

# Memristor Model Based on Generalized Boundary Condition

Sourabh Sethi<sup>#1</sup>

<sup>#</sup>Student, School of Electrical Engineering, Vellore Institute of Technology, Vellore, India

**Abstract** — Memristor is labelled as a noteworthy applicant for building better storage structure, higher capacity and more efficient performance. Research shows that the density of memristor array could be 100 times compared to DRAM. Postulated by L. Chua in 1971, a memristor(M) is a fourth elementary two terminal component along with Resistor(R), Capacitor(C) and Inductor(L). A memristor displays predictable relationship between its resistance and electrical charge traversing through it. To qualify as a memristor, the response of the dynamical system, corresponding to sinusoidal excitation, must be a hysteresis loop pinched at the origin. In this study a generic physics model based on Generalized Boundary Conditions (GBC) is explained which accommodates highly nonlinear and asymmetric switching behaviour. GBC model combines the variation of state variable (w) and dopant drift memristance (M) definition to adopt the theory of their participation in the memristive behaviour. The generic memristor model can be used for implementation of dense nano crossbar arrays for memory and storage. Computer simulations were completed to verify the model and it verifies L.Chua’s hypothesis. Results show that the proposed approximated model can be well applied

**Keywords** — Memristor, Modelling, Simulation, Linear Ion Drift

## I. INTRODUCTION

The memristor has drawn worldwide attention recently since HP labs, USA released its invention<sup>[6]</sup>. Since then researchers are finding its applications in various areas of Information Technology like storage, flash memory etc. A good model will help us to study and seize the behavior characteristics of the memristor device so the further research can be done.

## II. MEMRISTOR

Memristor is a portmanteau of “Memory resistor”. It is a passive device with two terminals, where magnetic flux is related to the amount of electric charge through the device<sup>[5]</sup>. Since it is not an active element, it cannot store or generate any power. The symbol of memristor is shown in Figure 1.

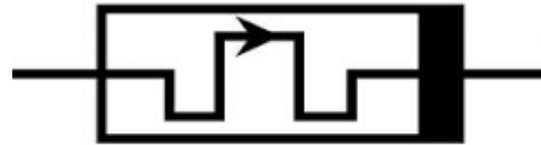


Figure 1 : Symbol of memristor

The memristance is defined as a functional relationship between magnetic flux linkage  $\Phi_m$  and the amount of electric charge that has flowed,  $q$

$$\Phi_m = F(q)$$

### A. Memristance

In the relationship between  $\Phi_m$  and  $q$ , the derivative of one with respect to the other depends on the value of one or the other, and so each memristor is characterized by its memristance(M) function describing the charge-dependent rate of change of flux with charge<sup>[8]</sup>.

$$M(q) = \frac{d\Phi_m}{dq}$$

Substituting the flux as the time integral of the voltage, and charge as the time integral of current, the more convenient form is

$$M(q(t)) = \frac{\frac{d\Phi_m}{dt}}{\frac{dq}{dt}} = \frac{V(t)}{I(t)}$$

To relate the memristor to the resistor, capacitor, and inductor, it is helpful to compare it with differential equations of resistor, capacitor and inductor<sup>[1]</sup>.

TABLE I  
DIFFERENTIAL EQUATIONS OF RESISTOR(R), CAPACITOR(C), INDUCTOR(I) AND MEMRISTOR(M).

Device	Units	Equation
Resistor (R)	V/A,ohm, $\Omega$	$R = dV/dI$
Capacitor (C)	C/V,Farad	$C = dq/dV$
Inductor (L)	Wb/A, Henry	$L = d\Phi_m/dI$
Memristor (M)	Wb/C, Ohm	$M(q) = d\Phi_m/dq$

### III. Memristor Modelling

It is assumed that the physical device of width  $D$  has two regions<sup>[5]</sup>. One side has oxygen vacancies or in other word, is doped with positive oxygen ions; and the other side is undoped. Each region is amodeled with a resistor. The doped region of width  $w$  (which acts as the state variable), has lower resistance and hence more conductive. And the undoped region has high resistance. It is also assumed ohmic conductance, and that the field is uniform, the ion drift is linear and the ions have equal average ion mobility  $\mu v$ . Figure 2 (a) shows the model of the memristor built by HP<sup>[6]</sup>.

