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Soret Effect on Chemically Reacting Natural Convection between Two Concentric Circular Cylinders in a Porous Medium

B.R.Sharma[1] and Debozani Borgohain[2]

[1,2]*Department of Mathematics, Dibrugarh University, Dibrugarh-786004, Assam, India*

Abstract: *The effects of Soret number, Dufour number and order of chemical reaction on separation of a binary fluid mixture of an incompressible, viscous, chemically reacting, thermally and electrically conducting fluid confined between two concentric stationary circular cylinders embedded in a porous medium in the presence of uniform radial magnetic field are investigated. The governing non linear partial differential equations in cylindrical polar coordinates are transformed into coupled non dimensional non linear ordinary differential equations by using non-dimensional quantities and are solved numerically by using MATLAB's built in solver bvp4c. The influence of the Dufour number, Soret number, dimensionless chemical reaction parameter and order of chemical reaction on temperature and concentration profiles as well as on Nusselt number and Sherwood number are illustrated graphically. It is concluded that the Soret number and the chemical reaction play a crucial role on the heat and mass transfer.*

Keywords*: Binary fluid mixture; porous medium; two concentric circular cylinders; Dufour and Soret effects; chemical reaction.*

I. INTRODUCTION

Many researchers have shown a great amount of interest in the field of heat and mass transfer over years due to their applications in chemical industries and hydro-metallurgical industries. The binary mixture in MHD through porous medium has many applications in geothermal reservoirs, drying of porous solids, thermal insulation, packed-bed catalytic reactors, the study of the stellar and solar structures, interstellar matter, radio propagation through the ionosphere, hydromagnetic flow and heat transfer in the earth's liquid core and others. Chemical reactions in combined heat and mass transfer problems play a significant role. Chemical reactions are either homogeneous or heterogeneous processes. The reaction is homogeneous, if it occurs uniformly through a given phase. In well mixed system, it takes place in the solution while a heterogeneous reaction occurs at the interface i.e. in a restricted region or within the boundary of a phase. If the rate of reaction is proportional to the n^{th} power of concentration then the chemical reaction is said to be of order 'n'. Chemical reaction between a foreign mass and the fluid occurs in many industrial applications such as polymer production, formation and dispersion of fog, damage of crops due to freezing and so on. Separation processes of components of a fluid mixture have many applications in science and technology, environmental engineering, chemical industry and separation of isotopes from their naturally occurring mixture. Soret effect is the tendency of a convection-free fluid mixture to separate under a temperature gradient. Soret effect also plays a crucial role in the hydrodynamic instability of mixtures, mineral migrations and mass transport in living matters.

II. RELATED WORK

Vasseur et. al. [1] have discussed the natural convection between horizontal concentric cylinders filled with a porous layer with internal heat generation. Flow pattern of natural convection in horizontal cylindrical annuli was studied by Rao et. al. [2]. Caltigirone[3] has analyzed the thermoconvective instabilities in a porous medium bounded by two concentric horizontal cylinders. Convective heat transfer in vertical cylindrical annuli filled with a porous medium was investigated by Havstad et. al.[4]. Sharma et. al. [5] have studied the thermal diffusion in a binary fluid mixture confined between two concentric circular cylinders in presence of a radial magnetic field. Vijaya et. al.[6] have examined the radiation effect on mixed convection flow through a porous medium in concentric circular annulus with constant heat sources.

The present paper deals with the steady incompressible flow of a viscous binary fluid mixture between the walls of two concentric circular annuli embedded in a porous medium in the presence of n^{th}

order chemical reaction and radial magnetic field. The aim of this paper is to study the influence of Soret effect, Dufour effect, dimensionless chemical reaction parameter and order of chemical reaction. The effects of material parameters on temperature and concentration as well as on Nusselt number and Sherwood number are investigated mathematically.

