Non Linear Dynamic Stability Analysis of Finite Flexible Oil Journal Bearings Including Fluid Inertia Effects

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Abstract: ***The aim of this study is to analyse the Non-linear transient stability of finite oil journal bearing including the effect of fluid inertia and bearing liner surface deformations. The inertia effect is usually ignored in view of its negligible contribution compared to viscous force. However, fluid inertia effect is to be taken in the analysis when modified Reynolds number is around one. This investigation deals with the stability of finite isoviscous oil journal bearing with fluid film inertia effect using finite difference method. An attempt has been made to evaluate the critical mass parameter. A non-linear time transient method is used to simulate the journal centre trajectory to estimate the stability parameter, which is a function of speed.***

In the present work, a modified form of Reynolds equation is developed to include the combined influence of fluid inertia and bearing liner surface deformation for the analysis of finite isoviscous oil journal bearing. The modified average Reynolds equation considering inertia effect with bearing liner surface deformation is solved by a finite difference method with a successive over-relaxation scheme (Gauss-Siedel),while the equation of motion of the journal is solved by the fourth-order Runge-Kutta method. The stability increases with the increase of eccentricity ratio and modified Reynold's numbers.

Keyword- Modified Reynolds number, slenderness ratio, attitude angle, sommerfeld number, eccentricity ratio, journal bearings, inertia, and deformation factor..

I INTRODUCTION

The fluid inertia effect cannot be neglected when the viscous and the inertia forces are of the same order of magnitude shown by Pinkus and Sterlincht [1], though the basic assumptions in the classical hydrodynamic theory include negligible fluid inertia forces in comparison to the viscous forces. In recent times synthetic lubricants, low viscosity lubricants, are used in industries and owing to high velocity it is possible to arrive at such a situation where flow is laminar but the fluid inertia effect cannot be neglected. In such cases the classical Reynolds equation is not valid.

Keeping in view of the above, consideration of inertia effect of a lubricant flow may be one of the areas of recent extension of the classical lubrication theory. Among the few studies related to effect fluid inertia effect, Constatinescu and Galetuse [2] evaluated the momentum equations for laminar and turbulent flows by assuming the velocity profiles is not strongly affected by the inertia forces. Banerjee et.al [3] introduced an extended form of Reynolds equation to include the effect of fluid inertia, adopting an iteration scheme. Chen and Chen [5] obtained the steady-state characteristics of finite bearings including inertia effect using the formulation of Banerjee et al.[3]. Kakoty and Majumdar [4, 13] used the method of averaged inertia in which inertia terms are integrated over the film thickness to account for the inertia effect in their studies. The above studies were mainly based on ideally smooth rigid bearing surfaces.

When the fluid-film thickness in a journal bearing system is of the order of few micrometres, the bearing surface is not rigid, rather deformable, then surface deformation due to elastic distortion has a profound effect on bearing performance. In the present study it has been consider that the journal bearing is a cylindrical sleeve bearing made of comparatively soft material than shaft material and a rigid circular shaft rotates inside. The bearing liner is actually a thin tube surrounded by a relatively rigid housing. Since the periphery of the bearing is much larger than its thickness the radial deformation of the latter at a point may be assumed to be proportional to the pressure at that point. Elastic deformation of the journal and the bearing material by hydrodynamic fluid pressure changes the fluid film profile, modifies the pressure distribution and therefore changes the performance characteristics of the journal bearings.

Theoretical research on flexible (soft shell) bearings with a rigid rotor was started with the work of Higginson [6] using a simplified method (the distortion is proportional to the pressure). Since then many workers notably Hooke, Brighton, and O'Donoghue [7-9], Conway and Lee [11], and Oh and Huebner [12] solved the journal bearing problem considering the effect of elastic distortions of the bearing liner.

In the present work, a modified average Reynolds equation and a solution algorithm are developed to include fluid inertia and bearing sleeve surface deformation effects in the analysis of lubrication problems. The developed model is being used to study the influence of fluid inertia and surface deformation effects on the steady state characteristics such as circumferential pressure, load carrying capacity, attitude angle and side leakage of a hydrodynamic oil journal bearing.

II BASIC THEORY

The modified average Reynolds equation for fully lubricated surfaces is derived starting from the Navier-Stokes equations and the continuity equation with few assumptions. The non-dimensional form of the momentum equations and the continuity equation for a journal bearing may be written as (Figure.1)

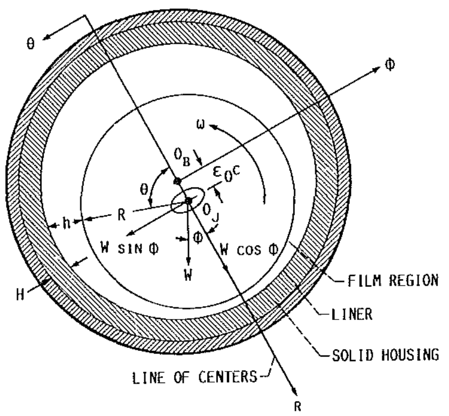

Figure.1 The schematic diagram of oil Journal Bearing

$$R^*_e\left[\Omega\frac{\partial\bar{u}}{\partial\tau}+\bar{u}\frac{\partial\bar{u}}{\partial\theta}+\bar{v}\frac{\partial\bar{u}}{\partial\bar{y}}+\bar{w}\left(\frac{D}{L}\right)\frac{\partial\bar{u}}{\partial\bar{z}}\right]=-\frac{\partial\bar{p}}{\partial\theta}+\frac{\partial^2\bar{u}}{\partial\bar{y}^2} \quad (1)$$

$$\frac{\partial\bar{p}}{\partial\bar{y}}=0 \quad (2)$$

$$R^*_e\left[\Omega\frac{\partial\bar{w}}{\partial\tau}+\bar{u}\frac{\partial\bar{w}}{\partial\theta}+\bar{v}\frac{\partial\bar{w}}{\partial\bar{y}}+\bar{w}\left(\frac{D}{L}\right)\frac{\partial\bar{w}}{\partial\bar{z}}\right]=-\left(\frac{D}{L}\right)\frac{\partial\bar{p}}{\partial\bar{z}}+\frac{\partial^2\bar{w}}{\partial\bar{y}^2} \quad (3)$$

$$\frac{\partial\bar{u}}{\partial\theta}+\frac{\partial\bar{v}}{\partial\bar{y}}+\left(\frac{D}{L}\right)\frac{\partial\bar{w}}{\partial\bar{z}}=0 \quad (4)$$

