Constrained Consumable Resource Allocation in GERT-type Networks with EXCLUSIVE-OR, PROBABILISTIC Nodes

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Abstract—*This research discusses constrained consumable resource allocation problem in special kinds of GERT-type networks (GTN). The Networks have only EXCLUSIVE-OR, PROBABILISTIC nodes. They don't have any loops. They have one source node and they can have more than one target nodes. Also, durations of activities are random variables with known probability density functions. A certain type of consumable resource is necessary for executing the activities. A new heuristic algorithm has been developed to achieve this purpose. Finally, three examples have been solved using the proposed method.*

Keywords—*constrained consumable resource allocation, GERT-type networks, Exclusive-or probabilistic.*

I. Introduction

In real world projects, the occurrence of activities and their durations are stochastic. This is why the real projects are formulated as stochastic networks [1]. Some of these projects are completed through the duration of one and only one path out of the various possible network paths. When the realization of activities is stochastic, activity times may be stochastic [2] or fuzzy [3]. Some of the researchers have studied the constrained resource allocation in alternative stochastic networks when the resource is renewable [4]-[5]-[6], and some others have studied the constrained consumable (non-renewable) resource allocation in alternative stochastic networks [7]-[8]-[9]. Constrained consumable (non-renewable) resource allocation has been studied in alternative stochastic networks with fuzzy times [10].

This paper studies the constrained consumable (non-renewable) resource allocation in GERT-type Networks. The new heuristic algorithm has been proposed by solving the 3 examples; we have shown that the proposed method works better than the method introduced in [7].

II. The problem

In GERT's classical calculation, it is assumed that there aren't any constraints related to resources. But in real word, we have constrained resources. So, here the main purpose is to constrain consumable resource allocation to GERT networks. In order for this to happen a new heuristic algorithm has been developed.

A. Assumptions

The studied GNT has the following characteristics:

- The network has a single start node and it can have one or more end nodes.
- The network contains only special kinds of GERT-type network nodes (nodes with exclusive-or receiver and exclusive-or emitter).
- The network does not contain any loops.
- To execute the activities, a consumable resource is needed (non-renewable).
- The amount of available resources is finite and defined.
- Resource allocation to activities is done discretely. In other words, the value of the resource allocated to each activity is limited to some specific levels.
- The time of the network activities is an arbitrary continuous random variable or it can be a constant value.
- Probability density function of activity durations is dependent on the amount of resource allocated to the activity and varies as this amount changes. By increasing the allocated resource to each activity, completion time of the activity gets shorter.

- **Notations**

Here, many of new algorithm notations are similar to the previous algorithm [7].

N Number of activities

R Number of paths

P_{ij} Occurrence probability of j-th path which terminates in i-th sink node

$\overline{P}_k$ Accomplishment probability of k-th activity, given that start node of this activity has already occurred

S_{ij} Activity set of j-th path which terminates in i-th sink node
t_k Duration time random variable of k-th activity
S_{l_k} The amount of resource allocated to k-th activity
$\bar{t}_k(S_{l_k})$ Average of duration time of the random variable of k-th activity when the amount allocated to k-th activity is S_{l_k}
RS The available amount of limited resource
$f_{t_k}(S_{l_k}, l_k)$ Probability density function of k-th activity completion time, when the allocated resource is S_{l_k}
H We sort the W_r values, the set of the first three Indices in this order is H.
$\bar{T}_b$ The average of completion time of the project after the reduction of the allocated resource to the activity b∈H.

B. Paths Matrix

Structure of the network can be shown by a matrix *M*. *M* is a matrix with *R* rows and *N* columns. Elements of matrix *M* are shown by m_{ijk} , $k = 1, 2, \dots, N$ and it is defined as $m_{ijk} = 1$ if $K \in S_{ij}$ otherwise $m_{ijk} = 0$.