III. FORMULATION OF THE PROBLEM

Consider the steady incompressible flow of a viscous, chemically reacting, thermally and electrically conducting fluid confined between the walls of two infinite concentric circular cylinders embedded in a porous medium in the presence of a uniform radial magnetic field of strength $\frac{B_0 a}{r}$ where r is the radial distance from the axis of the annulus. Homogeneous chemical reaction of order n takes place in the flow. Let r = a be the radius of the cylinder which is insoluble in the fluid and r = b be the radius of the cylinder which diffuses to the fluid to establish the equilibrium near the surface $(b > a)$. The plates are maintained at uniform constant temperatures T_0 at r = a and T_1 at r = b. The surface of the cylinder with radius r = a is considered to be impervious and the concentration of the rarer and lighter components is considered to be C_1 at r = b. The flow of the binary fluid mixture is considered to be in the axial direction. As the cylinders are of infinite length, so the flow depends only on r. Therefore the velocity vector is of the form $(0, 0, w(r))$.

Under these assumptions, the governing equations of continuity, momentum, energy and species conservation in the cylindrical polar coordinate system (r, θ, z) are given by

$$\frac{\partial w}{\partial z} = 0 \tag{1}$$

$$\frac{1}{r}\frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial r^2} - \frac{a^2 \sigma B_0^2 w}{r^2 \mu} - \frac{\nu}{\kappa} w = -\frac{P}{\mu} \tag{2}$$

$$\alpha_m \left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right) + \mu \left(\frac{\partial w}{\partial r}\right)^2 + \frac{a^2 \sigma B_0^2 w^2}{r^2} + \frac{D_m K_T}{C_S C_P}\left(\frac{\partial^2 C}{\partial r^2} + \frac{1}{r}\frac{\partial C}{\partial r}\right) = 0 \tag{3}$$

$$D_m \left(\frac{\partial^2 C}{\partial r^2} + \frac{1}{r}\frac{\partial C}{\partial r}\right) + \frac{D_m K_T}{T_m}\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right) - K_1 C^n = 0 \tag{4}$$

where B_0 is applied magnetic field, C is concentration, C_P is specific heat of the fluid at constant pressure, C_S is concentration susceptibility, D_m is mass diffusion coefficient, K_1 is dimensional chemical reaction parameter, K_T is thermal diffusion ratio, n is order of the chemical reaction, P is pressure, T is temperature, T_m is mean fluid temperature, w is fluid velocity in axial direction, α_m is thermal conductivity, κ is permeability of porous medium, μ is coefficient of viscosity of the binary fluid mixture, ν is kinematic viscosity and σ is electrical conductivity.

The boundary conditions are:

$$\left.\begin{array}{ll} w = 0, T = T_0, \frac{dC}{dr} + \frac{K_T}{T_m}\frac{dT}{dr} = 0 & \text{at } r = a \\ w = 0, T = T_1, C = C_1 & \text{at } r = b \end{array}\right\} \tag{5}$$

We now introduce the following dimensionless variables:

$$w = \frac{Pa^2}{\mu} W, T = T_1 + (T_0 - T_1)\theta, C = C_1 \phi, r = a\eta \tag{6}$$

Introducing the relation (6) into the equations (2), (3) and (4), we obtain the following equations:

$$\frac{d^2 W}{d\eta^2} + \frac{1}{\eta}\frac{dW}{d\eta} - \left(\frac{M^2}{\eta^2} + \frac{1}{Da}\right) W = -1 \tag{7}$$

$$\frac{d^2\theta}{d\eta^2} + \frac{1}{\eta}\frac{d\theta}{d\eta} + EcPr\left(\left(\frac{dW}{d\eta}\right)^2 + \frac{M^2}{\eta^2} W^2\right) + D_f Pr\left(\frac{d^2\phi}{d\eta^2} + \frac{1}{\eta}\frac{d\phi}{d\eta}\right) = 0 \tag{8}$$