Where, $\bar{z}=\frac{z}{L/2}$, $\bar{y}=\frac{y}{c}$, $\theta=\frac{x}{R}$, $\tau=\omega_p.t$, $\Omega=\frac{\omega_p}{\omega}$, $\bar{u}=\frac{u}{\omega R}$, $\bar{v}=\frac{v}{c\,\omega}$ $\bar{w}=\frac{w}{\omega R}$ $\bar{p}=\frac{p c^2}{\eta\,\omega R^2}$ and

$$R_e^*=R_e.\frac{c}{R}=\frac{\rho\omega c^2}{\eta}$$

The fluid film thickness in the case of flexible bearing can be written as

$$h=c+e\cos\theta+\delta \quad (5)$$

$$\bar{h}=1+\varepsilon\cos\theta+\bar{\delta} \quad (6)$$

where $\bar{\delta}=\frac{\delta}{c},\varepsilon=\frac{e}{c},\bar{h}=\frac{h}{c},$

where δ is the elastic deformation of the bearing surface and it is a function of θ and z.

Here the variation in the density with time is considered to be negligible. The momentum equations may be presented in the following form using equation of continuity. However, the second momentum equation is not used any further because there is no variation in pressure across the film. After Constantinescu and Galetuse [2] the velocity components are approximated by the parabolic profiles. The velocity components may be expressed in non-dimensional form as follows:

$$\bar{u}=\left[\frac{\bar{y}}{\bar{h}}+Q_\theta\left(\frac{\bar{y}^2}{\bar{h}^2}-\frac{\bar{y}}{\bar{h}}\right)\right] \quad (7)$$

$$\bar{w}=\left[Q_z\left(\frac{\bar{y}^2}{\bar{h}^2}-\frac{\bar{y}}{\bar{h}}\right)\right] \quad (8)$$

Q_θ and Q_z are dimensionless flow parameter in θ and $\bar{z}$ direction respectively.

Substituting these two into momentum equations and integrating give

$$Q_\theta=\frac{\bar{h}^2}{2}\left(\frac{\partial\bar{p}}{\partial\theta}\right)+R_e^*\times I_X \quad (9)$$

$$Q_z=\frac{\bar{h}^2}{2}\left(\frac{D}{L}\right)\left(\frac{\partial\bar{p}}{\partial\bar{z}}\right)+R_e^*\times I_Z \quad (10)$$

Where

$$I_X=\frac{\bar{h}}{2}\left[\frac{1}{2}\Omega\left(1-\frac{1}{3}Q_\theta\right)\frac{\partial\bar{h}}{\partial\tau}-\frac{1}{6}\Omega\bar{h}\frac{\partial Q_\theta}{\partial\tau}-\frac{1}{3}\left(1-\frac{1}{2}Q_\theta+\frac{1}{10}Q_\theta^{\,2}\right)\frac{\partial\bar{h}}{\partial\theta}+\frac{1}{3}\bar{h}\left(\frac{1}{5}Q_\theta-\frac{1}{2}\right)\frac{\partial Q_\theta}{\partial\theta}+\right.$$

$$\left.\frac{1}{30}\left(\frac{D}{L}\right)\bar{h}\,Q_z\frac{\partial Q_\theta}{\partial\bar{z}}+\frac{1}{6}\left(\frac{D}{L}\right)\bar{h}\left(\frac{1}{5}Q_\theta-\frac{1}{2}\right)\frac{\partial Q_z}{\partial\bar{z}}+\frac{1}{6}\left(\frac{D}{L}\right)\times Q_z\times\left(\frac{1}{5}Q_\theta-\frac{1}{2}\right)\frac{\partial\bar{h}_0}{\partial\bar{z}}\right] \quad (11)$$

$$I_Z=\frac{\bar{h}}{2}\left[-\frac{1}{6}\Omega\,Q_z\frac{\partial\bar{h}}{\partial\tau}-\frac{1}{6}\Omega\bar{h}\frac{\partial Q_z}{\partial\tau}-\frac{1}{6}Q_z\left(\frac{1}{5}Q_\theta-\frac{1}{2}\right)\frac{\partial\bar{h}}{\partial\theta}-\frac{1}{6}\bar{h}\left(\frac{1}{5}Q_\theta-\frac{1}{2}\right)\frac{\partial Q_z}{\partial\theta}+\right.$$

$$\frac{1}{30}\bar{h}Q_z\frac{\partial Q_\theta}{\partial\theta}+\frac{1}{15}\left(\frac{D}{L}\right)\bar{h}\,Q_z\frac{\partial Q_z}{\partial\bar{z}}+\frac{1}{30}\times\left(\frac{D}{L}\right)\times Q_z^{\;2}\times\frac{\partial\bar{h}_0}{\partial\bar{z}}\Bigg] \quad (12)$$

From continuity equation one can obtain the following form of modified Reynold's equation in rotating coordinate system

$$\frac{\partial}{\partial\theta}\left(\bar{h}^3\frac{\partial\bar{p}}{\partial\theta}\right)+\left(\frac{D}{L}\right)^2\frac{\partial}{\partial\bar{z}}\left(\bar{h}^3\frac{\partial\bar{p}}{\partial\bar{z}}\right)=6\left(1-2.\Omega\frac{\partial\phi}{\partial\tau}\right)\frac{\partial\bar{h}}{\partial\theta}+$$

$$12\,\Omega\frac{\partial\bar{h}}{\partial\tau}-2\times R_e^{*}\times\left[\frac{\partial}{\partial\theta}\left(\bar{h}\times I_X\right)+\left(\frac{D}{L}\right)\frac{\partial}{\partial\bar{z}}\left(\bar{h}\times I_z\right)\right] \quad (13)$$

Boundary conditions for equation (13) are as follows

1. The pressure at the ends of the bearing is assumed to be zero (ambient):

$$\bar{p}\,(\theta,\pm1)=0$$

2. The pressure distribution is symmetrical about the mid-plane of the bearing:

$$\frac{\partial\bar{p}}{\partial\bar{z}}(\theta,0)=0$$

3. Cavitation boundary condition is given by:

$$\frac{\partial\bar{p}}{\partial\theta}\left(\theta_2,\bar{z}\right)=0\ \ and\ \ \bar{p}\left(\theta,\bar{z}\right)=0\ \ for\ \theta_1\geq\theta\geq\theta_2$$

The equation (13) along with the equations (9) to (12) are first expressed in finite difference form and solved using Gauss-Siedel method in a finite difference scheme.