C. Improved algorithm steps

Steps of the improved algorithm are as follows:

Step-1: Allocate the maximum value for activities $\forall k, S_{l_k}$.
Step-2: If $\forall i, j, \sum_{k \in s_{ij}} S_{l_k} \le RS$ then stop, otherwise go to step 3.
Step-3: Determine the

$$P_{uv} = \min_{ij} \left\{ P_{ij} \middle| \sum_{k' \in S_{ij}} S_{l_{k'}} > RS \right\}$$

such that $P_{ij} = \prod_{k \in S_{ij}} \bar{P}_k$
Step-4: For all activities of S_{uv} compute The

$$W_r = \left\{ \sum_{ij, k \in S_{uv}} \left(P_{ij} \sum_{k' \in S_{uv}} \bar{T}_{k'}(S_{l_{k'}}) \right) m_{ijk} \right\} \quad (a)$$

then arrange the values of W_r in an ascending order. Consider the first three values of the ordered list; the set of the first three Indices in this order is H. If there are the same numbers in the ordered list of set H, the number of members of the set H may be four or more.
Step-5: Calculates the average completion times of the project by reducing the allocated resources to each activity of H. Suppose that $\bar{T}_b$ is the average completion time of the project after the reduction of the allocated resources of the activity b∈H. Then we have:

$$\bar{T}_k = \min_{b \in H} \{\bar{T}_b\}$$

So activity k is selected to reduce the allocated resources. Then go back to the step 2.

III. EXAMPLES

A. Example 1

Consider the network Fig. 1. Suppose that the available resource is RS and the RS can range from 13 to 17. For each of these values, resource must be allocated to network activities.

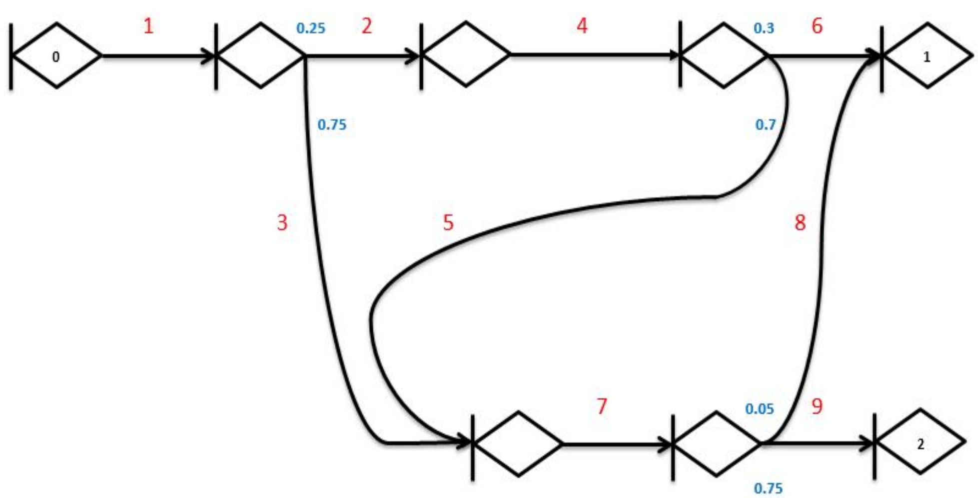

Fig 1: The network of example 1

Table 1 contains the information of the activity times.

Table 1. Information of the activity times of example 1

Activity	L_i	S_{Li}	$F_i(S_{Li},t_i)$	period	Activity	L_i	S_{Li}	$F_i(S_{Li},t_i)$	period
1	1	4	$4t^3$	$0 < t < 1$	2	1	1	$\frac{1}{2}e^{-\frac{1}{2}t}$	$t > 0$
	2	5	$4(1-t)^3$	$0 < t < 1$		2	2	e^{-t}	$t > 0$
3	1	5	$\frac{1}{2}e^{-\frac{1}{2}t}$	$t > 0$	6	1	5	1	$2 < t < 3$
	2	6	e^{-t}	$t > 0$		2	6	$\frac{1}{2}$	$1 < t < 3$
8	1	1	$3t^2$	$0 < t < 1$	9	1	1	$\frac{1}{3}$	$1 < t < 4$
	2	2	$3(1-t)^2$	$0 < t < 1$		2	2	$\frac{2}{3}$	$1 < t < 2.5$

Activity	L_i	S_{Li}	$t_{i,S_{li}}$	Activity	L_i	S_{Li}	$t_{i,S_{li}}$	Activity	L_i	S_{Li}	$t_{i,S_{li}}$
4	1	1	3	7	1	3	3	5	1	1	2
	2	2	2		2	4	2		2	2	1

The example has been solved with both methods (the method presented in the article [7] and the new proposed method). Then results are compared.

