# A Linear Programming approach for optimal scheduling of workers in a Transport Corporation

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**ABSTRACT**

*The scheduling of workers is very important in any organization as an excess or scarce of workers accounts for the loss of the company both in measures of time and money. Proper scheduling of workers will enhance the outcome of the company. In this paper the scheduling of drivers for a transport corporation in a metropolitan city has been considered. A mathematical model has been created by using the linear programming techniques. By this model the minimum number of drivers needed for each shift in a day has been calculated and this reduces the amount spends for the reserved drivers. Extending this model to a private transport agency, where the number of drivers required for each day varies and also each day has four shifts. Hence seven sub problems have been solved and the results are tabulated. By Linear programming techniques the real life problem has been mathematically formulated and solved analytically to get the optimal solution. As the number of variables increases, the problem becomes more complex and therefore the computational technique using MATLAB software has been applied. Here the constraints are obtained by taking the maximum number of drivers required for each shift and the objective is to minimize the allocation of drivers for each day. Mathematically both the constraints and objective function are assumed to be linear.*

**Keywords:** *Linear Programming, Workers Scheduling, Optimal Solution, MATLAB, Minimization*

**Introduction:**

Many organizations provide jobs that are to be done in shifts, it is very important to schedule the employers in proper shifts to benefit both employer and the employee to get optimum output from the employers so as to improve the output of the company. Linear programming is a method of finding the optimal solution for given real life problem. Linear programming techniques have been applied in many fields. In production management it is applied for determining the optimal allocation of resources like materials, machines, manpower, by a firm to optimize its revenue and also for product smoothing. In personnel management these techniques enable the personnel manager to solve problems relating to recruitment, selection, training, and deployment of manpower to different departments of the firm. It is also used to determine the minimum number of employees required in various shifts to meet production schedule within a fixed time interval. When a firm is faced with the problem of inventory management of raw materials and finished products then LP techniques have been applied to minimize the inventory cost. LP technique enables the marketing manager in analyzing the audience coverage of advertising based on the available media, given the advertising budget as the constraint. It also helps the sales executive of a firm in finding the shortest route for his tour. With its use, the marketing manager determines the optimal distribution schedule for transporting the product from different warehouses to various market locations in such a manner that the total transport cost is the minimum. The financial manager of a firm, mutual fund, insurance company, bank, etc. uses the LP technique for the selection of investment portfolio of shares, bonds, so as to maximize return on investment.Thesetechniques are also applicable in blending problem when a final product is produced by mixing a variety of raw materials. The blending problems arise in animal feed, diet problems, petroleum products, chemical products, etc. In all such cases, with raw materials and other inputs as constraints, the objective function is to minimize the cost of final product.

Emmanuel (2016) applied Linear Programming techniques to plan the production of the company for one year to maximize their profit and the profit has been optimized for a sample data collected form the manufacturing company compared with the manual computations[1]. The mathematical model used in the paper (Salvador 2016) would be made a tool of great importance for the construction of optimized treatment plans, this model provides a set of optimal solutions, which when related to the treatment, may enable a high quality therapy[2]. Snezana (2009) applied the LP technique which determines the optimum values for the process design variables, so that minimum cost would be achieved[3]. Niloo (2008) used linear programming combined with simulation and spreadsheet analysis which incase help to determine ideal space assignments, scheduling configurations and throughput targets for numerous other clinic services in a startup hospital[4]. Balogun (2012)made a case study on the application of Linear programming Technique in a firm named Nigeria Bottling Company which produces soft drinks and optimized the profit of the company by increasing two soft drinks production to the optimal level[5].Kourosh (2013)applied LP technique to solve the transportation problem of a service company[6].