Before trying to find the solution of equation (13) satisfying the appropriate boundary conditions, the elastic deformation δ is obtained in the following way:

The method is similar to that of Brighton et.al [8,9] and also Majumder et al. [7].In the present calculation the three displacement components *u*, *v*, and *w* are solved simultaneously satisfying the boundary conditions.

The oil film pressure between the shaft and the bearing can be expressed in a double Fourier series of the form as indicated by Brighton et.al [8,9] and Majumder et.al., [7]

$$p=\sum_m^{l}\sum_n^{l}p_{m,n}\cos\frac{2m\pi z}{L}\cos\left(n\theta+\alpha_{m,n}\right) \quad (14)$$

where $\sum^{l}$ indicates that the first term of the series is halved.

$p_{m,n}$ and $\alpha_{m,n}$ are given as follows,

$$p_{m,n}=\frac{4}{\pi L}\left[\left\{\int_0^{2\pi}\int_0^{\frac{L}{2}}p\cos\frac{2m\pi z}{L}\cos n\theta\,dz\,d\theta\right\}^2+\left\{\int_0^{2\pi}\int_0^{\frac{L}{2}}p\cos\frac{2m\pi z}{L}\sin n\theta\,dz\,d\theta\right\}^2\right]^{\frac{1}{2}} \quad (15)$$

$$\alpha_{m,n}=\tan^{-1}\left[\frac{-\int_0^{2\pi}\int_0^{\frac{L}{2}}p\cos\frac{2m\pi z}{L}\sin n\theta\,dz\,d\theta}{\int_0^{2\pi}\int_0^{\frac{L}{2}}p\cos\frac{2m\pi z}{L}\cos n\theta\,dz\,d\theta}\right] \quad (16)$$

$$p_{0,0}=\frac{2}{\pi L}\int_0^{2\pi}\int_0^{\frac{L}{2}}p\,d\theta\,dz \quad (17)$$

The first term of the right-hand side of equation (14) is $\frac{1}{2}p_{0,0}$.

Using the end condition of the bearing (i,e $p=0$ at $z=\frac{L}{2}$) we can obtain $p_{0,0}$. This term does not contribute any deformation at $z=\frac{L}{2}$. Its effect for the other values of z is included in the total deformation. The boundary conditions of the inner radius are

$$\sigma_r=-p,\ \ \tau_{r\theta}=0,\ \ \tau_{rz}=0 \quad (18)$$

After non-dimensionalisation, the equation (15), (16) and (17) becomes

$$\bar{p}_{m,n}=\frac{2}{\pi}\left[\left\{\int_0^{2\pi}\int_0^{1}\bar{p}\cos m\pi\bar{z}\cos n\theta\,d\bar{z}\,d\theta\right\}^2+\left\{\int_0^{2\pi}\int_0^{1}\bar{p}\cos m\pi\bar{z}\sin n\theta\,d\bar{z}\,d\theta\right\}^2\right]^{\frac{1}{2}} \quad (19)$$

$$\alpha_{m,n}=\tan^{-1}\left[\frac{-\int_0^{2\pi}\int_0^{1}\bar{p}\cos m\pi\bar{z}\sin n\theta\,d\bar{z}\,d\theta}{\int_0^{2\pi}\int_0^{1}\bar{p}\cos m\pi\bar{z}\cos n\theta\,d\bar{z}\,d\theta}\right] \quad (20)$$

and $$\bar{p}_{0,0}=\frac{1}{\pi}\int_0^{2\pi}\int_0^{1}\bar{p}\,d\theta\,d\bar{z} \quad (21)$$

where $\bar{p}_{m,n}=\frac{p_{m,n}c^2}{\eta_0\,\omega R^2}$, $\bar{p}=\frac{p\,c^2}{\eta_0\,\omega R^2}$, $\bar{z}=\frac{z}{L/2}$

The outer surface of the bearing is rigidly enclosed by the housing, preventing any displacement of the outer surface. The ends of the bearing are prevented from expanding axially, but are free to move circumferentially or radially.

The displacement components in r,θ and z directions are found from the pressure distribution, which has been expressed in a Fourier series. It is apparent that the displacements will also be harmonic functions.

These displacements were substituted in the stress-strain relationships using Lame's constants. The six components of stresses were then used in the equations of equilibrium to obtain the following three displacement equations.

$$C^* \frac{d^2 u^*}{d\bar{y}^2} + \frac{C^*}{\bar{y}}\frac{du^*}{d\bar{y}} - \left(C^* + n^2\right)\frac{u^*}{\bar{y}^2} + \left(C^* - 1\right)\frac{n}{\bar{y}}\frac{dv^*}{d\bar{y}}$$
$$-\left(C^*+1\right)\frac{n}{\bar{y}^2} v^* - k^2 u^* + \left(C^*-1\right)k\frac{dw^*}{d\bar{y}} = 0 \quad (22)$$

$$\frac{d^2 v^*}{d\bar{y}^2} + \frac{1}{\bar{y}}\frac{dv^*}{d\bar{y}} - \left(1 + C^* n^2\right)\frac{v^*}{\bar{y}^2} - k^2 v^* - \left(C^*-1\right)\frac{n}{\bar{y}}\frac{du^*}{d\bar{y}}$$
$$-\left(C^*+1\right)\frac{n}{\bar{y}^2}u^* - nk\left(C^*-1\right)\frac{w^*}{\bar{y}} = 0 \quad (23)$$

$$\frac{d^2 w^*}{d\bar{y}^2} + \frac{1}{\bar{y}}\frac{dw^*}{d\bar{y}} - \frac{n^2}{\bar{y}^2}.w^* - C^* k^2 w^{*/} - k\left(C^*-1\right)\frac{du^*}{d\bar{y}}$$
$$-k\left(C^*-1\right)\frac{u^*}{\bar{y}} - n\left(C^*-1\right).k\frac{v^*}{\bar{y}} = 0 \quad (24)$$

