Impact of Parametric Variations on Chaotic Behaviour of Indirect Field Controlled Induction Motor Drives

Mirza Abdul Waris Begh*1, Bharat Bhushan Sharma#2
*Chair of Power Electronics and Drives, Technical University of Munich, Germany
#Department of Electrical Engineering, NIT Hamirpur, H.P., India
1mirza.begh@tum.de, 2bharat.nit@gmail.com

***Abstract*—**Controlling complex chaotic systems and analyzing their behavior have emerged as an attractive field of exploration in different domains of engineering. Over the years, large number of mathematical tools are developed to identify and control the typical behaviour of these systems. The work presented in this manuscript explores chaos in nonlinear dynamics of an indirect field controlled induction motor drive system. For this exploration, impact of variation in rotor inductance is considered while assuming the load torque to be fixed. Chaotic attractors are first verified by investigating Lyapunov Exponents. The range of parametric variation is explored to check for the events where chaos can creep into the system again. Finally, an attempt is made to measure the transition point between stability and instability of the chaotic system. This is verified using the Lyapunov Exponent measure and the phase plots. The detailed simulation results highlight the efficacy of the methodology to identify the chaotic behaviour of the induction motor.

Keywords—*Chaotic behaviour, Field Controlled Induction Motor, Hopf Bifurcation, Chaos Control.*

I. Introduction

The general perception on chaos is equivalent to disorder or complete randomness. It should be noted that chaos is not exactly disordered, and its random-like behaviour is mathematically governed by a deterministic model or equations that contain no element of chance [1]. The advances in understanding and analyzing nonlinear systems have shown that the phenomenon of typical chaotic behavior may be attributed to certain systematic rules rather than linking it to arbitrariness. The intriguing aspect of chaotic systems is that even though their description can be made by following deterministic models, their sensitiveness to small perturbation in initial conditions renders them almost impossible to accurately predict their future behavior [2]. Along with that, plot of the trajectories of these systems in phase plane exhibit their important trait of non-settling to a particular equilibrium point or to a periodic orbit, but still remaining structurally stable while exhibiting strange attractors. Chaotic systems find reference in variety of fields like chemical systems, physical systems, electronic systems, electro-mechanical systems, biological systems, economical systems etc. Main thrust in all these areas is to identify chaotic behaviour, control and suppression of such behaviour and to utilize the benefits of this aspect in certain application fields like secure communication, encryption/decryption of information, pattern formation, chemical mixing etc. Various nonlinear control techniques are utilized to achieve all these objectives [3]-[12].

The investigation of chaotic behaviour in electric drives is an area of open interest due to direct applications of these drives in many areas, such as, industrial processes, electrical locomotives etc.[13]-[18]. For instance, feedback based linear and nonlinear controllers were proposed to control and synchronize the systems with chaotic dynamics by Rafikov et al. [19]. A global feedback control methodology utilizing measurement of the maximum amplitude of associated pattern in the domain of interest, which leads to stability of the system, was proposed by Golovin et al. [20]. The presence of chaos in an induction motor was investigated and a sliding mode technique (SMT) based controller was proposed by D. Chen et al. [21]. In this work, indirect field control based nonlinear dynamical model of induction motor drive was presented and the typical nonlinear behavior of the system model was analysed using bifurcation diagrams, phase portraits etc. Moreover, a sliding mode control method was also presented.