$$\frac{d^2\phi}{d\eta^2} + \frac{1}{\eta}\frac{d\phi}{d\eta} + SrSc\left(\frac{d^2\theta}{d\eta^2} + \frac{1}{\eta}\frac{d\theta}{d\eta}\right) - \gamma Sc\phi^n = 0 \tag{9}$$

where Da is porosity parameter, D_f is Dufour number, M is Hartmann number, Pr is Prandtl number, Sc is Schmidt number, Sr is Soret number, Ec is Eckert number and γ is dimensionless chemical reaction parameter and are defined as :

$$\left.\begin{array}{l} M = \frac{aB_0\sqrt{\sigma}}{\sqrt{\mu}}, Pr = \frac{\mu C_P}{\kappa} = \frac{\nu}{\kappa}, Sc = \frac{\nu}{D_m}, Da = \frac{\kappa}{\nu a^2}, \\ D_f = \frac{D_m K_T}{\nu C_S C_P}\frac{C_1}{(T_0 - T_1)}, Sr = \frac{D_m K_T}{\nu T_m}\frac{(T_0 - T_1)}{C_1}, Ec = \\ \frac{P^2 a^4}{\mu^2 C_P (T_0 - T_1)}, \gamma = \frac{K_1 a^2}{\nu} \end{array}\right\} \tag{10}$$

The boundary conditions (5) are now transformed to

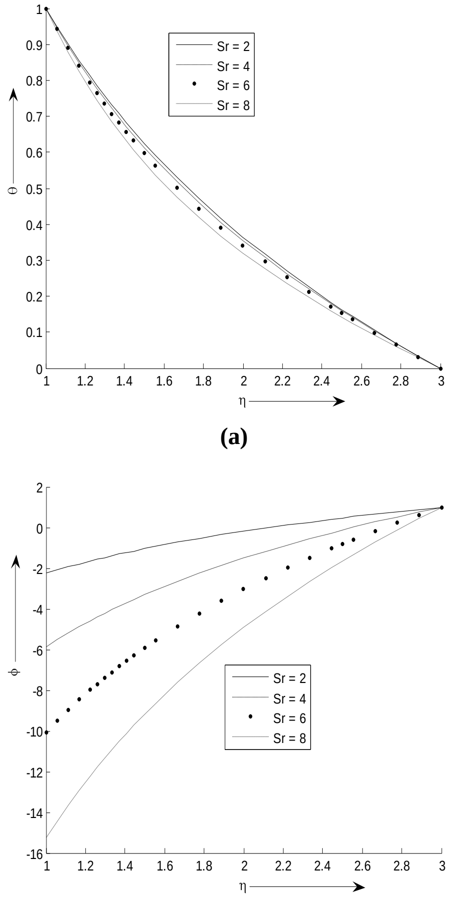
$$\left.\begin{array}{ll} W = 0, \theta = 1, \frac{d\phi}{d\eta} + SrSc\frac{d\theta}{d\eta} = 0 & \text{at } \eta = 1 \\ W = 0, \theta = 0, \phi = 1 & \text{at } \eta = \frac{b}{a} \end{array}\right\} \tag{11}$$

The system of ordinary differential equations (7) - (9) under the boundary conditions (11) are non-linear and highly coupled and so cannot be obtained in the closed form. Therefore these equations have been solved numerically by using MATLAB's built in solver bvp4c. Also the effects of the material parameters on the Nusselt and the Sherwood numbers which are respectively proportional to $(-\theta'(1), \phi'(1))$ are discussed graphically.

IV. RESULTS AND DISCUSSION

Numerical calculations have been carried out for concentration of the rarer component and temperature of the binary fluid mixture for various values of the parameters Sr, D_f, Da, γ and n and are plotted against η in Figures 1-8. The values of Schmidt number (Sc) are chosen for hydrogen (Sc = 0.22), water vapour (Sc = 0.62), carbon dioxide (Sc =1.14) and ethanol diffused in air (Sc = 1.6). The values of Prandtl number are chosen to be Pr = (0.7, 1.38, 4.2, 7) which represent air, gaseous ammonia, R-12 refrigerant and water at temperature 20^{o}C respectively.

