A Comparative Study between Numerical and Analytical Approaches to Load Carrying Capacity of Conical Shells under Axial Loading

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Abstract *In this study, loss of stability of the truncated conical shell structures under axial loading was investigated. Analytical calculations were made by means of analytical expressions derived from the linear theory. These expressions are presented in previous studies in the literature and so in ECCS regulations. A numerical study was performed with two finite element package programs; Cosmos/M and Abaqus. In all numerical simulations, nonlinear geometry effect was considered (GNA – Geometrically nonlinear analysis). Conical shells with a semi-vertex angle in the range of 10-80° and shell thickness in the range of 0.6-1mm were modeled. All results from both analytical and numerical studies were compared. Furthermore, an empirical expression which evaluates the load carrying capacity of the presented conical shells was exhibited in terms of the dimensionless parameter* r_e/t_{shell}.

Keywords — *Conical shell, Elastic buckling, Axial compression, Finite element method.*

I. Introduction

Conical shell structures have a very common usage in the industry such as space crafts, robots, shelters, domes, and tanks. Therefore, there are studies performed by many authors about standard shell structures consist of components which have an ordinary shape such as completely cylindrical shells, spheres and torus or conical shells with the semi vertex angle lower than 65° in the current literature. Since the membrane stress is exclusively dominated by this type structures, theoretical approaches can be used to obtain load carrying capacity of a structure. Also updated standards and recommendations provide useful approaches to obtain stability of the conical shells with the semi vertex angle which is lower than 65 degrees [1] and [2]. Nevertheless, the standard methods are not applicable for the shells which have semi vertex angle higher than 65° or a flexible boundary ring. The rules included in the recommendations can only be applied on conical shells which have clamped edges or edges with a stiff ring.

In order to obtain the elastic stability of unstiffened conical shells under compressive loading, Seide [3] has approached the problem analytically and developed an expression based on Donnell type shell theory as a classical solution for axisymmetric buckling of conical shells. Seide's equation may be written as:

$$P_{cr} = \frac{2\pi E t^2 \cos^2\beta_c}{\sqrt{3(1-\nu^2)}} = P_{cyl}\cos^2\beta_c \tag{1}$$

where:

P_{cr}	Critical load of the conical shell, [N]
E	Modulus of elasticity. [MPa]
β_c	Semi-vertex angle. [rad]
P_{cyl}	Critical load of the cylinder. [N]
ν	Poission's ratio.
t	Shell thickness. [mm]

The stability of truncated conical shells subjected to axial compression has been studied by many prominent authors.

Weingarten et. al. [4] studied the stability of cylindrical and conical shells under axial compression experimentally. Experiments were performed using specimens made of both Mylar polyester and steel. Results of the study indicated that buckling coefficient varied with the radius-to-thickness ratio. Also, lower bound curves for buckling coefficients were given.

Tani and Yamaki [5] studied the elastic stability of truncated conical shells under axial compression. Unlike previous studies, authors analyzed the problem under four sets of boundary conditions including both simply supported and clamped cases. After detailed calculations and clarification of the correlations to the buckling of equivalent cylindrical shells, critical load estimation for conical shells was expressed.

Pariatmono and Chryssanthopoulos [6] investigated the buckling of conical shells under axial compression in terms of critical buckling load and mode shapes. The study included numerical solutions of the problem with both simply supported and clamped boundary conditions. With the adaptation of F-W method, two different displacement functions were used to obtain and compare the results. As a result of the study, numerical difficulties at the simply supported case were reduced by introducing clamped boundary conditions. Clamping also changed buckling mode at both near the boundaries and center of the cone.

Marios et al [7] studied stiffened cones and some specific characteristics of conical shells in order to obtain detailed information about the particular case. Finite element analysis was used for evaluation critical elastic response and imperfection sensitivity in order to develop a design approach for stringer-stiffened cones subjected to axial compression. According to the current recommendations of European Shell Buckling Recommendations (ECCS), proposals for improvement of the design of both unstiffened and stiffened cones were made.

