4.5: Force - Time curve for three Joints

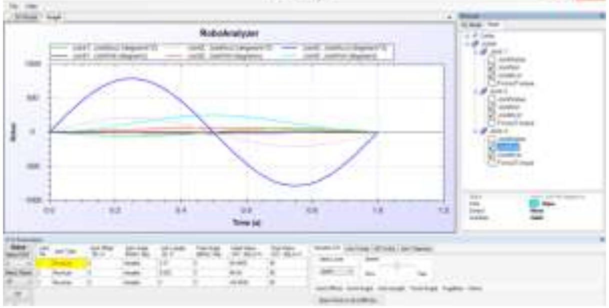

Figure 4.6: Velocity - Acceleration curve for three Joints

V. CONCLUSION

The kinematics models including direct kinematics, inverse kinematics, and differential kinematics for a wheeled mobile robot is established. For direct kinematics, the Euler angles are used to represent the posture of the end effector, which are more convenient for measurement and control than the posture vectors. During the analysis of differential kinematics, a direct differentiation method was used, which is more accurate than the traditional geometric Jacobian method. The critical criteria of the tip-over stability for the wheeled mobile robot has been established. In RoboAnalyzer the arm of WMR is simulated and path traced from initial to final end position is determined. By performing a stress analysis in Ansys shows that the mechanical structure is safe for working at extended position of arm. The tip over of WMR was checked by comparing the contact stress of two wheels. On the extended side the stress is greater and on opposite side it was found to be enough to have friction between wheel and ground to keep WMR stable.

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