Where, $C^* = 2 + \dfrac{\lambda}{\mu}$ & $k = \dfrac{2m\pi r_i}{L}$

The boundary conditions are, at $\bar{y} = 1$,

$$C^*\frac{du^*}{d\bar{y}} = -\frac{1}{\mu}p_{m,n} - \left(C^* - 2\right)\left(\frac{nv^*}{\bar{y}} + \frac{u^*}{\bar{y}} + kw^*\right) \quad (25)$$

$$\frac{dv^*}{d\bar{y}} = \frac{nu^*}{\bar{y}} + \frac{v^*}{\bar{y}} \quad (26)$$

$$\frac{dw^*}{d\bar{y}} = u^* k \quad (27)$$

and at $\bar{y} = \dfrac{b}{a}$,

$$u^* = v^* = w^* = 0 \quad (28)$$

The equations (22), (23) and (24) expressed first in finite difference form solving the displacement equations with the boundary conditions (25 to 28) the values of the distortion coefficient $d_{m,n}$ were obtained and expressed as,

$$d_{m,n} = \frac{\mu u^*}{Rp} \quad (29)$$

The radial deformation δ_0 of the bearing surface will be $\delta = u^*$

or, $\delta = d_{m,n}\dfrac{Rp}{\mu} r_i \cos\left(n\theta + \alpha_{m,n}\right)\cos\dfrac{2m\pi z}{L}$

Considering the bearing clearance is very small in compare to the diameter of the journal, the total radial deformation will be

$$\delta = \frac{R\, p_{0,0}\, d_{0,0}}{2\mu} + \sum_{\substack{m=0 \\ (m,n)\neq(0,0)}}^{\infty}\sum_{n=0}^{\infty} d_{m,n}\frac{R}{\mu}p_{m,n}\cos\left(n\theta + \alpha_{m,n}\right)\cos\frac{2m\pi z}{L} \quad (30)$$

Using $\bar{p}_{m,n} = \dfrac{p_{m,n} c^2}{\eta_0\, \omega\, R^2}$, $\bar{z} = L/2$ and μ is replaced by $\dfrac{E}{2(1+\nu)}$,

the radial deformation in the inner surface will be in the form,

$$\bar{\delta} = 2(1+\nu)\, F\left[\bar{p}_{0,0}\, d_{0,0} + \sum_{\substack{m=0 \\ (m,n)\neq(0,0)}}^{\infty}\sum_{n=0}^{\infty} \bar{p}_{m,n} d_{m,n}\cos\left(n\theta + \alpha_{m,n}\right)\cos m\pi\bar{z}\right] \quad (31)$$

In steady condition radial deformation in the inner surface may be written as,

$$\bar{\delta} = 2(1+\nu)\, F\left[\bar{p}_{0,0}\, d_{0,0} + \sum_{\substack{m=0 \\ (m,n)\neq(0,0)}}^{\infty}\sum_{n=0}^{\infty} \bar{p}_{m,n} d_{m,n}\cos\left(n\theta + \alpha_{m,n}\right)\cos m\pi\bar{z}\right] \quad (32)$$

Where $\bar{\delta} = \dfrac{\delta}{c}$ & $F = \dfrac{\eta_0\, \omega R^3}{E c^3}$

III METHOD OF SOLUTION

A. Steady-State Analysis.

To find out steady-state pressure all the time derivatives are set equal to zero and the non-dimensionalised all equations (13) and (9) to (12) and also equations (22) to (24) and equation (19) to (21) are written in finite difference form along with all required boundary conditions to proceed for calculation. For $\varepsilon_0 \le 0.2$ the pressure distribution and flow parameters Q_θ and Q_z are evaluated from inertia less $(\mathrm{Re}^* = 0)$ solution, i.e., solving classical Reynold's equation. These values are then used as initial value of flow parameters to solve Eqs.(9) and (10) simultaneously for Q_θ and Q_z Using Gauss-Siedel method in a finite difference scheme. Then updated I_x and I_z and then calculate Q_θ and Q_z for use to solve Eq.(13) with initial zero surface deformation for new pressure $\bar{p}$ with inertia effect by using a successive over relaxation scheme. The latest values of Q_θ and Q_z and $\bar{p}$ are used iteratively to solve the set of equations until all variables converges using a finite-difference method (Gauss-Seidel) with successive over relaxation scheme. The convergence criterion adopted for pressure is $\left|\left(1 - \left(\sum \bar{p}_{new} \middle/ \sum \bar{p}_{old}\right)\right) \le 10^{-5}\right|$ and also same criterion for Q_θ and Q_z.. The distribution was expressed as a double Fourier series as given by equation (20). The deformation equation (37) was then calculated for a given F using distortion coefficients from equation (35) after calculating displacement components solving equations (28), (29), and (30) using boundary conditions (31) to (35). The film thickness equation was then modified using equation (17). The fluid film pressure was again obtained from equation (14) simultaneously with equation (15), (16) and equation (18) and (19) and then get modified film shape. The process was repeated until a compatible film shape and pressure distribution was determined.

For higher eccentricity ratios ($\varepsilon_0 > 0.2$) the initial values for the variables are taken from the results corresponding to the previous eccentricity ratios. Very small increment in ε is to be provided as Re^* increases. The procedure converges up to a value of $\mathrm{Re}^* = 1.5$ which should be good enough for the present study.