Thinvongpituk and El-Sobky [8] investigated the buckling behavior of conical shells under axial loading both numerically and experimentally. Finite element analyses were performed with commercial package program Abaqus and experiments were carried out under quasi-static loading with three different end conditions: simple support, top constraint, and base constraint. It was found that type of the end constraint has a great effect on the buckling load of conical shell.

Jabareen and Sheinman [9] examined the effect of the pre-buckling nonlinearity on the bifurcation point of a conical shell. Three shell theories: Donnell's, Sanders' and Timoshenko's have been used as the basis. Authors developed a computer code to examine the effect of the pre-buckling nonlinearity on the buckling of the shell under axial compression. It was obtained that for structures which have a softening behavior have a lower buckling load. This case is caused by the pre-buckling nonlinearity when compared to the classical case.

Blachut et al [10] studied on the buckling of conical shells subjected to axial compression, lateral pressure and hydrostatic pressure both numerically and experimentally. Authors performed experiments on five laboratory scale models and obtained a good proximity between numerical estimations of collapse and axisymmetric bifurcation buckling.

Ifayefunmi and Blachut [11] investigated the elastic-plastic buckling of short and relatively thick unstiffened truncated conical shells under axial compression and external pressure. Thirteen nominally identical laboratory samples under various loading conditions were used for the experiments. Authors gained a good approximation between experimental results and numerical predictions and also another comparison study, between predictions of failure loads obtained from ASME code 2286-2 and ECCS design rules, was performed.

Ifayefunmi [12] studied plastic buckling of thick steel conical shells under combined loading of axial compression and internal pressure. According to the ASME case code 2286-2, numerical calculations and experiments were accomplished to validate the rule both experimentally and numerically. As a result of the present study, it is emphasized that concept of equivalent cylinder approach for thick cones under combined loading have unstable results and it is unsafe for design purposes.

In this study, loss of stability of truncated conical shell structures under axial loading was investigated. A numerical approach was conducted for the problem and compared with the equation (1) derived by Seide [1] by means of the linear theory. The equation is taken into consideration for the case of axisymmetric buckling of truncated conical shells subjected to axial loading. In this numerical approach, geometrical nonlinearities were considered in order to compare with the linear formulation. Conical shells with semi-vertex angles between 10° and 80° were modeled and analyzed with two different finite element package program COSMOS/M and ABAQUS. Moreover, the influence of the thicknesses of conical shells on the stability was investigated in the range of 0.6mm and 1mm.

II. Material and Method

Numerical simulations were performed using two different commercial finite element package programs: Cosmos/M and Abaqus. Basic sketch with two different views (front view and top view) for the models of conical shells are given in Figure 1 with geometrical parameters.

Geometrical parameters as seen in Figure 1 are named; r_1: upper radius, r_2: bottom radius, h: height of the stiff pipe, L : conical shell length, r_e : equivalent cylinder radius, α_c: angle of lower edge, β_c: semi-vertex angle, t_{shell}: shell thickness and F: axial load. Upper radius "r_1" and bottom radius "r_2" were defined as 50 mm and 250 mm, respectively. The height of the relatively stiff pipe "h", located at the top of the truncated conical shell was assigned as 10 mm.

In the present study, analytical calculations were made using equivalent cylinder radius r_e which is individually calculated by recommendations stated in "Buckling of steel shells European design recommendations (ECCS)". According to the aforementioned recommendation, it is needed to be determined either short or long conical shell based on the equations given below [1].

$$l_e = min\left[L; \left(\frac{r_2}{sin\beta_c}\right)(0.53 + 0.125\beta_c)\right] \quad (2)$$

where β_c is the semi vertex angle of conical shell in [Rad],

$$\beta_c = \frac{\pi}{2} - \alpha_c \quad (3)$$

if, $l_e = L$,, it means this structure is a short conical shell, equivalent radius is;

