Application of Fuzzy Soft Set in Selection Decision Making Problem

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Abstract: *In our daily life we often face some problems in which the right decision making is highly essential. But in most of the cases we become confused about the right solution. To obtain the best feasible solution of these problems we have to consider various parameters relating to the solution. For this we can use the best mathematical tool called Fuzzy soft set theory. In this paper we select a burning problem for the parents and successfully applied the fs-aggregation algorithm in decision making for selecting a suitable bride by the family.*

Keywords:
Fuzzy set, Fuzzy Soft set, fs-aggregation

I. INTRODUCTION:

In many complicated problems arising in the fields of engineering, social science, economics, medical science etc involving uncertainties. Classical methods are found to be inadequate in recent times. Molodtsov [13] pointed out that the important existing theories viz. Probability Theory, Fuzzy Set Theory, Intuitionistic Fuzzy Set Theory, Rough Set Theory etc. which can be considered as mathematical tools for dealing with uncertainties, have their own difficulties. He further pointed out that the reason for these difficulties is, possibly, the inadequacy of the parameterization tool of the theory. In 1999 he initiated the novel concept of Soft Set as a new mathematical tool for dealing with uncertainties. Soft Set Theory, initiated by Molodtsov [13], is free of the difficulties present in these theories.

Molodtsov applied this theory to several directions [13], [14], [15], and then formulated the notions of soft number, soft derivative, soft integral, etc. in [16]. The soft set theory has been applied to many different fields with great success. Maji et al. [11] worked on theoretical study of soft sets in detail, and [10] presented an application of soft set in the decision making problem using the reduction of rough sets [18]. Chen et al. [5] proposed parameterization reduction of soft sets, and then Kong et al. [7] presented the normal parameterization reduction of soft sets.

Recently, many scholars study the properties and applications on the soft set theory. Xiao et al. [22] studied synthetically evaluating method for business competitive capacity and also Xiao et al. [23] gave recognition for soft information based on the theory of soft sets. Pei and Miao [19] showed that the soft sets are a class of special information systems. Mushrif et al. [17] presented a new algorithm based on the notions of soft set theory for classification of the natural textures. Kovkov et al. [8] considered the optimization problems in the framework of the theory of soft sets which is directed to formalization of the concept of approximate object description. Zou and Xiao [25] presented data analysis approaches of soft sets under incomplete information. Majumdar and Samanta [12] studied the similarity measure of soft sets. Ali et al. [1] introduced the analysis of several operations on soft sets.

Maji et al. [9] presented the concept of the fuzzy soft sets (fs-sets) by embedding the ideas of fuzzy sets [24]. By using this definition of fs-sets many interesting applications of soft set theory have been expanded by some researchers. Roy and Maji [20] gave some applications of fs-sets. Som [21] defined soft relation and fuzzy soft relation on the theory of soft sets. Krishna Gogoi et al [27] applied fuzzy soft set and Bhardwaj et al. [28] used Reduct soft set for real life decision making problems.

The operations of the fs-sets and soft sets defined by Maji et al. [9],[11] are used in all the works mentioned above. But, Chen et al. [5], Pei and Miao [19], Kong et al. [6] and Ali et al. [1] pointed out that these works have some weak points. Therefore, to develop the theory, Cagman and Enginoglu [2] redefined operations of the soft sets which are more functional for improving several new results. By using these new operations, Cagman and Enginoglu [3] presented a soft matrix theory. Cagman et al. [4] defined a fuzzy parameterized soft set theory and its decision making method.

In this paper author is defining fuzzy soft set and applying fuzzy soft set aggregation method to solve decision making problems. The fs-aggregation algorithm is well defined by Cagman et al. [26] for decision making. We finally give an example which shows that the method can be successfully applied to many problems containing uncertainties.

II. PRELIMINARIES:

In this section, we present the basic definitions of soft set theory [13] and fuzzy set theory [24] that are useful for subsequent discussions. Throughout this work, U refers to an initial universe, E is a set of parameters, P (U) is the power set of U, and A ⊆E.