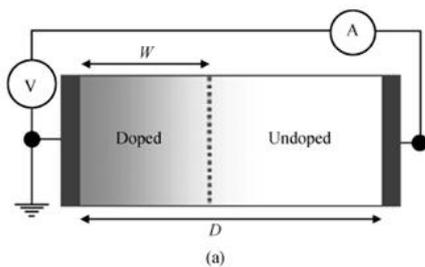


Figure 2(a) : Memristor model

The sum of resistances of both the doped and undoped regions of the device gives the total resistance.

$$V(t) = \left[ R_{on} \cdot \frac{w(t)}{D} + R_{off} \left( 1 - \frac{w(t)}{D} \right) \right] I(t)$$

where  $w(t)$  is the state variable and defined as

$$w(t) = \mu v R_{on} D q(t)$$

The state variable is normalised between  $0 < w < 1$  using boundary conditions with  $w = 0$  as off state and  $w = 1$  as on state and  $\mu v$  is the dopant mobility.

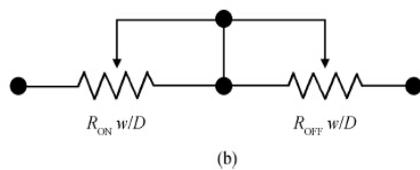


Figure 2(b) : Memristor regions modelled as resistor

### IV. Results and Discussion

One of the resulting properties of memristors and memristive systems is the existence of a pinched hysteresis effect. For a current-controlled memristive system, the slope of the curve represents the electrical resistance<sup>[8]</sup>. The change in slope of the pinched hysteresis curves demonstrates switching between different resistance states which is a phenomenon central to ReRAM and other forms of two-terminal resistance memory. The memristor parameter analysis is done by writing the MATLAB

code for above equations and giving sine wave, square wave and saw tooth voltage as input voltage and alternating current as input current. The memristor response is a hysteresis loop pinched at the origin, which confirms Chuna's hypothesis and validates our model.

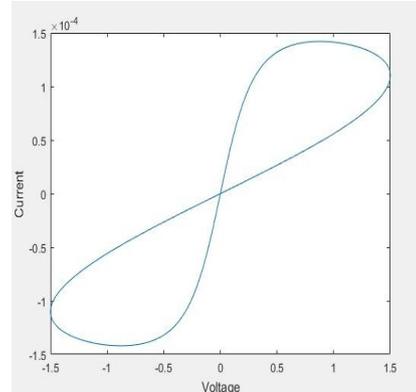


Figure 3 : Memristor current vs sine voltage characteristics shows pinched hysteresis loop.

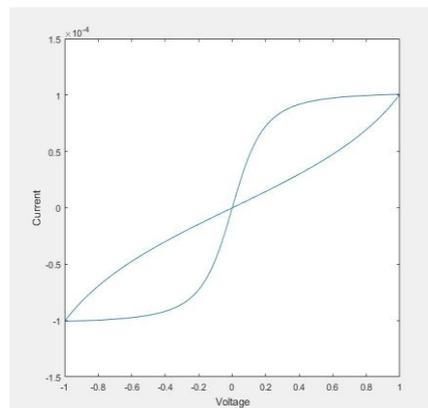


Figure 4 : Memristor current vs saw tooth voltage characteristics.

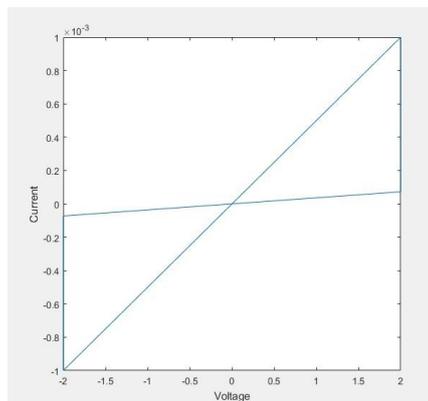


Figure 5 : Memristor current vs square wave characteristics.