In the present work, chaotic behaviour of indirect field controlled induction motor is analysed to explore possibility of chaos with parametric variations. The proposed approach uses the induction motor model presented in [21]. The core approach is based on computation of the Lyapunov exponents (LE) as a measure of quantifying chaos and thus identifying the region of chaotic behaviour. These Lyapunov exponents are one of the fundamental attributes associated with chaos in the literature [22]-[28]. These measures are evaluated by following different methodologies developed extensively over the period of time. Some of these approaches can be found in Geist et al. [22] where an early survey is presented. Wolf et al. [23] proposed an approach for calculating these measures from experimental data obtained from measurement. The Lyapunov exponents are predominantly used to characterize different behaviours exhibited by nonlinear systems. The number of Lyapunov exponents for a given dynamical system is equal to the number of associated state variables. These Lyapunov exponents may turn out to be positive, negative, or zero. Using Lyapunov exponent plots, we investigate the chaotic nature of the system and then by parameter variation, effort is made to either eliminate the system chaos or chaotify the system. For this purpose, impact of varying rotor inductance is considered while considering the load torque to be fixed. Further, bifurcation analysis is presented to identify the range of parameters for which the induction motor exhibit chaotic behaviour. Detailed simulations are presented in the end to highlight the efficacy of proposed methodology.

The paper is organised as follows. In Section 2, the nonlinear model of induction motor drive with current control, in a reference frame assumed to be rotating at synchronous speed, is discussed. Section 3 presents the method used to compute the Lyapunov exponents. Section 4 discusses the presence of chaos and its elimination from the system using Lyapunov plots. Section 5 discusses the chaotification of the system by parameter variation and this is verified using Phase plots, Hopf bifurcations and Lyapunov exponent plots. Finally, concluding remarks regarding the proposed work are presented in Section 6.

II. Dynamic Model of the Induction Motor

The model of a current-driven induction machine (developed by W. Leonhard in 1996) used for the analysis in [21] is reproduced here. This model is expressed in a reference frame which is assumed to be revolving at synchronous speed. The associated mathematical dynamic model is given as follows:

$$\begin{aligned}\dot{\psi}_{qr} &= -\frac{R_r}{L_r}\psi_{qr} - \omega_{sl}\psi_{dr} + \frac{L_m}{L_r}R_r i_{qs} \\ \dot{\psi}_{dr} &= -\frac{R_r}{L_r}\psi_{dr} - \omega_{sl}\psi_{qr} + \frac{L_m}{L_r}R_r i_{ds} \\ \dot{\omega}_r &= -\frac{R_\omega}{J}\omega_r + \frac{1}{J}\left[\frac{3}{2}\frac{L_m}{L_r}n_p(i_{qs}\psi_{dr} - i_{ds}\psi_{qr}) - T_L\right]\end{aligned} \quad (1)$$

where R_r is rotor resistance, L_r is rotor self-inductance, L_m is mutual inductance in a rotating reference frame, n_p is the number of pole pairs, ω_{sl} is slip frequency, ω_r is angular speed of the rotor, J is inertia coefficient, T_L is load, ψ_{qr} is rotor flux component associated with quadrature axis, ψ_{dr} is direct axis component of the rotor flux. The parameters of motor model presented in (1) are substituted with constants defined as below:

$$c_1 = \frac{R_r}{L_r}, c_2 = \frac{L_m}{L_r}R_r, c_3 = \frac{R_\omega}{J}, c_4 = \frac{1}{J}, c_5 = \frac{3}{2}\frac{L_m}{L_r}n_p \quad (2)$$

The state variables and control inputs are defined as:

$$\boldsymbol{x_1} = \psi_{qr}, \boldsymbol{x_2} = \psi_{dr}, \boldsymbol{u_1} = \omega_{sl}, \boldsymbol{u_2} = i_{ds}, \boldsymbol{u_3} = i_{qs} \quad (3)$$

Therefore, the dynamics of induction motor drive with indirect field control, in state space form, can be rewritten as follows:

$$\begin{aligned}\dot{x_1} &= -c_1x_1 - u_1x_2 + c_2u_3 \\ \dot{x_2} &= -c_1x_2 + u_1x_1 + c_2u_2 \\ \dot{\omega_r} &= -c_3\omega_r + c_4[c_5(x_2u_3 - x_1u_2) - T_L]\end{aligned} \quad (4)$$