Since the bearing is symmetrical about its central plane ($\bar{z}$ =0),only one half of the bearing needs to be considered for the analysis, Once the pressure distribution is evaluated fluid film forces and the load bearing capacity $\bar{W}_o$ and attitude angle (φ) are calculated

B. Fluid Film Forces.

The non-dimensional fluid film forces along line of centres and perpendicular to the line of centres are given by

$$\bar{F}_r = \int_0^1 \int_{\theta_1}^{\theta_2} \bar{p} \cos\theta \, d\theta \, d\bar{z} \quad (33)$$

$$\bar{F}_\phi = \int_0^1 \int_{\theta_1}^{\theta_2} \bar{p} \sin d\theta \, d\bar{z} \quad (34)$$

Where, $\bar{F}_r = \dfrac{F_r C^2}{\eta \omega R^3 L}$, $\bar{F}_\phi = \dfrac{F_\phi C^2}{\eta \omega R^3 L}$

where θ_1 and θ_2 are angular coordinates at which the fluid film commences and cavitates respectively.

C. Steady-State Load and Attitude Angle.

The steady state non-dimensional load and attitude angle are given by

$$\bar{W}_0 = \sqrt{\left(\bar{F}_{r_0}^{\,2} + \bar{F}_{\phi_0}^{\,2}\right)} \quad (35)$$

$$\phi_o = \tan^{-1}\left(\frac{-\bar{F}_{\phi_0}}{\bar{F}_{r_0}}\right) \quad (36)$$

Since the steady state film pressure distribution has been obtained at all the mesh points, integration of equations (33) and (34) can be easily performed numerically by using Simpson's 1/ 3 rd. rule to get $\bar{F}_r$ and $\bar{F}_\phi$. The steady state load $\bar{W}_0$ and the attitude angle (ϕ_0) are calculated using equations (35) and (36).

The present theoretical study has been done considering combine effect of fluid Inertia effects and bearing surface deformation. The results have been compared with available data of researchers.

D. Equations of motion

An axially symmetric system is considered for analysis and the system consists of a rigid rotor in a non-rotating fixed supported bearing. It is assumed that the rotor is perfectly balanced. Referring to figure 4.1 the equation of motion of the rigid journal, along the line of centres and its perpendicular direction respectively can be written as

Equation of motion of the rotor in " r " direction is given by

$$m_j \left[\frac{d^2 e}{dt^2} - e\left(\frac{d\phi}{dt}\right)^2\right] = F_r + W_0 \cos\phi \quad (37)$$

Equation of motion of the rotor in " ϕ " direction is given by

$$m_j \left[e\frac{d^2\phi}{dt^2} + 2\left(\frac{d\phi}{dt}\right)\left(\frac{de}{dt}\right)\right] = F_\phi - W_0 \sin\phi \quad (38)$$

where m_j is the mass of the journal per bearing, ϕ is the attitude angle, e is the bearing eccentricity, W_0 is the steady externally applied load on the journal, F_r & F_ϕ are the film forces in the r and ϕ directions respectively.

E. Stability analysis

The equations of motion (37) and (38) of the rigid rotor, along the line of centres and its perpendicular directions may be expressed in dimensionless form as below

$$\bar{M}\left[\varepsilon' - \varepsilon(\phi')^2\right] = \frac{1}{\bar{W}_0}\left[\bar{F}_r + \bar{W}_0 \cos\phi\right] \quad (39)$$

$$\bar{M}\left[\varepsilon\phi'' + 2\varepsilon'\phi'\right] = \frac{1}{\bar{W}_0}\left[\bar{F}_\phi - \bar{W}_0 \sin\phi\right] \quad (40)$$

Where, $\bar{M} = \dfrac{m_j c\omega^2}{W_0}$, $\bar{W}_0 = \dfrac{c^2 W_0}{\eta_0 \omega R^3 L}$,

The above two second order differential equations can be expressed as four first order differential equations as follows.

$$\varepsilon' = \frac{d\varepsilon}{d\tau} \quad (41)$$

$$\phi' = \frac{d\phi}{d\tau} \quad (42)$$

$$\varepsilon'' = \frac{d\varepsilon'}{d\tau} = \frac{1}{\bar{M}\bar{W}_0}\left[\bar{F}_r + \bar{W}_0 \cos\phi\right] + \varepsilon(\phi')^2 \quad (43)$$

$$\phi'' = \frac{d\phi'}{d\tau} = \frac{1}{\varepsilon \bar{M} \bar{W}_0}\left[\bar{F}_\phi - \bar{W}_0 \sin\phi\right] - \frac{2\varepsilon' \phi'}{\varepsilon} \quad (44)$$

Equations (41) through (44) can be solved for state space variables $\left(\varepsilon, \phi, \varepsilon', \phi'\right)$ by fourth order Runge-Kutta method and stability analysis can be carried out for the following three different types of loading.

(A) Unidirectional constant load:

The supplied load is assumed to be constant in both magnitude and direction and equal to the steady state load carrying capacity, i.e., $\bar{W} = \bar{W}_0$. Where, $\bar{W}_0$ is the steady state load carrying capacity.

(B) Unidirectional periodic load:

The applied load is assumed to be of the form $\bar{W}_0\left[1+\sin\left(\tau/2\right)\right]$, where, τ is the dimensionless time.

(C) Variable rotating load:

Considering a journal under the action of an external static load and an external centrifugal load P_c due to unbalance.

Equation of motion of the rotor in " r " direction is given by

$$m_j\left[\frac{d^2 e}{dt^2} - e\left(\frac{d\phi}{dt}\right)^2\right] = F_r + W_0\cos\phi + P_c\cos(\omega t - \phi) \quad (45)$$

Equation of motion of the rotor in " ϕ " direction is given by

$$m_j\left[e\frac{d^2\phi}{dt^2} + 2\left(\frac{d\phi}{dt}\right)\left(\frac{de}{dt}\right)\right] = F_\phi - W_0\sin\phi + P_c\sin(\omega t - \phi) \quad (46)$$

Substituting $\bar{M} = \frac{m_j c\omega^2}{W_0}$, $\bar{W}_0 = \frac{c^2 W_0}{\eta_0 \omega R^3 L}$, the equations of motion (45) and (46) of the rigid rotor with unbalance mass, along the line of centres and its perpendicular directions can be non-dimensionalised as follows.

$$\left[\varepsilon' - \varepsilon\left(\phi'\right)^2\right] = \frac{1}{\bar{M}\bar{W}_0}\left[\bar{F}_r + \bar{W}_0\cos\phi\right] + \frac{u^*}{c}\cos(\tau - \phi) \quad (47)$$

$$\left[\varepsilon\phi'' + 2\varepsilon'\phi'\right] = \frac{1}{\bar{M}\bar{W}_0}\left[\bar{F}_\phi - \bar{W}_0\sin\phi\right] + \frac{u^*}{c}\sin(\tau - \phi) \quad (48)$$

where, u^* is the eccentricity of mass unbalance.