Table 2. shows the resources allocated to activities based on the previous algorithm and the proposed algorithm for example 1.

Table 2. Allocated resource based on two algorithm

RS	Allocated Resource in Previous Algorithm	Allocated Resource in proposed Algorithm
13	(4,1,5,1,1,6,3,1,1)	(4,1,5,1,1,6,3,1,1)
14	(4,1,5,1,1,6,3,1,2)	(4,1,5,1,1,6,3,1,2)
15	(5,1,5,1,1,6,3,1,2)	(**4**,1,5,1,1,6,**4**,1,2)
16	(5,1,5,1,1,6,4,1,2)	(**4**,1,**6**,1,1,6,4,1,2)
17	(5,2,6,2,2,6,4,2,2)	(5,2,6,2,2,6,4,2,2)

Table 3. shows the expected value of completion time of the network based on the previous algorithm and the proposed algorithm for example 1.

Table 3. Expected value of completion time of project based on two algorithm

RS	T(RS) in Previous Algorithm	T(RS) in Proposed Algorithm
13	9.0565	9.0565
14	8.3975	8.3975
15	7.7975	7.4725
16	6.8725	6.7225
17	5.4243	5.4243

B. Example 2

Consider the network Fig. 2. Suppose that the available resource is RS and the RS can range from 19 to 27. For each of these values, resource must be allocated to network activities.

Table 4 contains the information of the activity times.