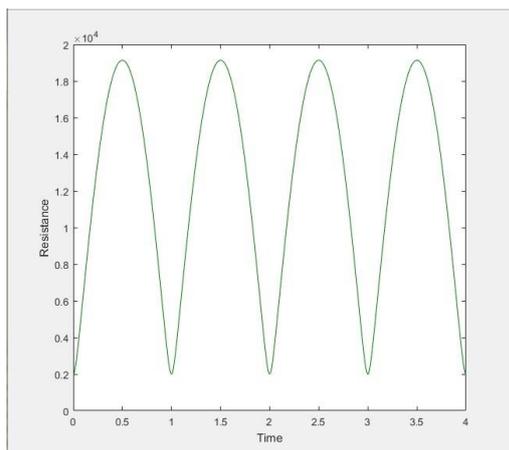


Figure 6 : Memristance vs time.

Memristor should return pinched hysteresis loop for any initial condition and any amplitude<sup>[1]</sup>. This is demonstrated in the next figure.

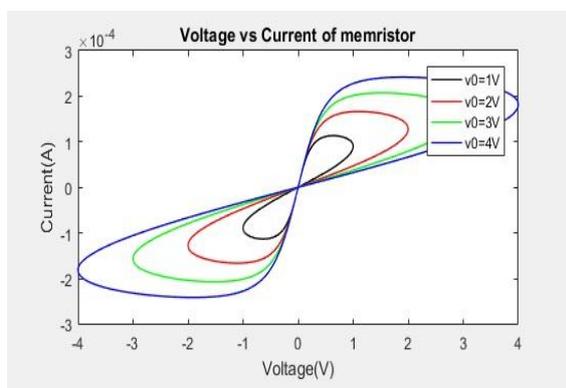


Figure 7: Memristor current vs sine wave at different amplitudes.

The output of the memristor is same as ideal memristor characteristics postulated by L.Chua. This generic memristor model can be used for implementation of logic memory and dense nano crossbar arrays for memory and storage.

## V. CONCLUSIONS

Generic memristor device model based on generalized boundary condition is described in this paper. This model is flexible and convenient model that can be used to characterize a variety of different practical memristive devices. This model suggests

a memristive device should exhibit a non linear dependence on the charge, as well as a dependence on the state variable<sup>[5]</sup>. While the simplicity of this model improves the efficiency of the simulation process, the model is sufficiently accurate. This model fits practical memristive devices better than previously proposed models. This model is suitable for memristive device-based circuit design and has been implemented in Verilog-A for SPICE simulations.

## ACKNOWLEDGMENT

This research was supported by Department of Microelectronics, SELECT School, VIT University. I thank our Head of Department Prof. Ramesh Babu and Prof. Sivabalan who provided insight and expertise that greatly assisted the research, although they may not agree with all of the interpretations of this paper.

I thank Mr. Selvaganeshan, Digital Simulation Lab for assistance with Pspice and MATLAB..

I would also like to show gratitude to Sakcham Bairoliya, Ankit, Saidullu and others in IC Design Lab for their moral support and constant guidance.

## REFERENCES

- [1] L. Chua, "Memristor-The missing circuit element," IEEE Transactions on Circuit Theory, vol. 18, no. 5, pp. 507–519, 1971.
- [2] L.Chua and S. M. Kang, "Memristive devices and systems," Proceedings of the IEEE, vol. 64, no. 2, pp.209–223, 1976.
- [3] A. Beck, J. G. Bednorz, C. Gerber, C. Rossel, and D. Widmer, "Reproducible switching effect in thin oxide films for memory applications," Applied Physics Letters, vol. 77, no. 1, pp. 139–141,2000.
- [4] J. R. Heath, P. J. Kuekes, and R. S. Williams, "Molecular wire crossbar memory", US Patent 6,128,214,Oct. 2000.
- [5] S. Kvatinsky, E. G. Friedman, A. Kolodny, and U. C. Weiser, "TEAM: ThrEshold Adaptive Memristor Model," IEEE Transaction on circuit and systems I: Regular Papers, vol.23, p.14, 2012.
- [6] D. B. Strukov, G. S. Snider, D. R. Stewart, and R. S. Williams, "The missing memristor found," *Nature*, vol. 453, pp. 80–83, May 2008.
- [7] Adhikari, Shyam Prasad, Maheshwar Pd. Sah, Hyongsuk Kim, and Leon O. Chua. "Three Fingerprints of Memristor", IEEE Transactions on Circuits and Systems I Regular Papers, 2013.
- [8] Memristor.From Wikipedia, the free encyclopedia.[Online]. Available: <https://en.wikipedia.org/wiki/Memristor>