IV RESULTS AND DISCUSSIONS

For stability analysis, a non-linear time transient analysis is carried out using the equations of motion to compute a new set of ε, ϕ & their derivatives for the next time step for a given set of. $Re^*, H/R, \nu, L/D, \varepsilon, \bar{M}$ (Mass parameter) for a particular deformation factor, F. The forth order Runge-Kutta method is used for solving the equations of motion. The hydrodynamic forces are computed for every time step by solving the partial differential equation for pressure satisfying the boundary conditions.

To study the combined effect of fluid inertia and bearing surface deformation on journal centre trajectory a set of trajectories of journal centre and bearing has been studied and it is possible to construct the trajectories for numbers of complete revolution of the journal the plots shows the stability of the journal when the trajectory of journal and bearing centre ends in a limit cycle. Critical mass parameter for a particular eccentricity ratio, Poisson ratio, H/R ratio, slenderness ratio, modified Reynolds number, bearing liner surface deformation is found when the trajectory ends with limit cycle (Fig. 8) & (Fig. 9 and Fig.10) shows Instability and point stability of journal trajectory.

Fig. 2: Variation of Critical Mass Parameter with Deformation Factor for various Eccentricty Ratio

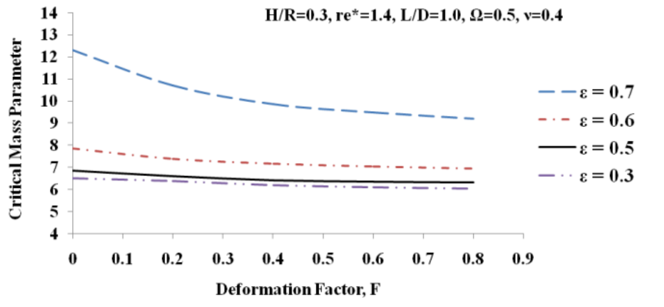

Figure 2 shows the variation of critical mass parameter with elasticity parameter for four eccentricity ratios (i.e., $\varepsilon = 0.3$, 0.5, 0.6, 0.7). Although there is little variation of mass parameter with F at low eccentricity ratios, the load drops sharply with F at $\varepsilon = 0.7$. The Critical mass parameter increases as Eccentricity ratio increases.

Fig. 3: Variation of Critical Mass Parameter with Deformation Factor for various Modified Reynolds Number

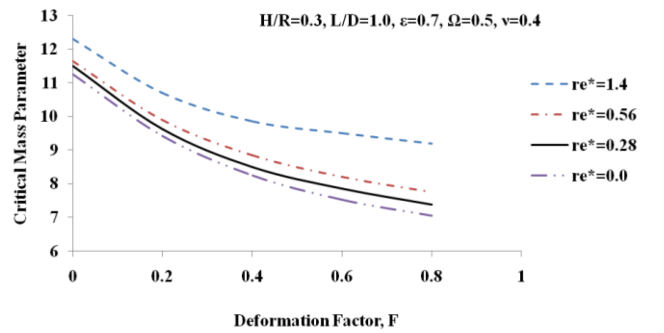

Figure 3 shows the variation of critical mass parameter with elasticity parameter / deformation factor for four Reynolds number (i.e., $Re^*=0.0$, 0.28, 0.56, 1.4). The critical mass parameter decreases as deformation factor increases and critical mass parameter increases as Reynolds number increases. The overall trend of the curve drops sharply with F .

Fig.4: Variation Critical Mass Parameter with Deformation Factor for various L/D ratio

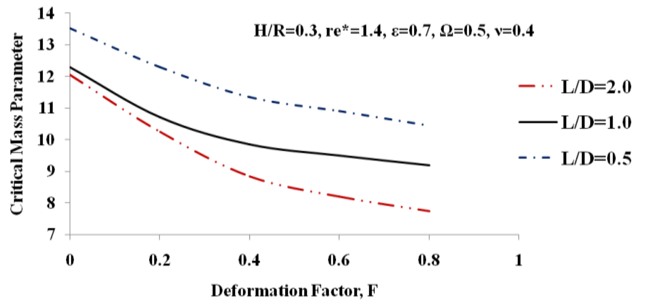

Figure 4 shows the variation of critical mass parameter with elasticity parameter / deformation factor for various L/D ratio (i.e., $L/D=0.5$, 1.0, 2.0). The critical mass parameter decreases as deformation factor increases and critical mass parameter decreases as Slenderness ratio increases. The overall trend of the curve drops sharply with F .

Fig.5: Variation of Critical Mass Parameter with Deformation Factor for Various H/R Ratio

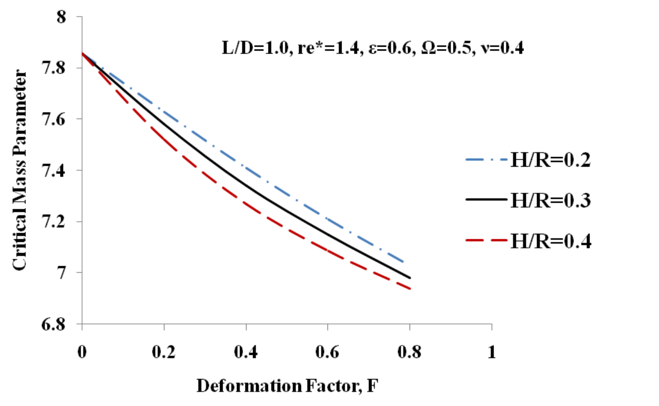

Figure 5 shows the variation of critical mass parameter with elasticity parameter / deformation factor for various H/R ratio (i.e., $H/R=0.2$, 0.3, 0.4). The critical mass parameter decreases as deformation factor increases and critical mass parameter decreases as H/R ratio increases. The overall trend of the curve drops very sharply with F .