In industrial applications associated with regulating the speed of motor, the indirect field oriented control is usually used. This scheme employs a proportional plus integral (PI) speed loop. This control scheme is described as follows:

$$\begin{aligned}u_1 &= \widehat{c_1}\frac{u_3}{u_2} \\ u_2 &= u_2^0 \\ u_3 &= K_p(\omega_{ref} - \omega_r) + K_i\int_0^t \omega_{ref}(\zeta) - \omega_r(\zeta))d\zeta\end{aligned} \quad (5)$$

where $\widehat{c_1}$ is the estimate for the inverse rotor time constant c_1, ω_{ref} is the constant reference velocity, u_2^0 is the constant reference for the rotor flux magnitude, K_p is the proportional gain and K_i is the gain associated with integral part of PI speed regulating element.

If $\widehat{c_1}$ = c_1 i.e. $\widehat{c_1}$ is a correct estimation of time constant of rotor, then control is said to be tuned. Otherwise, control is termed to be detuned. In such case, degree of tuning is taken as $k = \widehat{c_1}/c_1$. Obviously, the controller is tuned and one sets $k = 1$. Let us assume $x_3 = \omega_{ref} - \omega_r$ and $x_4 = u_3$. The above selections lead to a new fourth dimensional model, which is derived from the model of close loop system represented by equation (4) and the corresponding control strategy (5). The new model can be written as follows:

$$\begin{aligned}\dot{x_1} &= -c_1x_1 + c_2x_4 - \frac{kc_1}{u_2^0}x_2x_4 \\ \dot{x_2} &= -c_1x_2 + c_2u_2^0 + \frac{kc_1}{u_2^0}x_1x_4 \\ \dot{x_3} &= -c_3x_3 - c_4\left[c_5(x_2x_4 - x_1u_2^0) - T_L - \frac{c_3}{c_4}\omega_{ref}\right] \\ \dot{x_4} &= (k_i - k_pc_3) - k_pc_4\left[c_5(x_2x_4 - x_1u_2^0) - T_L - \frac{c_3}{c_4}\omega_{ref}\right]\end{aligned} \quad (6)$$

This model is used for chaotic behaviour analysis in subsequent sections.

III. Computation of the Lyapunov Exponents (LEs)

In case of chaotic systems, Lyapunov exponents are considered as average exponentially convering or diverging rates of nearby trajectories of a given system in phase space. The qualitative behavior of the systems dynamics can be linked to the signs of the Lyapunov exponents. One dimensional systems have a single characteristic Lyapunov exponent which is positive for chaotic regime, zero for a marginally stable orbit, and negative for a periodic attractor. Lyapunov exponents as measure of chaotic nature have been utilized to analyze nonlinear phenomena which are localized in time and/or space. For example, these measures have been used in analyzing fluid mixing along the mixing chamber's border as highlighted in [29]. The methodology used for the computation of the LEs was motivated by the approach proposed by Bennetin et al [30] and the work highlighted by Shimada et al. [31] for obtaining a complete spectrum from a given differential description. Overall, Lyapunov exponent gives us the average rate of separation of two adjacent orbits of the dynamical system which are very close to each other at time t = 0. Quantitatively, the rate of divergence of two nearby trajectories in phase space with small separation in initial conditions is given as

$$|\delta \boldsymbol{x}(t)| = e^{\lambda t}|\delta \boldsymbol{x}_o| \tag{7}$$

where λ is the Lyapunov exponent.

The above rate of divergence can be different for different orientations of initial separation vector. Thus, the spectra of Lyapunov exponents exists with number of LEs equal to the dimension of the associated phase space. The largest Lyapunov exponent is commonly referred as the Maximal Lyapunov exponent (MLE), because it determines a notion of predictability for a given system. A positive MLE is a indicator of underlying chaotic behaviour of the system.