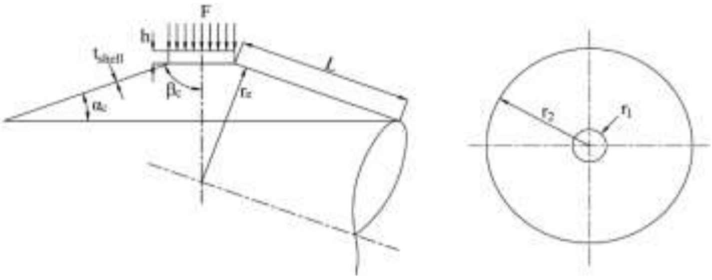

Fig. 1 Front (on the left) and top view (on the right) of the conical shell.

$$r_e = \frac{0.55 r_1 + 0.45 r_2}{\cos\beta_c} \qquad (4)$$

if, $l_e = \left(\frac{r_2}{\sin\beta_c}\right)(0.53 + 0.125\beta_c)$, it means this structure is a long conical shell and the equivalent radius r_e is;

$$r_e = 0.71 r_2 \frac{1 - 0.1\beta_c}{\cos\beta_c} \qquad (5)$$

According to the abovementioned equations taken from ECCS, for the current case, all of the models were determined as long conical shells and equivalent cylinder radius values were calculated using equation (5). Calculated equivalent cylinder radius r_e values were used to calculate the r_e/t_{shell} dimensionless parameter. The range of the shell thickness t_{shell} values and dimensionless the parameter r_e/t_{shell} values are presented in Table 1 for each individual semi vertex angle β_c.

Tab. 1 Variable geometrical parameters of the models.

Semi Vertex Angle β_c [°]	Shell Thickness t_{shell} [mm]	r_e/t_{shell} [-]
10	0.6 – 1	177 – 295
20	0.6 – 1	182 – 304
30	0.6 – 1	194 – 324
40	0.6 – 1	215 – 359
50	0.6 – 1	252 – 420
60	0.6 – 1	317 – 530
70	0.6 – 1	455 – 759
80	0.6 – 1	879 – 1465

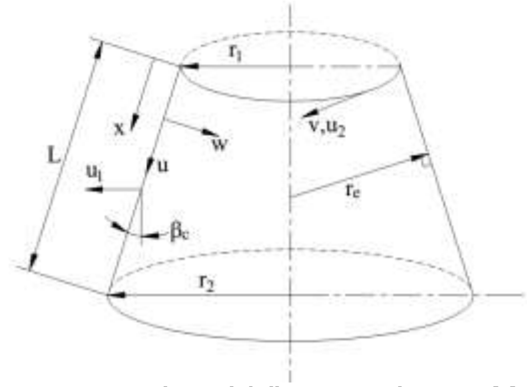

Fig. 2 Basic truncated conical shell geometry and notations [7].

In all models, upper and lower circular ends of the conical shells were constrained in the radial direction. In other words, these constraints can be defined as below using coordinates given in Figure 2. Boundary conditions assigned on the models are also shown in Figure 3 in detail.

$$u_1 = u\sin\beta_c - w\cos\beta_c = 0 \quad and \quad v = 0 \qquad (6)$$

The material used for the models was considered as S235 steel assumed to have a linear and isotropic material behavior. Properties of the material in analytical and numerical analysis are; modulus of elasticity "E" of 200 GPa, Poisson's ratio "υ" of 0.3, and mass density "ρ" of $7850\ kg/m^3$.

In all simulations, geometrical nonlinearity was taken into consideration, so large displacement formulation was enabled in both FEA solutions.

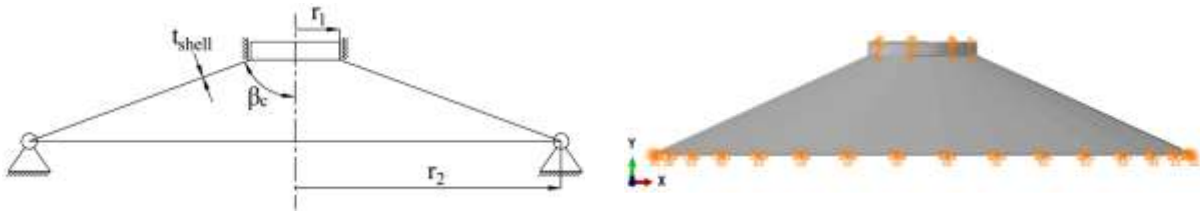

Fig. 3 Schematic representation of boundary conditions of numeric models.