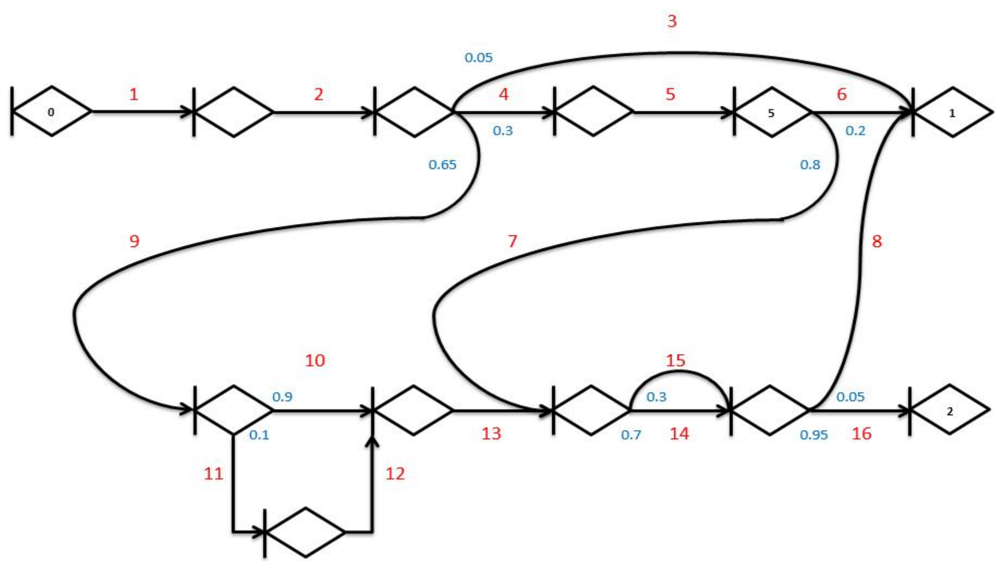

Fig 2: The network of example 2

Table 4. Information of the activity times of example 2

Activity	L_i	S_{Li}	$F_i(S_{Li},t_i)$	period	Activity	L_i	S_{Li}	$F_i(S_{Li},t_i)$	period
1	1	5	$4t^3$	$0<t<1$	2	1	4	$\frac{1}{2}e^{-\frac{1}{2}t}$	$t>0$
	2	6	$4(1-t)^3$	$0<t<1$		2	5	e^{-t}	$t>0$
3	1	3	$\frac{1}{2}e^{-\frac{1}{2}t}$	$t>0$	6	1	2	1	$2<t<3$
	2	4	e^{-t}	$t>0$		2	3	$\frac{1}{2}$	$1<t<3$
8	1	2	$3t^2$	$0<t<1$	9	1	2	$\frac{1}{3}$	$1<t<4$
	2	3	$3(1-t)^2$	$0<t<1$		2	3	$\frac{2}{3}$	$1<t<2.5$
10	1	1	$\frac{1}{2}$	$1<t<3$	11	1	1	$\frac{1}{3}e^{-\frac{1}{3}t}$	$t>0$
	2	2	$\frac{2}{3}$	$1<t<2.5$		2	2	$\frac{1}{2}e^{-\frac{1}{2}t}$	$t>0$
12	1	1	$\frac{1}{4}$	$1<t<5$	13	1	2	$\frac{1}{4}e^{-\frac{1}{4}t}$	$t>0$
	2	2	$\frac{1}{3}$	$1<t<4$		2	3	$\frac{1}{3}e^{-\frac{1}{3}t}$	$t>0$
14	1	2	1	$3<t<4$	16	1	1	$5t^4$	$0<t<1$
	2	3	$\frac{2}{5}$	$1<t<3.5$		2	2	$5(1-t)^4$	$0<t<1$
Activity	L_i	S_{Li}	$t_{i,}\ s_{li}$		Activity	L_i	S_{Li}	$t_{i,}\ s_{li}$	
4	1	4	3		5	1	1	1.5	
	2	5	2			2	2	1	
7	1	1	3		15	1	1	5	
	2	2	2			2	2	4	

The example has been solved with both methods (the method presented in [7] and the new proposed method) .Then results were compared.

Table 5. shows the resources allocated to activities based on the previous algorithm and the proposed algorithm for example 2.

Table 5. Allocated resource based on two algorithm

RS	Allocated Resource in Previous Algorithm	Allocated Resource in proposed Algorithm
19	(5,4,4,4,1,3,1,2,2,2,1,1,2,2,1,2)	(5,4,4,4,1,3,1,2,2,2,1,1,2,2,1,2)
20	(5,4,4,4,2,3,1,2,2,2,1,1,2,2,1,2)	(5,4,4,5,1,3,1,2,2,2,1,1,2,2,1,2)
21	(5,4,4,5,2,3,1,2,2,2,1,1,2,2,1,2)	(5,4,4,5,1,3,2,2,2,2,1,1,2,2,1,2)
22	(5,4,4,5,2,3,2,2,2,2,1,1,2,2,1,2)	(5,4,4,5,2,3,2,2,2,2,1,1,2,2,1,2)
23	(6,4,4,5,2,3,2,2,2,2,1,1,2,2,1,2)	(5,5,4,5,2,3,2,2,2,2,1,1,2,2,1,2)
24	(6,4,4,5,2,3,2,2,2,2,1,1,2,3,1,2)	(5,5,4,5,2,3,2,2,2,2,1,1,3,3,1,2)
25	(6,5,4,5,2,3,2,2,3,2,1,2,3,3,2,2)	(6,5,4,5,2,3,2,2,3,2,2,1,3,3,2,2)
26	(6,5,4,5,2,3,2,2,3,2,2,2,3,3,2,2)	(6,5,4,5,2,3,2,2,3,2,2,2,3,3,2,2)
27	(6,5,4,5,2,3,2,3,3,2,2,2,3,3,2,2)	(6,5,4,5,2,3,2,3,3,2,2,2,3,3,2,2)

Table 6. shows the expected value of completion time of network, based on Previous algorithm and the proposed algorithm for example 2.