Definition 2.1. A soft set F_A over U is a set defined by a function f_A representing a mapping

$f_A : E \rightarrow P(U)$ such that $f_A(x) = \emptyset$; if $x \notin A$

Here, f_A is called approximate function of the soft set F_A, and the value $f_A(x)$ is a set called x-element of the soft set for all x € E. It is worth noting that the sets $f_A(x)$ may be arbitrary, empty, or have nonempty intersection. Thus a soft set over U can be represented by the set of ordered pairs

$F_A = \{(x, f_A(x)) : x \in E, f_A(x) \in P(U)\}$

the set of all soft sets over U will be denoted by S(U).

Example 2.1. Let $U = \{u_l, u_2, u_3, u_4, u_5\}$ be a universal set and $E = \{x_1, x_2, x_3, x_4\}$ be a set of parameters. If $A = \{x_1, x_2, x_4\} \subseteq E$, $f_A(x_1) = \{u_2, u_4\}$, $f_A(x_2) = U$ and $f_A(x_4) = \{u_1, u_3, u_5\}$, then the soft set F_A is written by
$F_A = \{(x_1, \{u_2, u_4\}), (x_2, U), (x_4, \{u_1, u_3, u_5\})\}$.

Definition 2.2. Let U be a universe. A fuzzy set X over U is a set defined by a function μ_X representing a mapping

$\mu x: U \rightarrow [0, 1]$

μx is called the membership function of X, and the value $\mu_X(u)$ is called the grade of membership of $u \in U$. The value represents the degree of u belonging to the fuzzy set X. Thus, a fuzzy set X over U can be represented as follows:

$X = \{(\mu_X(u)/u), u \in U, \mu_X(x) \in [0, 1]\}$.

The set of all the fuzzy sets over U will be denoted by F(U).

In the soft sets, the parameter sets and the approximate functions are crisp. But in the fs-sets, while the parameters sets are crisp, the approximate functions are fuzzy subsets of U. From now on, we will use $\Gamma_A, \Gamma_B, \Gamma_C$,..., etc. for fs-sets and $\gamma_A, \gamma_B, \gamma_C$,..., etc. for their fuzzy approximate functions, respectively.

Definition 2.3. An fs-set Γ_A over U is a set defined by a function γ_A representing a mapping

$\gamma A: E \rightarrow F(U)$ such that $\gamma_A(x) = \emptyset$; if $x \notin A$:

Here, γ_A is called fuzzy approximate function of the fs-set Γ_A, and the value $\gamma_A(x)$ is a set called x-element of the fs-set for all x€ E. Thus, an fs-set Γ_A over U can be represented by the set of ordered pairs

$\Gamma_{A=} \{(x, \gamma_A(x)) : x \in E; \gamma_A(x) \in F(U)\}$:

Note that the set of all fs-sets over U will be denoted by FS(U).

Example 2.2. Let $U = \{u_l, u_2, u_3, u_4, u_5\}$ be a universal set and $E = \{x_1, x_2, x_3, x_4\}$ be a set of parameters. If $A = \{x_1, x_2, x_4\} \subseteq E$, $\gamma_A(x_1) = \{0.9/u_2, 0.5/u_4\}$, $\gamma_A(x_2) = U$, and $\gamma_A(x_4) = \{0.2/u_1, 0.4/u_3, 0.8/u_5\}$ then the soft set F_A is written by

$F_A = \{(x_1, \{0.9/u_2, 0.5/u_4\}), (x_2; U), (x_4, \{0.2/u_1, 0.4/u_3, 0.8/u_5\})\}$.

Definition 2.4. Let $\Gamma_A \in FS(U)$. If $\gamma_A(x) = \emptyset$; for all $x \in E$, then Γ_A is called an empty fs-set, denoted by $\Gamma\Phi$.

Definition 2.5. Let $\Gamma_A \in FS(U)$. If $\gamma_A(x) = U$ for all $x \in A$, then Γ_A is called A-universal fs-set, denoted by $\Gamma\tilde{A}$.
If A = E, then the A-universal fs-set is called universal fs-set, denoted by $\Gamma\tilde{E}$.