Figures 1 (a)-(b) depict temperature and concentration profiles for various values of $\mathrm{Sr}=(2, 4, 6, 8)$ by taking $\mathrm{Da}=1, \mathrm{D_f}=1, \mathrm{Pr}=0.7$, $\mathrm{M}=10, \mathrm{Sc}=1.6, \mathrm{Ec}=0.2, \gamma=0.02, \mathrm{n}=2$. It is noticed that with an increase in the values of Soret number Sr, the temperature of the binary fluid mixture and the concentration of the rarer and lighter components decrease. It is evident that the temperature of the binary fluid mixture rises and the species separation can be enhanced by reducing the Soret effect. Thus the species separation can be promoted and the binary fluid mixture gets heated by decreasing the temperature difference between the surfaces of the two cylinders.

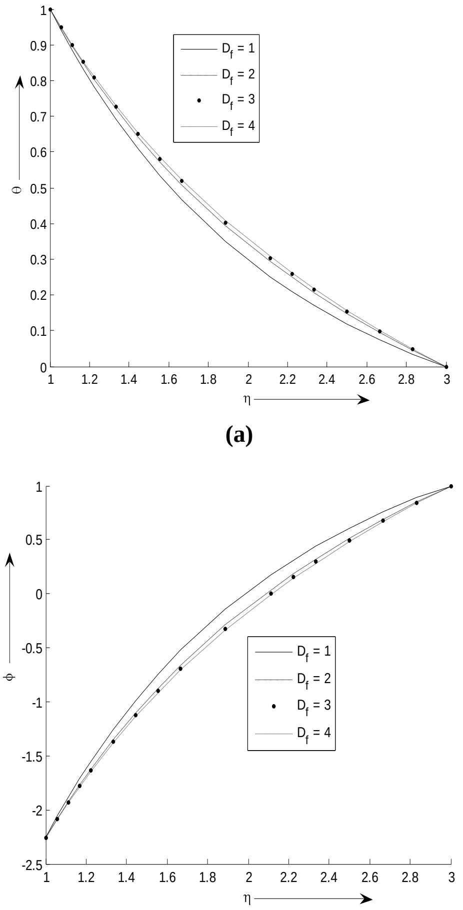


(a)

(b)

Fig. 1: Effects of Soret number Sr on (a) temperature profiles (b) concentration profiles

Figures 2 (a)-(b) exhibit temperature and concentration profiles for various values of $\mathrm{D_f}=(1, 2, 3, 4)$by taking $\mathrm{Sr}=2, \mathrm{Da}=1, , \mathrm{Pr}=0.7$, $\mathrm{M}=5, \mathrm{Sc}=1.6, \mathrm{Ec}=3, \gamma=0.02, \mathrm{n}=2$. It is noticed that with an increase in the values of Dufour number D_f, the concentration of the rarer and lighter components decreases while temperature of the binary fluid mixture increases. It can be established that the Dufour number has positive effect on temperature of the binary mixture but negative effect on species separation.



(a)

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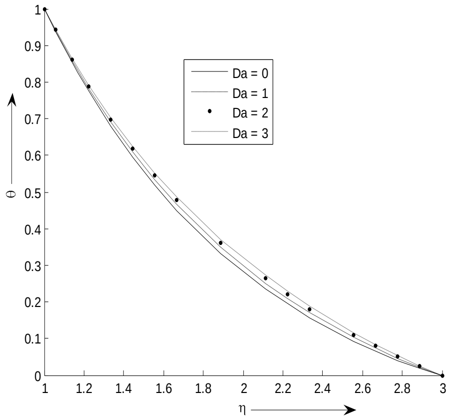
Fig. 2: Effects of Dufour number D_f on (a) temperature profiles (b) concentration profiles

Figure 3 (a)-(b) show the temperature and concentration profiles for various values of Da = (0, 1, 2, 3) by considering $\mathrm{Sr}=2, D_f=1, \mathrm{Pr}=0.7$, $\mathrm{M}=5, \mathrm{Sc}=1.6, \mathrm{Ec}=3, \gamma=0.02, \mathrm{n}=2$. It is observed that the porosity parameter has got reverse effect on temperature and concentration profiles. The temperature of the binary fluid mixture rises whereas the concentration of the components reduces with the increase in porosity.