Table 6. Expected value of completion time of project based on two algorithm

RS	T(RS) in Previous Algorithm	T(RS) in Proposed Algorithm
19	14.3629	14.3629
20	14.2129	14.0629
21	13.9129	13.8229
22	13.6729	13.6729
23	13.0729	12.6729
24	12.2941	11.2441
25	9.8571	9.8246
26	9.7921	9.7921
27	9.7699	9.7699

C. *Example 3*

Consider the network Fig. 3. Suppose that the available resource is RS and the RS can range from 20 to 32. For each of these values, resource must be allocate to network activities.

Table 7 contains the information of the activity times.

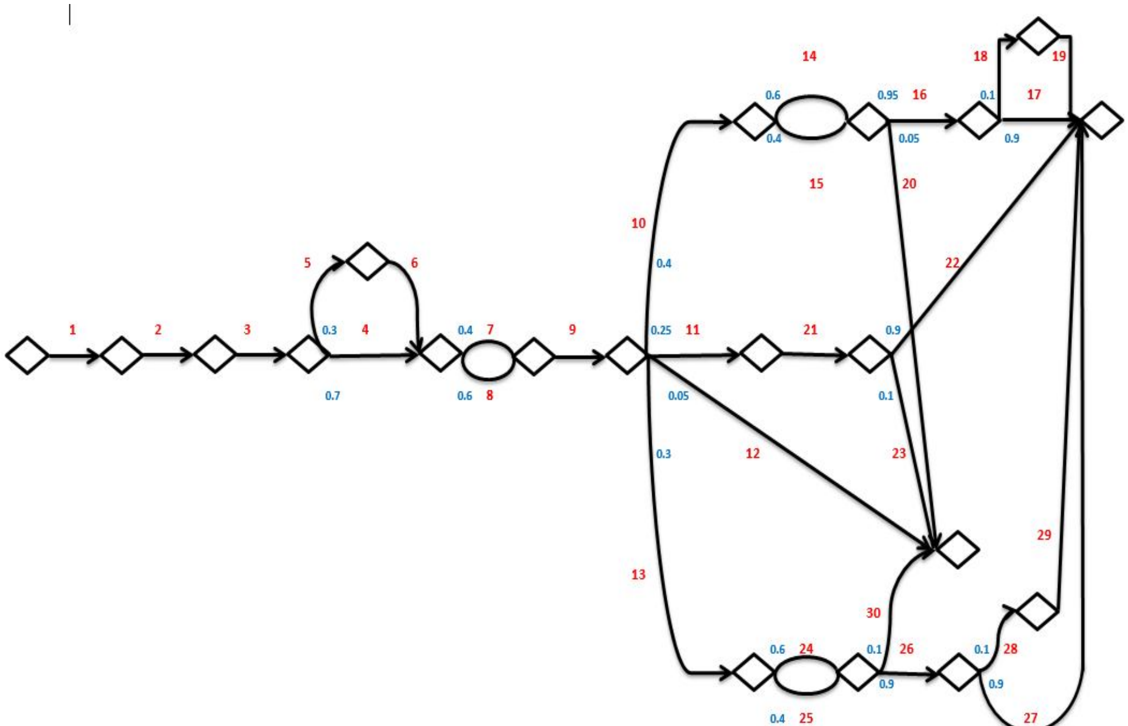

Fig 3: The network of example 3

Table 7. Information of the activity times of example 3

Activity	L_i	S_{Li}	$F_i(S_{Li},t_i)$	period	Activity	L_i	S_{Li}	$F_i(S_{Li},t_i)$	period	Activity	L_i	S_{Li}	t_i, s_{li}
2	1	2	$\frac{1}{2}e^{-\frac{1}{2}t}$	t > 0	3	1	1	$4t^3$	0 < t < 1	1	1	4	1.5
	2	3	e^{-t}	t > 0		2	2	$4(1-t)^3$	0 < t < 1		2	5	1
5	1	1	$5t^4$	0 < t < 1	7	1	2	$3t^2$	0 < t < 1	4	1	2	3
	2	2	$5(1-t)^4$	0 < t < 1		2	3	$3(1-t)^2$	0 < t < 1		2	3	2
9	1	3	$\frac{1}{3}$	1 < t < 4	10	1	1	$\frac{1}{2}e^{-\frac{1}{2}t}$	t > 0	6	1	1	5
	2	4	$\frac{2}{3}$	1 < t < 2.5		2	2	e^{-t}	t > 0		2	2	4