Example 2.3. Assume that $U = \{u_1, u_2, u_3, u_4, u_5\}$ is a universal set and $E = \{x_1, x_2, x_3, x_4\}$ is a set of all parameters.

If $A = \{x_2, x_3, x_4\}$, $\gamma_A(x_2) = \{0.5/u_2, 0.9/u_4\}$, $\gamma_A(x_3) = \emptyset$; and $\gamma_A(x_4) = U$, then the fs-set Γ_A is written by $\Gamma_A = \{(x_2, \{0.5/u_2, 0.9/u_4\}), (x_4, U)\}$.

If $B = \{x_1, x_3\}$, and $\gamma_B(x_1) = \emptyset$, $\gamma_B(x_3) = \emptyset$, then the fs-set Γ_B is an empty fs-set, i.e. $\Gamma_B = \Gamma\Phi$.

If $C = \{x_1, x_2\}$, $\gamma_C(x_1) = U$, and $\gamma_C(x_2) = U$, then the fs-set Γ_C is a C-universal fs-set, i.e., $\Gamma C = \Gamma_C^{\sim}$.

If D = E, and $\gamma_D(x_i) = U$ for all x_i € E, where i = 1,2,3,4, then the fs-set Γ_D is a universal fs-set, i.e., $\Gamma D = \Gamma\tilde{E}$.

III. FS-AGGREGATION ALGORITHM :

we define an fs-aggregation operator that produces an aggregate fuzzy set from an fs-set and its cardinal set. The approximate functions of an fs-set are fuzzy. An fs-aggregation operator on the fuzzy sets is an operation by which several approximate functions of an fs-set are combined to produce a single fuzzy set which is the aggregate fuzzy set of the fs-set. Once an aggregate fuzzy set has been arrived at, it may be necessary to choose the best single crisp alternative from this set.

Therefore, we can make a decision by the following algorithm.

Step 1: Construct an fs- set Γ_A over U.
Step 2. Find the cardinal set $c\Gamma_A$ of Γ_A.
Step 3: Find the aggregate fuzzy set Γ^*_A of Γ_A.
Step 4: Find the best alternative from this set that has the largest member-ship grade by $\max \Gamma^*_A(u)$.

A. Step 1

Let Γ_A€ FS(U). Assume that $U = \{u_1; u_2; \ldots; u_m\}$, $E = \{x_1; x_2; \ldots; x_n\}$ and $A \subseteq E$, then the Γ_A can be presented by the following table.

ΓA	X1	X2	. . .	Xn
u1	μγA(x1)(u1)	μγA(x2)(u1)	. . .	μγA(xn)(u1)
u2	μγA(x1)(u2)	μγA(x2)(u2)	. . .	μγA(xn)(u2)
.	.	.	.	.
.	.	.	.	.
.	.	.		.
um	μγA(x1)(um)	μγA(x2)(um)	. . . .	μγA(xn)(um)

Where μγA(x) is the membership function of ΓA.

If $[aij]$ = μγA(xj) (ui) for i=1,2,….m and j=1,2,…n then the fs-set ΓA is uniquely characterized by the matrix

$$[aij]_{mxn} = \begin{bmatrix} a11 & a12 & \dots & a1n \\ a21 & a22 & \dots & a2n \\ . & . & & . \\ . & . & \ddots & . \\ . & . & & . \\ am1 & am2 & \dots & amn \end{bmatrix}$$

This matrix is called an m x n fs-matrix of the fs-set ΓA over U.