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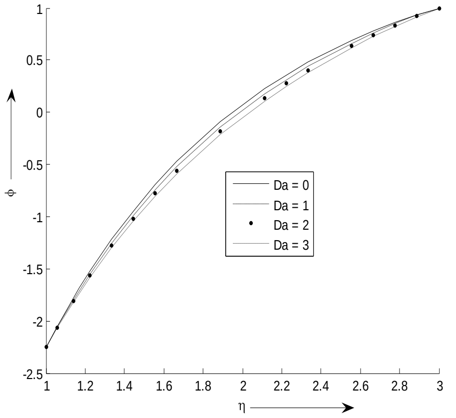
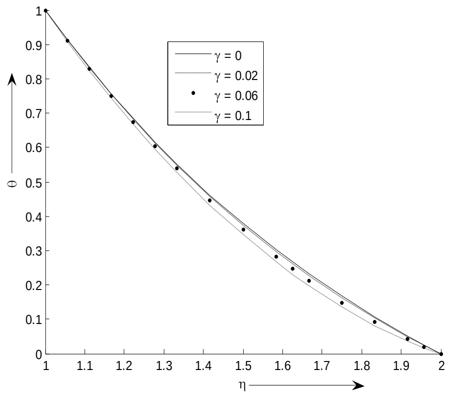


(b)

Fig. 3: Effects of Porosity parameter Da on (a) temperature profiles (b) concentration profiles

Figures 4 (a)-(b) depict temperature and concentration profiles for various values of $\gamma=(0, 0.02, 0.06, 0.1)$ by taking $Sr=1, Da=1, D_f=1, Pr=0.7, M=10, Sc=1.6, Ec=3, , n=2$. It is observed that with an increase in the values of dimensionless chemical reaction parameter γ, the concentration of the rarer and lighter components increases while the temperature decreases thereby suggesting that chemical reaction favours the concentration but opposes the temperature of the binary mixture.



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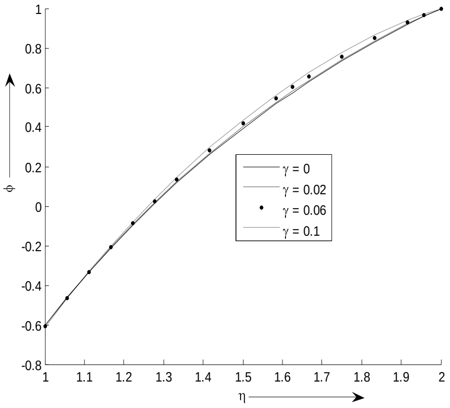
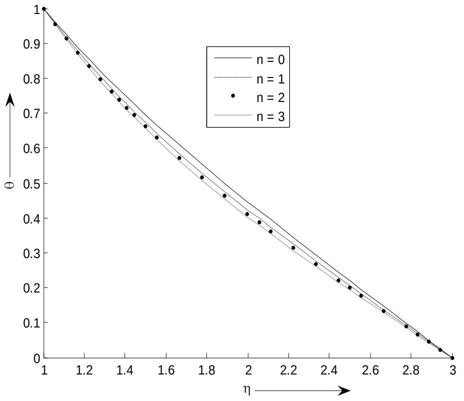


(b)

Fig. 4: Effects of dimensionless chemical reaction parameter γ on (a) temperature profiles (b) concentration profiles

The temperature and concentration profiles for various values of n = (0, 1, 2, 3) and considering the other values as $Sr=1, Da=1, D_f=1, Pr=0.7, M=10, Sc=0.6, Ec=0.2, \gamma=0.2$ are shown in figure 5 (a)-(b). It is noticed that with an increase in the values of order of chemical reaction n, the concentration of the rarer and lighter components increases while temperature decreases. Thus the order of chemical reaction has positive effect on concentration but reverse effect on temperature.