Fig.6: Variation of Critical Mass Parameter with Deformation Factor for Various Poisson Ratio

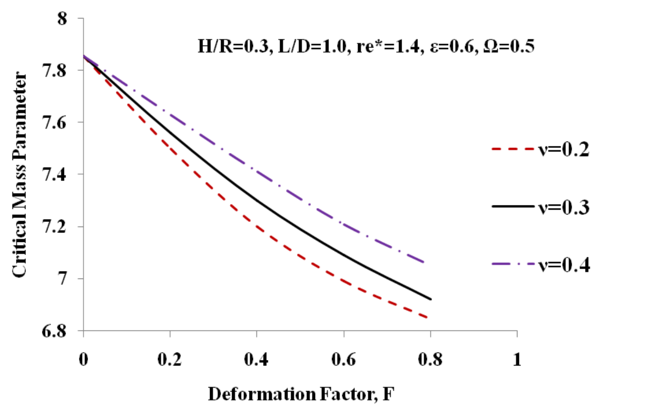

Figure 6 shows the variation of critical mass parameter with elasticity parameter / deformation factor for various Poisson ratio ν (i.e., $\nu=0.2$, 0.3, 0.4). The critical mass parameter decreases as deformation factor increases and critical mass parameter increases as H/R ratio increases. The overall trend of the curve drops very sharply with F .

Fig.7: Variation of Critical Mass Parameter with Eccentricity Ratio for various Deformation Factor

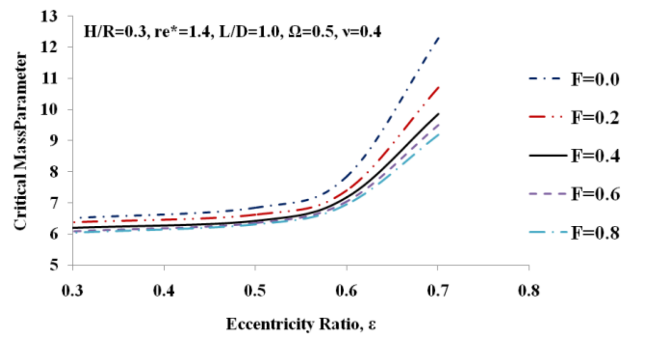

Figure 7 shows the variation of critical mass parameter with Eccentricity ratio for various elasticity parameter F (i.e., $F=0.0$, 0.2, 0.4, 0.6, 0.8). The critical mass parameter increases as deformation factor increases and critical mass parameter increases as F decreases. The increase is more sharp when eccentricity ratio $\varepsilon>0.6$.

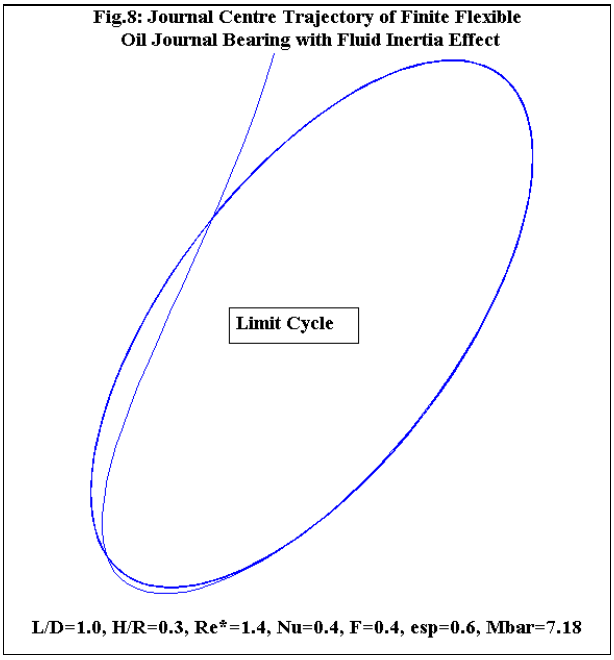

Fig.8: Journal Centre Trajectory of Finite Flexible Oil Journal Bearing with Fluid Inertia Effect

Figure 8 shows Journal trajectory ends with a limit cycle.

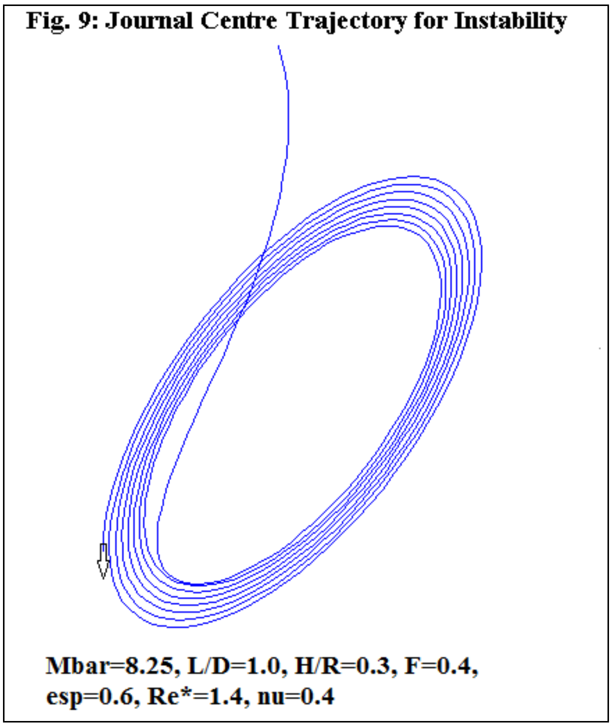

Fig. 9: Journal Centre Trajectory for Instability

Figure 9 shows Journal trajectory proceeds towards instability.