In case of nonlinear system with evolution dynamics $\boldsymbol{f}(\boldsymbol{x})$ in an n-dimensional phase space, the Lyapunov exponents measures $\lambda_1, \lambda_2, ..., \lambda_n$ generally are dependent on initial condition $\boldsymbol{x}_o$. These Lyapunov exponents characterize the behaviour of state trajectories in the tangent space of phase space. These are described in terms of Jacobian matrix computed as follows:

$$Q(\boldsymbol{x}_o) = \left[\frac{d\boldsymbol{f}(\boldsymbol{x})}{d\boldsymbol{x}_o}\right]_{x=x_o} \tag{8}$$

The Q matrix describes how a small perturbation in the initial point x_o propagates in the long run to final point $\boldsymbol{f}(\boldsymbol{x_o})$.The limit

$$\lim_{t\to\infty}(Q.Q^T)^{1/2t} \tag{9}$$

defines a matrix $L(\boldsymbol{x}_o)$. If $\Lambda_i(\boldsymbol{x}_o)$ denotes the eigenvalues of $L(\boldsymbol{x}_o)$, then the LEs are expressed as follows [32]:

$$\lambda_i(\boldsymbol{x}_o) = log\Lambda_i(\boldsymbol{x}_o) \tag{10}$$

IV. Analysis of Chaos in the System

In the work presented in [21] the authors have emphasized the presence of chaos using phase portraits, bifurcation diagrams and Poincare maps. Lyapunov exponents, which may efficiently indicate whether a system is chaotic or not, were not used. We first present the Lyapunov exponents plot depicting the systems chaotic nature. To simulate the system of equations obtained in (6) the parameters are chosen as, $c_1 = 13.67, c_2 = 1.56, c_3 = 0.59, c_4 = 1176, c_5 = 2.86, u_0^2 = 4, k_p = 0.001, k_i = 1, k = 1.5, \omega_{ref} = 181.1$ with the load torque $T_L = 0.5$ and the initial states set to $x_1 = 0, x_2 = 0.4, x_3 = -200, x_4 = 6$. The simulations and results are shown in Fig. (1). From Fig. (1), it is clearly

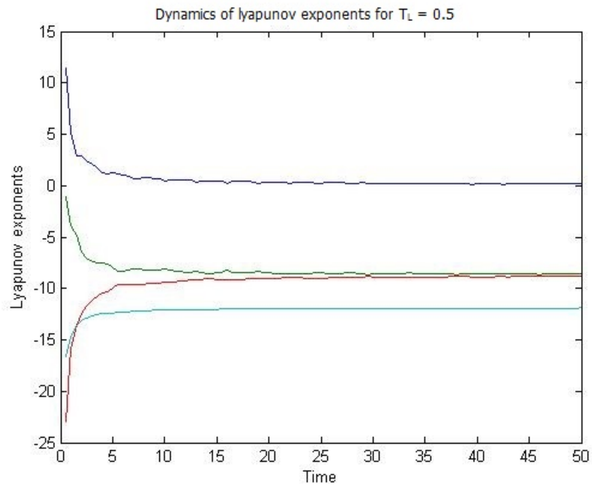

Fig. 1: Dynamics of Lyapunov exponents at $T_L = 0.5$.

concluded that one of the Lyapunov exponent measure is positive in the given range, irrespective of the time, which implies that the trajectories are diverging continuously with time. It indicates that the system is behaving in a chaotic manner for very small loads. When the load is increased to a higher value ($> T_L = 8.5$), then as indicated by the Lyapunov exponent plot in Fig. (2) all the values are negative depicting nonchaotic behaviour.

From Fig. (2) we observe that by increasing the motor load significantly all the Lyapunov exponents turn out to be negative thereby concluding that system chaos can be affected by increment in the load.

V. Chaotification of the System using Parameter Variation

In this section, the chaotic behaviour onset is explored for the induction motor by using parameter variation. To induce chaos, the Lyapunov exponents of the closed loop system should evolve in such a way that at least one of these Lyapunov exponent measures remain positive.