B. ***Step 2***

let ΓA € FS(U) then the cardinal set ΓA denoted by cΓA and defined by cΓA={ μcΓA(x)/x :x∈E is a fuzzy set over E. The membership function μcΓA of cΓA is defined by

μcΓA :E→[0,1], μcΓA(x)=$\frac{|\gamma A(x)|}{|U|}$

where $|U|$ is the cardinality of universe U, and $|\gamma A(x)|$ is the scalar cardinality of fuzzy set $\gamma A(x)$. The set of all cardinal sets of the fs-sets over U will be denoted by cFS(U)⊆F(E).

now let ΓA ∈ FS(U) and cΓA ∈ cFS(U). Assume that E={x1,x2,…..,xn} and A⊆E, then cΓA can be presented by the following table

E	x1	x2		xn
μcΓA	μcΓA(x1)	μcΓA(x2)	. . .	μcΓA(xn)

If a1j= μcΓA(xj) for j=1,2,…..n, then the cardinal set cΓA is uniquely characterized by a matrix,

[a1j]1xn = [a11, a12, … a1n]

Which is called the cardinal matrix of the cardinal set cΓA over E.

C. ***Step 3***

let ΓA ∈ FS(U) and cΓA ∈ cFS(U). then fs-aggregation operator, denoted by FSagg, is defined by

FSagg :cFS(U) x FS(U) →F(U), FSagg(cΓA, ΓA)= Γ*A

Where Γ*A ={ μΓ*A(u)/u :u€U} is a fuzzy set over U. Γ*A is called the aggregate fuzzy set of the fs-set ΓA. The membership function μΓ*A of Γ*A is denoted as follows:

μΓ*A : U→[0,1], μΓ*A(u)=$\frac{1}{|E|}\sum_{x\,\epsilon E} \mu c\Gamma A(x)\mu\gamma A(x)(u)$

where $|E|$ is the cardinality of E.

Now assume that U={u1, u1, …um}, then the Γ*A can be presented by the following table.

ΓA	μΓ*A
u1	μΓ*A(u1)
u2	μΓ*A(u2)
.	.
.	.
.	.
um	μΓ*A(um)

If ai1= μΓ*A (ui) for i=1,2,….m then Γ*A is uniquely characterized by the matrix ai1

$$]mx1=\begin{bmatrix} a11 \\ a21 \\ . \\ . \\ . \\ am1 \end{bmatrix}$$

Which is called the aggregate matrix of Γ*A over U.

If MΓA, McΓA, MΓ*A are representation matrices of ΓA, cΓA and Γ*A, respectively then

$$\text{IEI x M}\Gamma^*\text{A} = \text{M}\Gamma\text{A x } M^T_{c\Gamma A}$$

$$\text{Then M}\Gamma^*\text{A} = \frac{1}{IEI}\text{M}\Gamma\text{A x } M^T_{c\Gamma A}$$

Where $M^T_{c\Gamma A}$ is the transposition of $M_{c\Gamma A}$ and IEI is the cardinality of E.

D. Step 4

Find the best alternative from this aggregate fuzzy set Γ*A that has the largest member-ship grade by max $\Gamma^*_A(u)$.

IV. APPLICATION:

A family wants to choose a bride groom among five girls The family give different weight in terms of fuzzy set to the girls according to their gentry, education, beauty, job and elegancy. So the five girls who form the set of alternatives, U= {u1, u2, u3, u4, u5} and the selection parameters of the family make a set of parameters, E={x1, x2, x3, x4, x5}. The parameter xi for i= 1,2,3,4,5 stand for gentry, education, beauty, job and elegancy respectively. For making a right decision for selecting the suitable bride, the fs-aggregation algorithm is applied here as follows.

Step 1.: Construct an fs-set ΓA over U as given in the following table

ΓA	x1	x2	x3	x4	x5
u1	0.5	0.7	0.6	0.8	0.8
u2	0.6	0.5	0.5	0.7	0.4
u3	0.4	0.6	0.7	0.5	0.7
u4	0.9	0.5	0.5	0.6	0.4
u5	0.4	0.6	0.7	0.5	0.7

Then $[aij]_{mxn}$ is called an m x n fs-matrix of the fs- set ΓA over U as given below.

$$[aij]_{mxn} = \begin{bmatrix} 0.5 & 0.7 & 0.6 & 0.8 & 0.8 \\ 0.6 & 0.5 & 0.5 & 0.7 & 0.4 \\ 0.4 & 0.6 & 0.7 & 0.5 & 0.7 \\ 0.9 & 0.5 & 0.5 & 0.6 & 0.4 \\ 0.4 & 0.6 & 0.7 & 0.5 & 0.7 \end{bmatrix}$$