The main objective of this paper is used to minimize the number of drivers allocated for each shift and hence the excess money spends for the reserved drivers have been minimized. The same mathematical model has been used in the places wherever the driver allocation is taken place in a rotational basis. The assumption made in this paper is the functions considered in the mathematical model are linear in nature. While solving the problem, when the results obtained are non-integer then the problem would be extended as Integer Programing Problem by including one more integer constraints on the variables. The data has been taken from one bus major depo of the transport corporation in a metropolitan city.With the data given the real world problem has been formulated as mathematical model and applying LP technique the optimal solution of the problem has been obtained. The same model has been considered to obtain the optimal allocation of the drivers from Monday to Friday for a private transportation company. Using computation technique with MATLAB software all the sub problems are solved and optimal solutions are tabulated. The pseudo code has been given in appendix A.

## 2. Mathematical Formulation:

### 2.1. Model 1:

The scheduling of bus drivers in BMTC depo-3 which operates all the days in a weekhas been considered. For each day there are five shifts. The data collected from the bus depo has been given in Table 1. Every day the minimum number of drivers required is 209. The allocation of drivers in each shift has been determined using LP technique.

| Shifts | No. of drivers |
|---|---|
| 1st shift (6am – 8pm) | 60 |
| 2nd shift (8am – 8pm) | 56 |
| 3rd shift (2pm – 10pm) | 60 |
| 4th shift (9pm [Day 1] – 6am[Day 2]) | 07 |
| 5th shift (1pm [Day 1] – 2pm[Day 2]) | 93 |

Table1: Data from a BMTC bus depo

The drivers in 4th shift will start their shift at 9pm on day 1 and end at 6am the next day or day 2.The drivers in 5th shift will start their shift at 1pm on day 1 and end at 2pm the next day or day 2. The drivers in the 5th shift will work on alternative days.

Let$x_i$be the number of drivers starting the duty in ith shift (i=1 to 5). As the shifts in the data are overlapping let us divide them into non-overlapping shifts and assumed that the driver continue three shifts consecutively in a day so as to cover the 8 hours working time.

The allocation of the drivers in the seven non-overlapping shifts is given in table 2.

| | 1st shift | 2nd shift | 3rd shift | 4th shift | 5th shift | 6th shift | 7th shift |
|---|---|---|---|---|---|---|---|
| $x_1$ | * | * | * | - | - | - | - |
| $x_2$ | - | * | * | * | * | - | - |
| $x_3$ | - | - | - | * | * | * | - |
| $x_4$ | - | - | - | - | - | * | * |
| $x_5$ | * | * | * | * | * | * | * |

Table:2 Allocation of drivers in the non-overlapping shifts

The problem has been formulated to minimize the total number of drivers as

$$\text{Minimize } Z = \sum_{i=1}^{7} x_i \qquad (1)$$

The constraints on the minimum number of drivers required perday and per shift as

$x_1+x_2+x_3+x_4+x_5 \geq 209$

$x_1+x_5 \geq 153$

$x_1+x_2+x_5 \geq 209$

$x_2+x_3+x_5 \geq 209$

$x_3+x_5 \geq 153$

$x_3+x_4+x_5 \geq 160$

$x_4+x_5 \geq 100$ (2)

The non-negativity constraint as

$x_i \geq 0$ (i=1:5) (3)

**2.2. Model 2:**

The scheduling of drivers for a private travel agency which operates on all the days of the week is considered and the data has been provided in table 3. The assumptions made are every driver works 6 days consecutively and each day has four shifts.

| Days | Number of drivers | | | | |
|---|---|---|---|---|---|
| | | Shifts | | | |
| | | 1 | 2 | 3 | 4 |
| **Monday** | 100 | 30 | 20 | 30 | 20 |
| **Tuesday** | 80 | 25 | 15 | 25 | 15 |
| **Wednesday** | 90 | 25 | 20 | 25 | 20 |
| **Thursday** | 85 | 30 | 15 | 30 | 10 |
| **Friday** | 95 | 30 | 15 | 35 | 15 |
| **Saturday** | 110 | 25 | 25 | 40 | 20 |
| **Sunday** | 70 | 10 | 25 | 25 | 10 |

Table 3. Data from a private transport company

Let $y_i$be the number of drivers starting the duty on $i^{th}$ (i=Monday to Sunday). Let us consider the allocation of drivers as in table.4.

| | Monday | Tuesday | Wednesday | Thursday | Friday | Saturday | Sunday |
|---|---|---|---|---|---|---|---|
| $y_1$ | * | * | * | * | * | * | - |
| $y_2$ | - | * | * | * | * | * | * |
| $y_3$ | * | - | * | * | * | * | * |
| $y_4$ | * | * | - | * | * | * | * |
| $y_5$ | * | * | * | - | * | * | * |
| $y_6$ | * | * | * | * | - | * | * |
| $y_7$ | * | * | * | * | * | - | * |

Table 4 Allocation of drivers

The main objective of the problem is to minimize the total number of drivers.