11	1	1	1	2 < t < 3	12	1	1	$\frac{1}{2}$	1 < t < 3	8	1	1	3
	2	2	$\frac{1}{2}$	1 < t < 3		2	2	$\frac{2}{3}$	1 < t < 2.5		2	2	2
13	1	1	$\frac{1}{3}e^{-\frac{1}{3}t}$	t > 0	14	1	2	$\frac{1}{4}$	1 < t < 5	15	1	2	3.75
	2	2	$\frac{1}{2}e^{-\frac{1}{2}t}$	t > 0		2	3	$\frac{1}{3}$	1 < t < 4		2	3	3
18	1	1	$\frac{1}{4}e^{-\frac{1}{4}t}$	t > 0	22	1	1	$\frac{1}{3}e^{-\frac{1}{3}t}$	t > 0	16	1	1	3
	2	2	$\frac{1}{3}e^{-\frac{1}{3}t}$	t > 0		2	2	$\frac{1}{2}e^{-\frac{1}{2}t}$	t > 0		2	2	2
23	1	1	$\frac{1}{3}$	1 < t < 4	24	1	1	$4t^3$	0 < t < 1	17	1	1	2.5
	2	2	$\frac{2}{3}$	1 < t < 2.5		2	2	$4(1-t)^3$	0 < t < 1		2	2	1
26	1	2	1	3 < t < 4	27	1	1	$3t^2$	0 < t < 1	19	1	1	2.5
	2	3	$\frac{2}{5}$	1 < t < 3.5		2	2	$3(1-t)^2$	0 < t < 1		2	2	2
28	1	1	$5t^4$	0 < t < 1	30	1	2	$\frac{1}{2}$	1 < t < 3	20	1	2	4
	2	2	$5(1-t)^4$	0 < t < 1		2	3	$\frac{2}{3}$	1 < t < 2.5		2	3	2
Activity	L_i	S_{Li}	t_i, s_{li}		Activity	L_i	S_{Li}	t_i, s_{li}		29	1	1	5
21	1	1	2		25	1	1	4					
	2	2	1			2	2	3			2	2	4

The example has been solved with both methods (the method presented in [7] and the new proposed method) .Then the results were compared.

Table 8. shows the resources allocated to activities based on the previous algorithm and the proposed algorithm for example 3.