Step 2: The cardinal set cΓA of ΓA is computed as fallows:

$$\mu c\Gamma a(x1) = \frac{0.5+0.6+0.4+0.9+0.4}{5} = 0.56$$

$$\mu c\Gamma a(x2) = \frac{0.7+0.5+0.6+0.5+0.6}{5} = 0.58$$

$$\mu c\Gamma a(x3) = \frac{0.6+0.5+0.7+0.5+0.7}{5} = 0.60$$

$$\mu c\Gamma a(x4) = \frac{0.8+0.7+0.5+0.6+0.5}{5} = 0.62$$

$$\mu c\Gamma a(x5) = \frac{0.8+0.4+0.7+0.4+0.7}{5} = 0.60$$

So cardinal set cΓA = {0.56/x1, 0.58/x2, 0.60/x3, 0.62/x4, 0.60/x5}

Step 3: The aggregate fuzzy set MΓ*A is computed as follows:

$$\text{M}\Gamma^*\text{A} = \frac{1}{5}\begin{bmatrix} 0.5 & 0.7 & 0.6 & 0.8 & 0.8 \\ 0.6 & 0.5 & 0.5 & 0.7 & 0.4 \\ 0.4 & 0.6 & 0.7 & 0.5 & 0.7 \\ 0.9 & 0.5 & 0.5 & 0.6 & 0.4 \\ 0.4 & 0.6 & 0.7 & 0.5 & 0.7 \end{bmatrix}\begin{bmatrix} 0.56 \\ 0.58 \\ 0.60 \\ 0.62 \\ 0.60 \end{bmatrix} = \begin{bmatrix} 0.208 \\ 0.144 \\ 0.174 \\ 0.141 \\ 0.174 \end{bmatrix}$$

That means,

Γ*A ={0.208/u1, 0.144/u2, 0.174/u3, 0.141/u4, 0.174/u5}

Step 4: Finally the largest membership grade is chosen by max μΓ*A(u) = 0.208

Which means that that the bride u1 has the largest membership grade, hence u1 is the best suitable match among five girls.

V. CONCLUSION:

In the present paper we aim to give an alternative way for computation of fuzzy soft decision making problem in more precisely qualitative than the existed methods. By using fs-aggregation method, we obtain the optimum logical results in an easier and faster way. To develop the theory, in this work, we first defined fs-sets and their operations. We then presented the decision making method for the fs-set theory. Finally, we provided an example demonstrating the successfully application of this method. It may be applied to many fields with problems that contain uncertainty, and it would be beneficial to extend the proposed method to subsequent studies.

References:

[1] M. I. Ali, F. Feng, X. Liu, W. K. Min and M. Shabir, "On some new operations in soft set theory," Comput. Math. Appl., vol. 57, pp. 1547-1553, 2009.

[2] N. Cagman and S. Enginoglu, "Soft set theory and uni-int decision making," Eur. J. Oper. Res., vol. 207, pp. 848-855, 2010.

[3] N. Cagman and S. Enginoglu, "Soft matrix theory and its decision making, Comput. Math. Appl.," vol. 59(10), pp. 3308-3314, 2010.

[4] N. Cagman, F. C tak and S. Enginoglu, "Fuzzy parameterized fuzzy soft set theory and its applications," Turk. J. Fuzzy Syst., vol. 1(1), pp. 21-35, 2010.

[5] D. Chen, E. C. C. Tsang, D. S. Yeung and X. Wang, "The parameterization reduction of soft sets and its applications," Comput. Math. Appl., vol. 49, pp. 757-763, 2005.

[6] Z. Kong, L. Gao, L. Wang and S. Li, "The normal parameter reduction of soft sets and its algorithm," Comput. Math. Appl., vol. 56, pp. 3029-3037, 2008.

[7] Z. Kong, L. Gao and L. Wang, "Comment on A fuzzy soft set theoretic approach to decision making problems," J. Comput. Appl. Math.,vol. 223, pp. 540-542, 2009.