Minimize $Z = \sum_{i=1}^{7} y_i$ (4)

The constraints are formulated with the data provided in the table 3.

$y_1+y_3+y_4+y_5+y_6+y_7 \geq 100$
$y_1+y_2+y_4+y_5+y_6+y_7 \geq 80$
$y_1+y_2+y_3+y_5+y_6+y_7 \geq 90$
$y_1+y_2+y_3+y_4+y_6+y_7 \geq 85$
$y_1+y_2+y_3+y_4+y_5+y_7 \geq 95$
$y_1+y_2+y_3+y_4+y_5+y_6 \geq 110$
$y_2+y_3+y_4+y_5+y_6+y_7 \geq 70$ (5)

The non-negativity constraints on the variables

$y_i \geq 0$ (6)

As each day has four shifts, by applying the model 1 with i=4 and each driver would continue two shifts consecutively, the seven sub problems are formulated with the common objective function

Minimize $Z = \sum_{i=1}^{4} z_i$ (7)

Sub problem:1(Monday)

$z_1+z_4 \geq 30$
$z_1+z_2 \geq 20$

$z_2+z_3 \geq 30$
$z_3+z_4 \geq 20$ (8)

Sub problem:2 (Tuesday)
$z_1+z_4 \geq 25$
$z_1+z_2 \geq 15$
$z_2+z_3 \geq 25$
$z_3+z_4 \geq 15$ (9)

Sub problem:3 (Wednesday)
$z_1+z_4 \geq 25$
$z_1+z_2 \geq 20$
$z_2+z_3 \geq 25$
$z_3+z_4 \geq 20$ (10)

Sub problem:4 (Thursday)
$z_1+z_4 \geq 30$
$z_1+z_2 \geq 15$
$z_2+z_3 \geq 30$
$z_3+z_4 \geq 10$ (11)

Sub problem:5 (Friday)
$z_1+z_4 \geq 30$
$z_1+z_2 \geq 15$
$z_2+z_3 \geq 35$
$z_3+z_4 \geq 15$ (12)

Sub problem:6 (Saturday)
$z_1+z_4 \geq 25$
$z_1+z_2 \geq 25$
$z_2+z_3 \geq 40$
$z_3+z_4 \geq 20$ (13)

Sub problem:7 (Sunday)
$z_1+z_4 \geq 10$
$z_1+z_2 \geq 25$
$z_2+z_3 \geq 25$
$z_3+z_4 \geq 10$ (14)

The non-negative constraints
$z_i \geq 0$where $z_i$ refers the number of drivers starting at the $i^{th}$ shift.

**3. RESULTS AND DISCUSSION:**

The model 1 has been solved by the LP technique and the final optimal table with the optimal solutions have been given in table 5.