It is observed that if the value of rotor inductance is decreased (which may happen due to a decrease in the number of turns in case a short circuit occurs or with the increase in

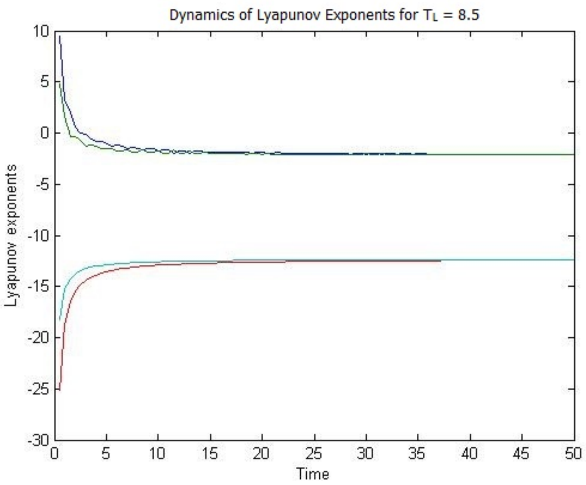

Fig. 2: Dynamics of Lyapunov exponents at $T_L = 8.5$.

reluctance) the system behaves chaotically irrespective of the load T_L. To decrease the rotor inductance (L_r), parameters c_1, c_2, c_5 must change as L_r is common in all of them. If the modified rotor inductance is $L_r^{'}$, the new set of parameters will be:

$$c_1 = \frac{R_r}{L_r^{\prime}}, c_2 = \frac{L_m}{L_r^{\prime}} R_r, c_3 = \frac{R_\omega}{J}, c_4 = \frac{1}{J}, c_5 = \frac{3}{2}\frac{L_m}{L_r^{\prime}} n_p \quad (11)$$

Let us define a new ratio $r = L_r/L_r^{'}$. To decrease the rotor inductance ratio r is set as 10 and the modified set of parameters used for analysis becomes $c_1 = 136.7, c_2 = 15.6, c_3 = 0.59, c_4 = 1176, c_5 = 28.6, u_0^2 = 4, k_p = 0.001, k_i = 1, k = 1.5, \omega_{ref} = 181.1$ with the load torque $T_L = 8.5$ and the initial states set to $x_1 = 0, x_2 = 0.4, x_3 = -200, x_4 = 6$.

The phase portraits obtained by simulating the system (6) using the parameters given above are shown in the figures (3)-(6):

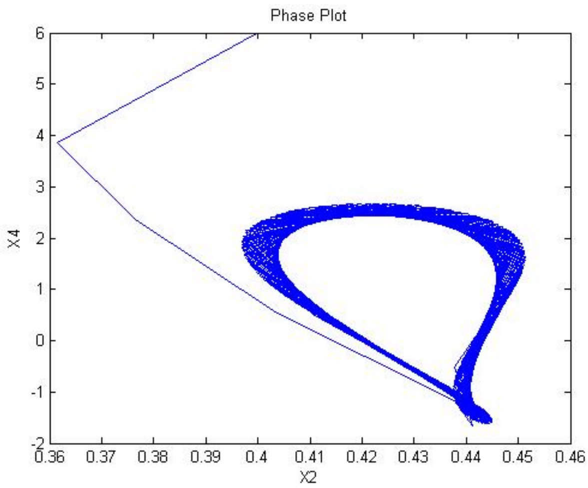

Fig. 3: 2-dimensional phase portrait between x_2 and x_4.