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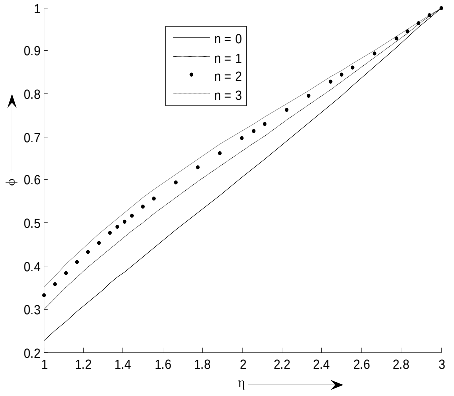
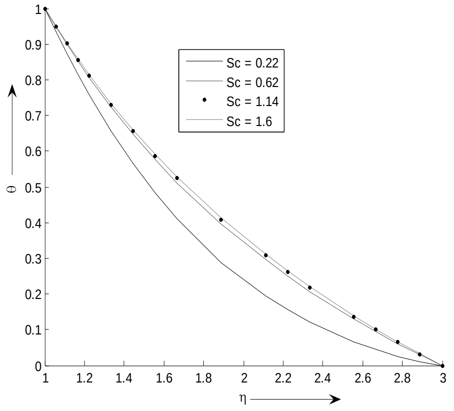


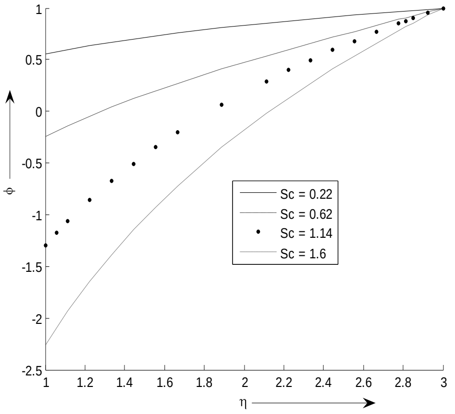
(b)

Fig. 5: Effects of order of chemical reaction n on (a) temperature profiles (b) concentration profiles

Figure 6 (a)-(b) exhibit the temperature and concentration profiles for various values of Sc = (0.22, 0.62, 1.14, 1.6) by taking $Sr=1, Da=1, D_f=1, Pr=7, M=5, Ec=0.2, \gamma=0.02, n=2$.



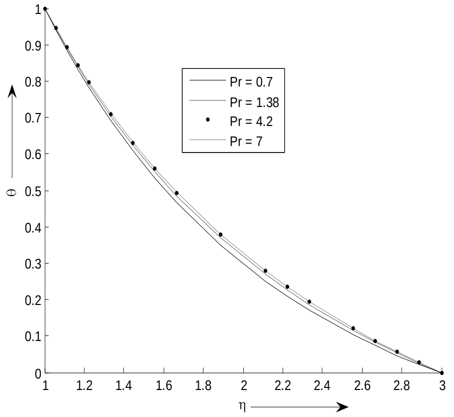
(a)



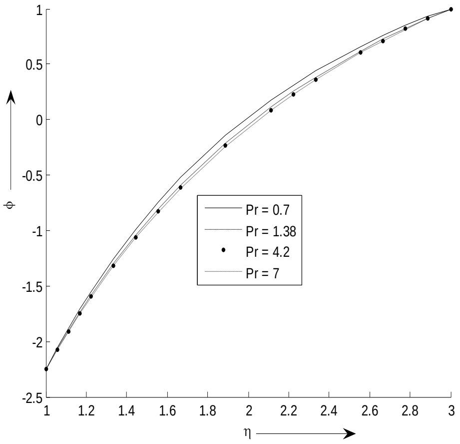
(b)

Fig. 6: Effects of Schmidt number Sc on (a) temperature profiles (b) concentration profiles

Figure 7 (a)-(b) show the temperature and concentration profiles for various values of Pr = (0.7, 1.38, 4.2, 7) by taking $Sr=2, Da=1, D_f=1, M=5, Sc=1.6, Ec=3, \gamma=0.02, n=2$.