| $c_j$ | | | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **$C_B$** | **B** | **$X_B$** | **$x_1$** | **$x_2$** | **$x_3$** | **$x_4$** | **$x_5$** | **$s_1$** | **$s_2$** | **$s_3$** | **$s_4$** | **$s_5$** | **$s_6$** | **$s_7$** |
| 1 | $x_1$ | 113/2 | 1 | 0 | 0 | 0 | 0 | -1/2 | 0 | 1/2 | 0 | 0 | 0 | 0 |
| 0 | $s_7$ | 7/2 | 0 | 0 | 0 | 0 | 0 | 1/2 | -1 | 1/2 | 0 | -1 | 0 | 1 |
| 0 | $s_4$ | 7/2 | 0 | 0 | 0 | 0 | 0 | -1/2 | 0 | -1/2 | 1 | 0 | 0 | 0 |
| 1 | $x_3$ | 60 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 1 | 0 |
| 1 | $x_4$ | 7/2 | 0 | 0 | 0 | 1 | 0 | 1/2 | 0 | 1/2 | 0 | -1 | 0 | 0 |
| 1 | $x_5$ | 193/2 | 0 | 0 | 0 | 0 | 1 | 1/2 | 0 | 1/2 | 0 | -1 | 0 | 0 |
| 1 | $x_2$ | 56 | 0 | 1 | 0 | 0 | 0 | 1 | -1 | 1 | 0 | 0 | 0 | 0 |
| **$z_j$ - $c_j$** | | | 0 | 0 | 0 | 0 | 0 | 3/2 | -1 | 5/2 | 0 | -3 | 1 | 0 |

Table 5 Optimal table for model 1

For the given data it has been observed that the results obtained are non-integer values. As the number of workers to integers either we can round off the values to the nearest integer of by including the integrity constraints on the variables the problem has been formulated as integer programing problem which in turn produce an improved optimal solution.If the transport corporation spend an amount of x for each driver per day then by using the above optimal solution the excess amount spend

$$A=\sum_{i=1}^{5}\left(x_i - x_i^o\right)x \qquad (15)$$

where $x_i$-the number of drivers required for each shift
$x_i^o$-the optimal solution obtained.
This is the amount saved by the transport corporation by using LP technique.

As the number of variables increase would result in a complexity to solve the problem using LP technique. Hence using computation technique with MATLAB software the model 2 has been solved and the results obtained are tabulated in table.6. The pseudo code using MATLAB has been given in appendix A. By using these optimal solutions all the seven subproblems are solved and the results are tabulated (Table 6).

| | Monday | Tuesday | Wednesday | Thursday | Friday | Saturday | Sunday |
|---|---|---|---|---|---|---|---|
| $z_1$ | 15 | 12.5 | 12.5 | 15.4 | 15 | 13 | 6.34 |
| $z_2$ | 15 | 12.5 | 12.5 | 15.4 | 17 | 20.99 | 18.66 |
| $z_3$ | 15 | 12.5 | 12.5 | 14.6 | 17 | 19 | 6.34 |
| $z_4$ | 15 | 12.5 | 12.5 | 14.6 | 15 | 11.99 | 3.66 |
| $y_i$ | 30 | 9.8 | 22.54 | 15.73 | 18.88 | 13.4 | 0 |

Table.6 Solution for model 2 and the seven sub problems

For the given data the above result infers that no drivers start their shift on Sunday. Also from Monday to Tuesday all the shifts the same number of drivers are allocated. Friday and Saturday there shows a huge variation in the number of drivers starting the second shift and fourth shift. By using the LP technique the transport company can save the amount given in Eqn.15 by allocating the optimal allocation on each day (here i=1:7).

## 4. CONCLUSION

In both the models, the respective organizations would save more revenue than before if the organization implements the optimality obtained from the linear programming technique. The same model would be applied in scheduling of nurses for hospitals, scheduling workers in a firm which has more shifts. This also save the energy, the amount spend on wages and time of the organisation. It also reduces the stress on the workers who work in shifts. In real life some of the assumptions made would not be pertained then the linear programming nature would be extended to non-linear programming approach. Overall by applying LP technique many complex real world problems would be optimized with some basic assumptions.

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**Appendix A**

**MATLAB CODE:**

```
f=[yi]      // Coefficient of yi in the objectivefunction
A= - [yi]   // Coefficient of yi in the constraints
b= [bi]     // The constants in the constraints
Aeq=[ ]   // Coefficient of yi if any equality constraints
Beq=[ ]   // The constants in the equality constraints if any
l=[ zero]  // The lower limit for the yi values
u=[ M ]   // The upper limit for the yi values (Here M is any positive real value and M>0)
y=linprog(f,A,b,Aeq,Beq,l,u)
```