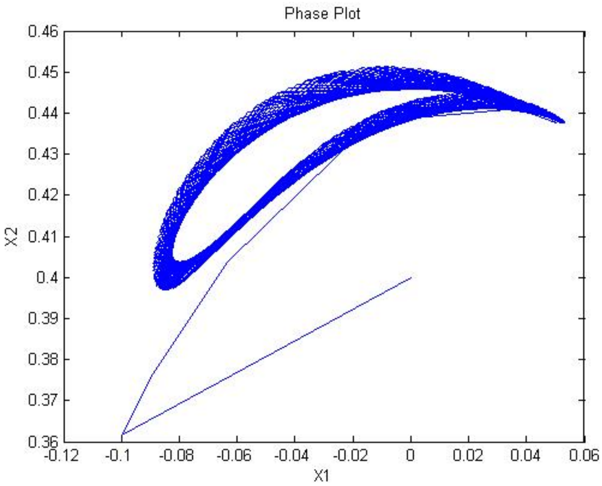

Fig. 4: 2-dimensional phase portrait between x_1 and x_2.

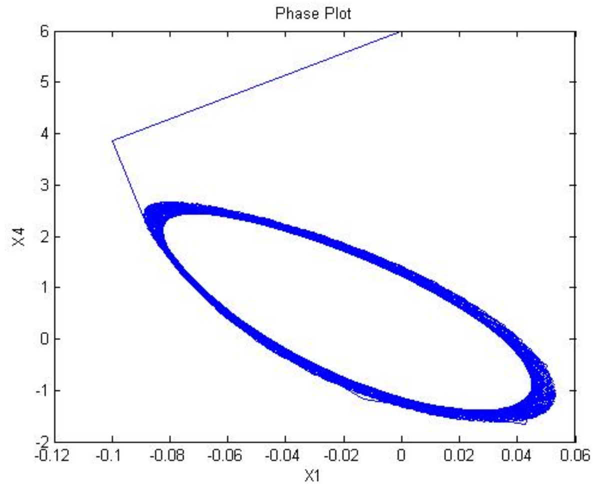

Fig. 5: 2-dimensional phase portrait between x_1 and x_4.

From figures (3)-(6), we clearly infer that the system exhibits chaotic nature because the response settles in an attractor, which is a distinctive feature of chaotic systems. The analysis presented here considers the parametric variations arising from assuming rotor inductance as variable with load torque as fixed whereas the chaotic behaviour of field oriented control based induction motor model for variable load and by varying parameters of PI controller can be found in [13]. To justify the chaotic nature of the system as investigated above, using phase portraits, we now try to present the analysis using Hopf Bifurcations.

Hopf bifurcation gives us idea about the appearance or disappearance of the periodic orbits through a local change in the stability properties of a steady point. Thus it serves as an essential tool for observing nonlinear dynamic response. A distinctive feature of Hopf bifurcations is that the system starts behaving unpredictably, thereby losing its periodicity, for those values where the system is chaotic.

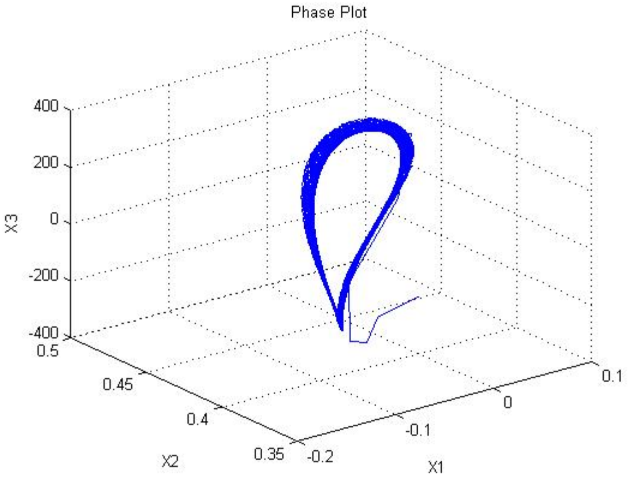

Fig. 6: 3-dimensional phase portrait between x_1 and x_2 and x_3.