All analyses were performed with a mesh consisting of quadrilateral shell elements. These elements are called “SHELL4” [13] in Cosmos/M and “S4” [14] in Abaqus. Mesh structures of the model with a 60° of semi-vertex angle are illustrated in Figure 4.

III. RESULTS AND DISCUSSION

Numerical analyses were carried out from the beginning to the point at which the loss of stability occurs under an increasing axial load applied on top of the edges of conical shells. This state is illustrated in Figure 5 as a force vs. end-shortening curve for one case ($\beta_c = 60°$ and $t_{shell} = 1mm$) from the data obtained with both Cosmos/M and Abaqus. All data were recorded from a reference point placed on the top of the conical shell.

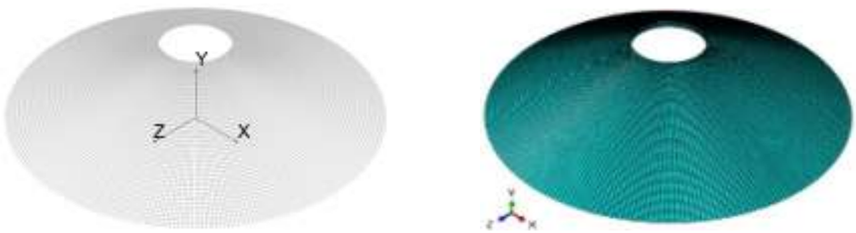

Fig. 4 Mesh structure of the models in Cosmos/M (on the left) and Abaqus (on the right).

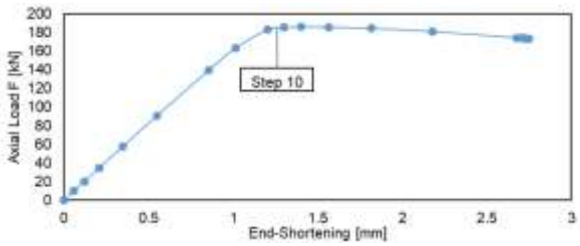

Fig. 5 F – End-shortening curve obtained from Abaqus for $\beta_c = 60°$ and $t_{shell} = 1mm$.

It is observed that geometric stiffness of the conical shell reduces with increasing load. When the slope of the F vs. end-shortening curve becomes zero, the structure loses its structural stability and the load carrying capacity of structure decreases correspondingly. Accordingly, it is seen that during the analysis after the loss of stability point, applied load exhibits a decreasing trend while end-shortening is still increasing. Deformed shapes of the same structure are also illustrated in Figure 6 at the nonlinear collapse point.

Axial load values and deformation shapes were seemed to be very consistent in Cosmos/M and Abaqus. For the structure depicted in Figures 5 and 6, at the loading of the loss of stability point ($F \cong 186\ kN$), four waves were appeared along the top and bottom edge of the shell geometry with a total end shortening of $1.29mm$. These waves are caused by the axisymmetric loss of stability of the structure and have a circumferential pattern along the shell geometry. Analyses for all semi-vertex angles and shell thicknesses were performed and the values of limit loads are plotted in Figure 7 and 8.