Table 8. Allocated resource based on two algorithm

RS	Allocated Resource in Previous Algorithm	Allocated Resource in proposed Algorithm
20	(4,2,1,2,1,1,2,2,3,1,2,2,1,2,2, 1,2,1,1,2,2,2,2,1,1,2,2,1,1,3)	(4,2,1,2,1,1,2,2,3,1,2,2,1,2,2, 1,2,1,1,2,2,2,2,1,1,2,2,1,1,3)
21	(4,2,1,3,1,1,2,2,3,1,2,2,1,2,2, 1,2,1,1,2,2,2,2,1,1,2,2,1,1,3)	(4,2,1,3,1,1,2,2,3,1,2,2,1,2,2, 1,2,1,1,2,2,2,2,1,1,2,2,1,1,3)
22	(4,2,1,3,1,1,2,2,3,1,2,2,1,3,2, 1,2,1,1,2,2,2,2,1,2,2,2,1,1,3)	(4,2,1,3,1,1,2,2,3,1,2,2,1,3,2, 1,2,1,1,2,2,2,2,1,2,2,2,1,1,3)
23	(4,2,1,3,1,1,2,2,3,1,2,2,1,3,2, 2,2,1,1,2,2,2,2,1,2,2,2,1,1,3)	(4,2,1,3,1,1,2,2,3,1,2,2,1,3,2, 2,2,1,1,2,2,2,2,1,2,2,2,1,1,3)
24	(4,2,1,3,1,1,2,2,3,1,2,2,1,3,2, 2,2,1,2,2,2,2,2,1,2,2,2,1,1,3)	(4,2,1,3,1,1,2,2,3,1,2,2,1,3,2, 2,2,2,1,2,2,2,2,1,2,2,2,1,1,3)
25	(4,2,1,3,1,1,2,2,3,1,2,2,1,3,2, 2,2,2,2,2,2,2,2,1,2,2,2,1,1,3)	(4,2,1,3,1,1,2,2,3,1,2,2,1,3,2, 2,2,2,2,2,2,2,2,1,2,2,2,1,1,3)
26	(4,2,2,3,1,1,2,2,3,1,2,2,2,3,2, 2,2,2,2,2,2,2,2,2,2,3,2,2,1,3)	(4,3,1,3,1,1,2,2,3,1,2,2,2,3,2, 2,2,2,2,2,2,2,2,2,2,3,2,1,2,3)
27	(4,2,2,3,1,1,2,2,3,1,2,2,2,3,2, 2,2,2,2,2,2,2,2,2,2,3,2,2,2,3)	(4,3,1,3,1,1,2,2,3,1,2,2,2,3,2, 2,2,2,2,2,2,2,2,2,2,3,2,2,2,3)
28	(5,2,2,3,1,1,2,2,3,1,2,2,2,3,2, 2,2,2,2,2,2,2,2,2,2,3,2,2,2,3)	(4,3,1,3,1,1,2,2,4,1,2,2,2,3,2, 2,2,2,2,2,2,2,2,2,2,3,2,2,2,3)
29	(5,2,2,3,1,1,2,2,4,1,2,2,2,3,2, 2,2,2,2,2,2,2,2,2,2,3,2,2,2,3)	(4,3,2,3,1,1,2,2,4,1,2,2,2,3,2, 2,2,2,2,2,2,2,2,2,2,3,2,2,2,3)
30	(5,3,2,3,1,1,2,2,4,2,2,2,2,3,2, 2,2,2,2,2,2,2,2,2,2,3,2,2,2,3)	(5,3,2,3,1,2,2,2,4,2,2,2,2,3,2, 2,2,2,2,2,2,2,2,2,2,3,2,2,2,3)
31	(5,3,2,3,2,1,3,2,4,2,2,2,2,3,2, 2,2,2,2,2,2,2,2,2,2,3,2,2,2,3)	(5,3,2,3,2,2,2,2,4,2,2,2,2,3,2, 2,2,2,2,2,2,2,2,2,2,3,2,2,2,3)
32	(5,3,2,3,2,2,3,2,4,2,2,2,2,3,3, 2,2,2,2,3,2,2,2,2,2,3,2,2,2,3)	(5,3,2,3,2,2,3,2,4,2,2,2,2,3,3, 2,2,2,2,3,2,2,2,2,2,3,2,2,2,3)

Table 9. shows the expected value of completion time of the network based on the previous algorithm and the proposed algorithm for example 3.

Table 9. Expected value of completion time of project based on two algorithm

RS	T(RS) in Previous Algorithm	T(RS) in Proposed Algorithm
20	20.3209	20.3209
21	19.6209	19.6209
22	19.3209	19.3209
23	18.9409	18.9409
24	18.9219	18.9029
25	18.8839	18.8839
26	17.5563	17.1474
27	17.5293	17.1293
28	17.0293	16.3793
29	16.2793	15.7793
30	14.8793	14.5793
31	14.4783	14.3783
32	14.0383	14.0383

IV. CONCLUSIONS AND RECOMMENDATIONS

In this paper, a new algorithm has been developed for constrained resource allocation in special kinds of GERT networks. In the proposed method, there is no limitation for size of the project. The proposed algorithm for constrained consumable resource allocation in this research works better than previous algorithms. Also, in this research, cumulative distribution function of completion time of the network in due date increases.

In this research, the GERT networks have a special kinds of nodes (EXCLUSIVE-OR, PROBABILISTIC). Other nodes can be studied in future researches. The research will be useful when several types of limited resources are available. A limited resource can be of just renewable type or even a combination of various types.

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