The bifurcation plots in figures (7)-(10) are plotted for varying ratio of rotor inductance w.r.t. state variables. From figures (7)-(10) we observe that the system loses its periodicity and shows chaotic behaviour whenever the rotor inductance is decreased by the ratio lying within $\{5, 15\}$. This implies that the system is not chaotic when ratio is higher than 15.

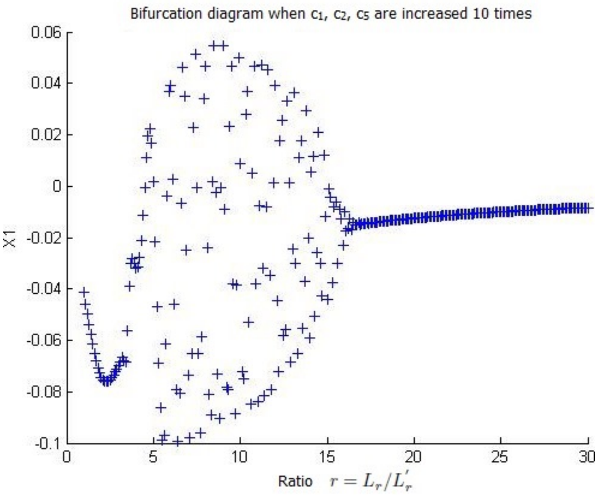

Fig. 7: Hopf bifurcation for state x_1 v/s r with c_1, c_2, c_5 increased 10 times.

The analysis is further extended by observing the Lyapunov exponents plot for the system with new set of parameters and we can see from Fig. (11) that one of the exponents is always positive proving that by parameter variation chaos can creep into a non-chaotic system. To justify the inexistence of chaos whenever the rotor inductance is decreased by a ratio higher than 15, we present a Lyapunov exponent plot in Fig. (12) which clearly shows that system is not chaotic.

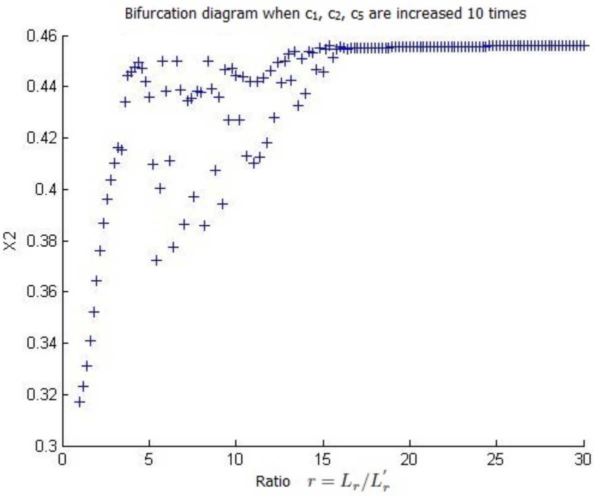

Fig. 8: Hopf bifurcation for state x_2 v/s r with c_1, c_2, c_5 increased 10 times.

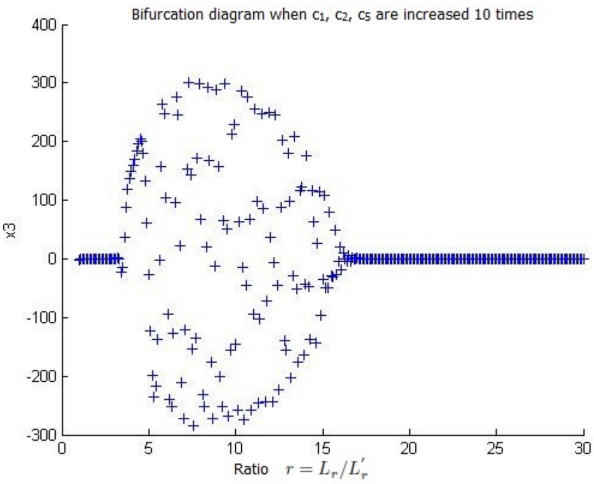

Fig. 9: Hopf bifurcation for state x_3 v/s r with c_1, c_2, c_5 increased 10 times.