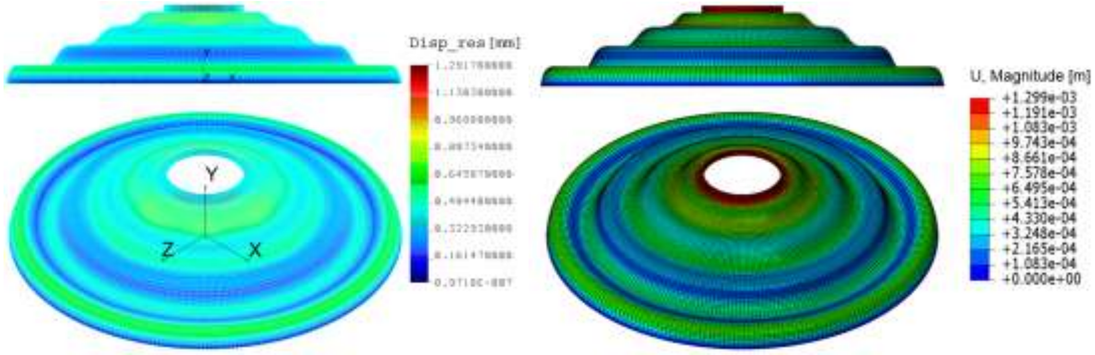

Fig. 6 F – Deformed shapes from Cosmos/M (on the left) and Abaqus (on the right). (Deformation scale factor = 20)

The results taken from both package programs have a good match in the range of 2% deviation with respect to the nonlinear collapse load. After a parametric study, it is obvious that the limit load is directly related to shell thickness and inversely related to the semi-vertex angle.

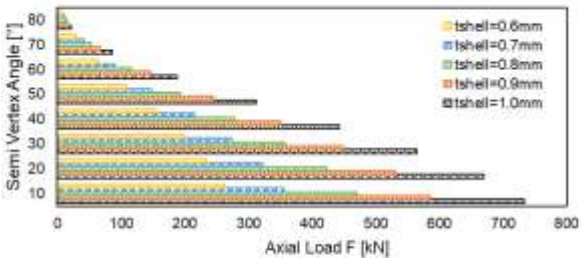

Fig.7 GNA results obtained from Cosmos/M

A change in the semi-vertex angle of the structure results a change in the components of the load applied on the conical shell. Increasing semi-vertex angle causes a decrease in the meridional load component (F_u) and an increase in the load attempts to bend the side surface of the conical shell inward (F_w). Aforementioned forces are illustrated in Figure 9 with necessary notations.

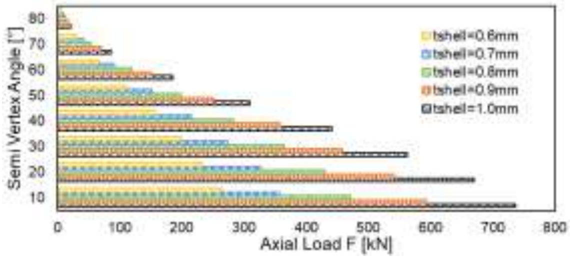

Fig. 8 GNA results obtained from Abaqus

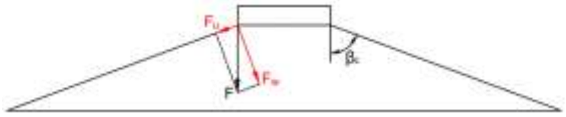

Fig. 9 Force components of the applied load affected on the conical shell structure

Conical shell structures are known to be more resistant to membrane stresses. Because of the increase in the load which causes the bending stress, load carrying capacity of the structure decreases with increasing semi-vertex angle. Also the load component meridional to the side surface of the conical shell (F_u) which causes the membrane stress on the structure decreases. This case leads to decrease the number of axisymmetric waves along the conical shell at the point of loss of stability. A number of the waves propagated on the structures decreased with increasing semi-vertex angle in all analyses.

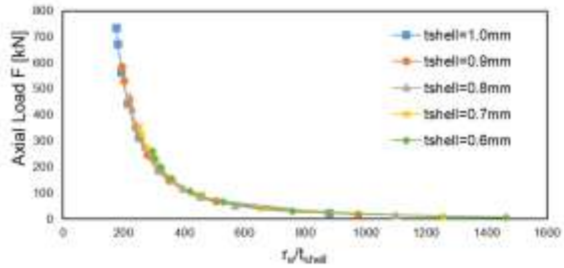

Fig. 10 F vs. r_e/t_{shell} curves for the results of Cosmos/M

A dimensionless parameter r_e/t_{shell} is introduced as a function of geometrical parameters (β_c, t_{shell}, r_e, etc.) of the conical shell structure. When the dimensionless parameter r_e/t_{shell} is identical at different semi-vertex angles and shell thicknesses, then their limit loads also become equal.