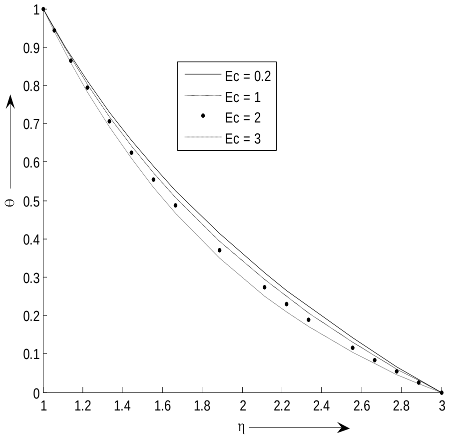
(a)



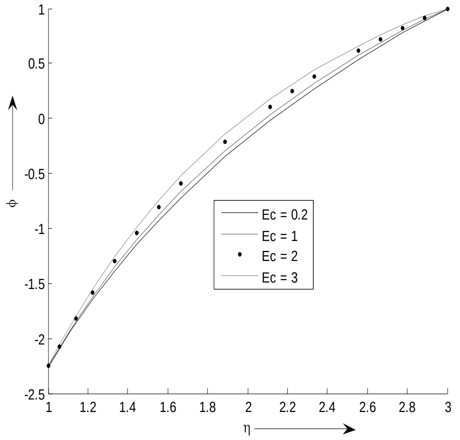
(b)

Fig. 7: Effects of Prandtl number Pr on (a) temperature profiles (b) concentration profiles

The temperature and concentration profiles for various values of Ec = (0.2, 1, 2, 3) by taking $Sr=2, Da=1, D_f=1, M=5, Sc=1.6, Pr=0.7, \gamma=0.02, n=2$ are shown in figure 8(a)-(b).



(a)



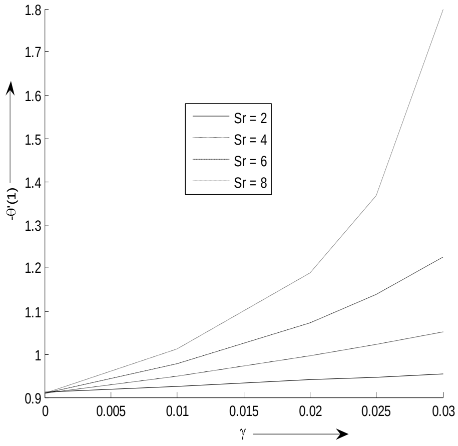
(b)

Fig. 8: Effects of Eckert number Ec on (a) temperature profiles (b) concentration profiles

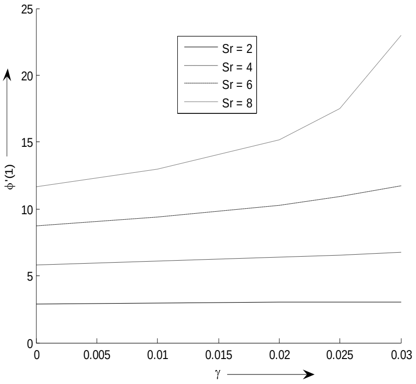
From the figures 1 to 8, it is also clear that for all the parameters with the increase in η the temperature of the binary fluid mixture through the concentric cylinders embedded in porous medium decreases exponentially from its maximum value at the surface of inner cylinder to its minimum value at the surface of other cylinder. This makes the cylinder heated from inside but colder from outside. But the result gets reversed for the concentration of the rarer and lighter components i.e. more particles get accumulated near the surface of the outer cylinder thereby throwing away the lighter particles towards the inner cylinder, so that, they can be easily separated. Also it is evident that with the increase in the values of Soret number, dimensionless chemical reaction parameter, order of chemical reaction and Eckert number, the temperature of the binary fluid mixture reduces while the concentration of the components increases but the effects get reversed with the enhancement of Dufour number, Porosity parameter, Schmidt number and Prandtl number.