VI. Conclusion

This paper presents analysis of chaos in an indirect field controlled induction motor using Lyapunov exponents and Hopf bifurcation methodology. The chaotification due to parameter variation indicates that chaos can again creep in the motor if proper steps are not ensured to prevent change in parameters. Several simulations were presented from which it can be concluded that due to change in the internal parameters of an induction motor chaos can reappear, which may not be desirable for the operation of motor. The further work in this direction could the development of a control algorithm to maintain the system chaos-free even during parametric variation.

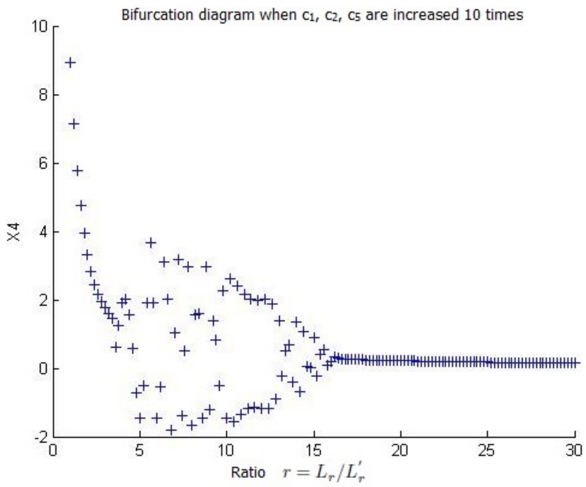

Fig. 10: Hopf bifurcation for state x_4 v/s r with c_1, c_2, c_5 increased 10 times.

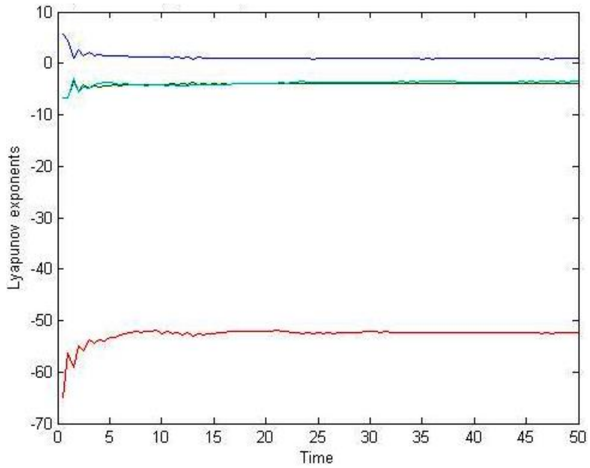

Fig. 11: Variation of Lyapunov exponents with time at $T_L = 8.5$ and ratio $r = 10$.

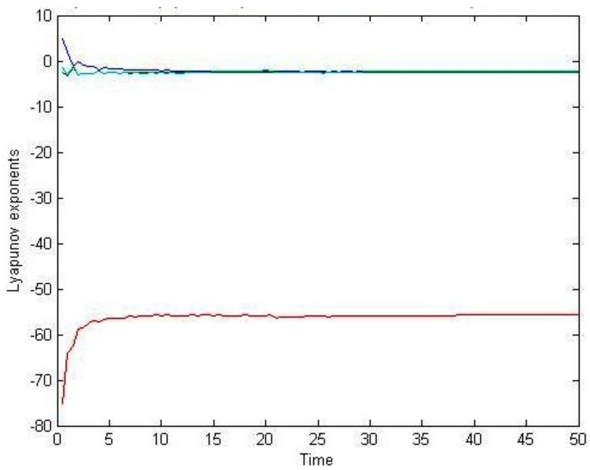

Fig. 12: Variation of Lyapunov exponents with time at $T_L = 8.5$ and ratio $r > 15$.

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