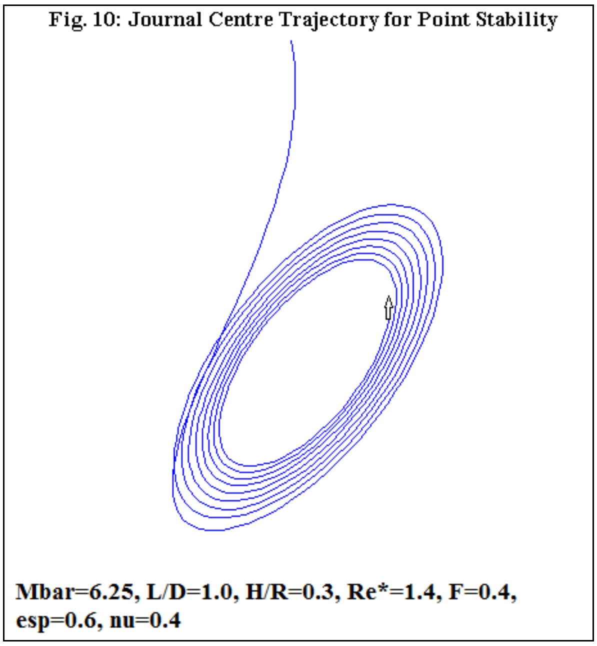

Fig. 10: Journal Centre Trajectory for Point Stability

Figure 10 shows Journal trajectory proceeds towards point stability.

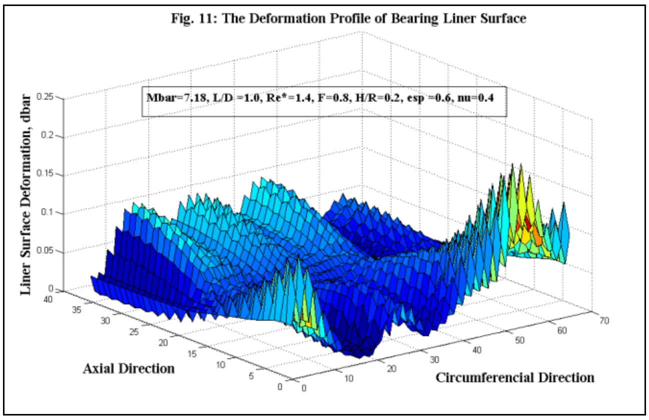

Fig. 11: The Deformation Profile of Bearing Liner Surface

Figure 11 shows the variation of bearing liner surface deformation in circumferential and axial direction for elasticity parameter $F = 0.8$ and when the journal trajectory ends with limit cycle.

V CONCLUSIONS

1. The effect of inertia on the stability is affected considerably at higher L/D ratios and eccentricity ratios (Figure 2 and 4).The probable reason may be that higher L/D ratios and eccentricity ratios, the circumferential component of flow will be overtaking the axial flow. The inertia effect of the circumferential flow will possibly add more stiffness in the film, thereby improving the stability. It is also noted that higher Re* means higher surface speed of the shaft (when other parameter remain constant).This will further increase couette flow which is a part of circumferential flow.

2. The stability decreases as the bearing liner is made more flexible for high eccentricity ratios (i.e., $\varepsilon_0 > 0.6$). For $\varepsilon_0 < 0.6$, the flexibility of the bearing liner had little or no effect on stability.
3. Bearing is highly stable when $L\ D$ is small but drops as $L\ D$ increases from 0.5 to 2.0. This stability drops as deformation factor increases.
4. The hydrodynamic pressure and hence the stability is reduced as the bearing liner becomes more flexible, especially at eccentricities greater than 0.6.
5. As the Poisson ratio increases the stability increases but drop sharply when bearing liner is made more flexible.
6. As the liner thickness to radius ratio increases the stability decreases but drop when bearing liner is made more flexible
7. Critical mass Parameter increases when elasticity parameter decreases.

NOMENCLATURE

$a = r_i$	Inner radius of the bearing liner [m]
$b = r_0$	Outer radius of the bearing liner [m]
c	Radial clearance [m]
R	Journal radius [m]
D	Journal diameter [m]
$d_{m,n}$	Distortion coefficient of $_{m,n}$ harmonic
m,n	Axial and circumferential harmonics
e	Eccentricity [m]
e_0	Steady state eccentricity [m]
E	Young's modulus [N/m^2]
F	Elasticity parameter or deformation factor,
$\bar{F}_r$	Nondimensional fluid film force along the line of centers
$\bar{F}_\phi$	Nondimensional fluid film force perpendicular the line of centres
$\bar{F}_{r_0}$, $\bar{F}_{\phi_0}$	Non-dimensional steady state fluid film forces
h	Oil film thickness [m]
h_0	Steady state oil film thickness [m]
$\bar{h}_0$	Non-dimensional steady state oil film thickness
H	Thickness of bearing liner [m]
J	Mechanical equivalent of heat
L	Length of bearing [m]
p	Oil film pressure [Pa]
p_0	Steady state film pressure [Pa]
$\bar{p}_0$	Dimensionless oil pressure
Q	End flow of oil [m^3/s]
$\bar{Q}$	Nondimensional End flow
$\bar{u}, \bar{v}, \bar{w}$	Components of fluid velocity in the x, y, and z direction, respectively. [m/s]
U	Shaft peripheral speed [m/s]
W_0	Steady state load [N]
$\bar{W}_0$	Dimensionless steady state load
x, y, z	Circumferential, radial and axial coordinates
$\theta, \bar{y}, \bar{z}$	Dimensionless coordinates in circumferential, radial and axial directions
η_0	Viscosity at inlet condition [Pa s]
ρ	Density [kg/m^3]
ν	Poisson's ratio
ε	Eccentricity ratio
ε_0	Steady state eccentricity ratio
ϕ	Attitude angle [rad]
ϕ_0	Steady state attitude angle [rad]
θ_1	Angular coordinates at which the fluid film commences [rad]
θ_2	Angular coordinates at which the fluid film cavitates [rad]
ω	Angular velocity of journal [rad / s]
$\bar{M}$	Mass Parameter
Ω	Whirl ratio. [$\frac{\omega_p}{\omega}$]
$\tau = \omega_p t$	Non dimensional time.
δ	Deformation of bearing surface. [m]
δ_0	Steady state deformation of bearing surface. [m]
$\bar{\delta}_0$	Non-dimensional deformation of bearing surface
λ, μ	Lame's constants
R_e =	Reynolds number, $\frac{\rho c R \omega}{\eta}$
R^*_e =	Modified Reynolds number, $\left(\frac{c}{R}\right) R_e$

Q_θ = Dimensionless flow parameter in θ direction

Q_z = Dimensionless flow parameter in $\bar{z}$ direction

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