[8] D. V. Kovkov, V. M. Kolbanov and D. A. Molodtsov, "Soft sets theory-based optimization," J. Comput. Sys. Sc. Int., vol. 46(6), pp. 872-880, 2007.

[9] P. K. Maji, R. Biswas and A. R. Roy, "Fuzzy soft sets," J. Fuzzy Math., vol. 9(3), pp. 589-602, 2001.

[10] P. K. Maji, A. R. Roy and R. Biswas, "An application of soft sets in a decision making problem," Comput. Math. Appl., vol. 44, pp. 1077-1083, 2002.

[11] P. K. Maji, R. Biswas and A. R. Roy, "Soft set theory," Comput. Math. Appl., vol. 45, pp. 555-562, 2003.

[12] P. Majumdar and S. K. Samanta, "Similarity measure of soft sets," New. Math. Nat. Comput., vol. 4(1), pp. 1-12, 2008.

[13] D. A. Molodtsov, "Soft set theory- rst results," Comput. Math. Appl., vol. 37, pp. 19-31, 1999.

[14] D. A. Molodtsov, "The description of dependence with the help of soft sets," J. Comput. Sys. Sc. Int., vol. 40(6), pp.977-984, 2001.

[15] D. A. Molodtsov, The theory of soft sets (in Russian), URSS Publishers, Moscow, 2004.

[16] D. A. Molodtsov, V. Yu. Leonov and D. V. Kovkov, "Soft sets technique and its application," Nechetkie Sistemi I Myakie Vychisleniya, vol. 1(1), pp. 8-39, 2006.

[17] M. M. Mushrif, S. Sengupta and A. K. Ray, "Texture classification using a novel, soft-set theory based classification, Algorithm." Lecture Notes In Computer Science, 3851, pp. 246-254, 2006.

[18] Z. Pawlak, "Rough sets," Int. J. Comput. Inform. Sci., vol. 11, pp. 341-356, 1982.

[19] D. Pei and D. Miao, From soft sets to information systems, In: X. Hu, Q. Liu, A. Skowron, T. Y. Lin, R. R. Yager, B. Zhang ,eds., "Proceedings of Granular Computing," IEEE, vol. 2, pp. 617-621, 2005.

[20] A. R. Roy and P. K. Maji, "A fuzzy soft set theoretic approach to decision making problems," J. Comput. Appl. Math., vol. 203, pp. 412-418, 2007.

[21] T. Som, "On the theory of soft sets, soft relation and fuzzy soft relation," Proc. of the National Conference on Uncertainty: A Mathematical Approach, UAMA-06, Burdwan, 2006, pp. 1-9.

[22] Z. Xiao, Y. Li, B. Zhong and X. Yang, Research on synthetically evaluating method for business competitive capacity based on soft set, Stat. Methods. Med. Res., pp. 52-54, 2003.

[23] Z. Xiao, L. Chen, B. Zhong and S. Ye, "Recognition for soft information based on the theory of soft sets," In: J. Chen ,eds., Proceedings of ICSSSM-05, 2, 2005, pp. 1104-1106.

[24] L. A. Zadeh, Fuzzy Sets, Information and Control, 8 , pp.338-353, 1965.

[25] Y. Zou and Z. Xiao, Data analysis approaches of soft sets under incomplete information, Knowl. Base. Syst., 21, pp. 941-945, 2008.

[26] N. Cagman and S. Enginoglu, and F. Citak, "Fuzzy soft set theory and its applications," Iranian Journal of Fuzzy Systems, Vol. 8, No. 3, pp. 137-147, 2011.

[27] Krishna Gogoi, Alok Kr. Dutta and Chandra Chutia, "Application of Fuzzy Soft Set in day to day Problems," International J. of Computer Applications, Vol. 85, No. 7, pp. 27-31, 2014.

[28] R.K. Bhardwaj, S.K. Tiwari and Kailash Chandra Nayak, "A Study of Solving Decision Making Problem using soft set," IJLTEMAS, Vol. IV, Issue IX, pp. 26-32, 2015 .