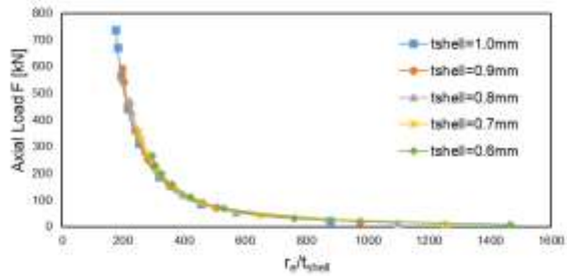

Fig. 11 F vs. r_e/t_{shell} curves for the results of Abaqus

The limit loads from FEA can be fitted with a power equation with a maximum deviation of 8%. Power equation in a relation with limit load values can be written as;

$$F_{lim} = 6 * 10^7 * (r_e/t_{shell})^{-2.19} \ [kN] \qquad (7)$$

As seen in the equation (7), in order to evaluate the load carrying capacity of the conical shells proposed in this study, it is enough to have information about the equivalent radius r_e and the shell thickness t_{shell}.

Based on the results and as seen in Figure 10 and 11, the dimensionless parameter r_e/t_{shell} has an inverse proportionality with the load carrying capacity of the structure. When the value of this parameter goes to zero, structure becomes to behave like infinitely stiff and increasing values of the dimensionless parameter causes the load carrying capacity of the structure to approach zero asymptotically.

Results obtained from analyses performed with two different commercial finite element package programs were compared to the values calculated with the analytical expression (equation 1) developed by Seide [3]. The main purpose of this comparison was to assess the effect of the geometrical nonlinearities on the load carrying capacity of conical shells. The difference between analytical calculations and numerical simulations are plotted in Figure 12.

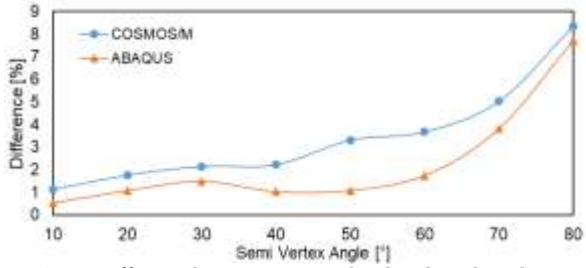

Fig. 12 Difference between numerical and analytical results.

The differences between the analytical and numerical results are up to 14% for different semi-vertex angles and shell thicknesses. Average differences of the results were calculated up to 9% as seen in Figure 12. The equation proposed by Seide [1] gives similar results with numerical simulations for $\beta_c \leq 65°$ which are named as "standard structure" in ECCS regulations. However, for larger β_c attaining high geometrical nonlinearity influences, implementing of critical load expression (equation 1) increases the percentage of error between analytical and numerical results. Load carrying capacities calculated by analytical method observed to be higher than those of numerical method.

IV. CONCLUSIONS

In the present study, the effect of semi-vertex angle and shell thicknesses on the load carrying capacity of truncated conical shells were investigated. Also, critical load values of the conical shells were calculated using the equation derived by Seide [1]. All results obtained from analytical calculations and numerical simulations (GNA-geometrically nonlinear analysis) were compared to each other. Main concluding remarks obtained from the current study are listed below.

The load carrying capacity of a truncated conical shell has a direct proportionality with the shell thickness and an inverse proportionality with the semi-vertex angle.

For the structures investigated in this study, aforementioned analytical expression (equation 1) gave consistent results for $\beta_c \leq 65°$. Besides, for higher semi-vertex angles $\beta_c > 65°$, the maximum difference between the results was calculated 14% depending on the shell thicknesses and semi-vertex angles.

In accordance with the numerical simulations, load carrying capacity of the conical shells can be represented by an empirical expression. This expression is based on the dimensionless parameter r_e/t_{shell} and characterised by a power function.

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