Finally, with the help of the figures 9 and 10, the behaviour of the Nusselt number $(-\theta'(1))$ and Sherwood number $(\phi'(1))$ at $\eta = 1$ i.e. when $a = b$ are observed.

Figure 9 show the effects of Soret number Sr and dimensionless chemical reaction parameter γ on the Nusselt number and Sherwood number at $\eta = 1$. It is noticed that the Nusselt number $(-\theta'(1))$ and the Sherwood number $(\phi'(1))$ at $\eta = 1$ i.e. when $a = b$ increase with the increasing values of either the Soret number Sr or the dimensionless chemical reaction parameter γ.



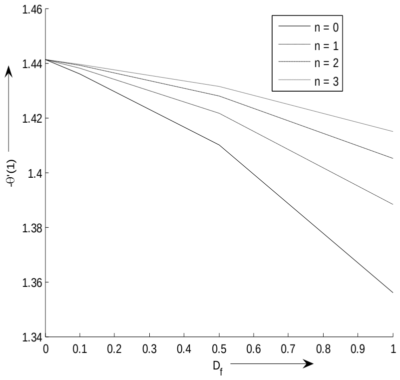
(a)



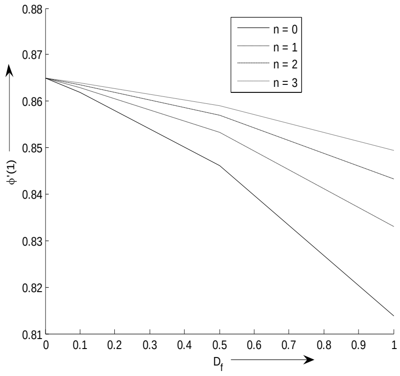
(b)

Fig. 9: Effects of Soret number Sr and dimensionless chemical reaction parameter γ on (a) Nusselt number $-\theta'(1)$ and (b) Sherwood number $\phi'(1)$.

Figure 10 exhibit the effects of Dufour number D_f and order of chemical reaction n on the Nusselt number and Sherwood number at $\eta = 1$. It is noticed that the Nusselt number $(-\theta'(1))$ and the Sherwood number $(\phi'(1))$ at $\eta = 1$ i.e. when $a = b$ decrease with the increasing values of the Dufour number D_f and decreasing order of chemical reaction n.



(a)



(b)

Fig. 10: Effects of order of chemical reaction n and Dufour number D_f on (a) Nusselt number $-\theta'(1)$ and (b) Sherwood number $\phi'(1)$.

V. CONCLUSION

From the above discussion, it can be concluded that

- The concentration of the rarer and lighter components is favoured while the temperature of the binary fluid mixture flowing through the concentric cylinders is reversely affected by chemical reaction, Soret number and Eckert number.
- Prandtl number, Schimdt number, Dufour number and porosity parameter favour the temperature of the binary fluid mixture whereas the concentration is opposed by these parameters.
- By reducing the Dufour effect and porosity of the medium and by enhancing the Soret effect and chemical reaction, the temperature of the binary fluid mixture in such situations can be controlled.
- The rates heat and mass transfer are enhanced by the chemical reaction and Soret effect whereas the Dufour number reduces them.

  The results of the work can be helpful in some practical applications such as the insulation of aircraft cabin or horizontal pipes, cryogenics, the underground cable systems, gas separating instruments and the storage of thermal energy. It can also be utilised in nuclear waste disposal research, chemical engineering and petroleum engineering.

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