

# Towards New Analytical Straight Line Definitions and Detection with the Hough Transform method

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**Abstract**—This paper establishes theoretically new analytical straight line definitions in using hexagons, octagons included in pixels. Octagon dual and hexagon dual lead to the detection of these new analytical straight lines with the Hough transform method.

**Keywords**—Hough Transform, Reconstruction, Analytical Straight Line

## I. INTRODUCTION

During long years, different models of digital objects have been defined in making a relation between euclidian geometry and digital ones. Many digital lines have been established through the definition of euclidian lines noticely analytical straight lines.

In image analysis, object definition and object detection are important. For instance, object definition is useful to draw lines in image simulation while object recognition is necessary in reconstruction and also in computer vision to detect object in the space. The first phase in recognition is to establish object definition. One of the first method to create digital lines was the algorithm of Bresenham line in 1965. There was also Bresenham circle algorithm.

In 1989, Reveilles in [9] proposed analytical straight line. Later, Reveilles definition has been extended to obtain naif, standard, supercover, thick and thin straight hyperplane in [5], [7].

Most of images are based on square pixel construction. For instance, voronoi diagram, Delaunay triangles and quasi-affine applications are different techniques to obtain various gridsn, particularly, irregular grids, used in image simulation. Hexagonal grid brings several advandages. In [16],[18], Techniques may convert square pixels to hexagonal ones.

Methods allows to obtain straight line detection, noticely the Hough Transform method. This method introduced by Paul Hough in 1962 is based on two spaces, an image space and a parameter space.

Standard Hough Transform associates a point  $(x, y)$  in an image space to a sinusoid curve  $p = x * \cos \theta + y * \sin \theta$  in a parameter space. The Hough Transform method has been

adapted to the recognition of discrete circles, ellipses and generalised shapes in [3], [4], [8].

In [8], Henri Maître unified the definitions of the Hough Transform method.

A survey on the Hough Transform method has been proposed by Priyanka Mukhopadhyay and others in [15].

The applications of Hough transform exist such as Mouth localization for audio-visual speech recognition [10], action recognition[11]. Programs also use OpenCV or Cimg libraries in implementing the Hough Transform method.

SERE and others in [1],[4] introduced extensions of standard Hough transform method, precisely the application of the standard Hough transform based on squares, triangles and rectangles in [14].

This paper proposes new analytical straight line definitions through virtual octagons and virtual hexagons. The computing of dual is necessary for analytical straight line recognition, precisely the dual of hexagons and octagons.

The main motivation is the interest of building speed methods to detect straight lines in an hexagonal grid or in an octogonal grid in order to shorten processing time.

The paper is organized as follows : section 2 named preliminaries concerns with the advantages of an hexagonal grid . It also defines naif, standard and supercover straight line. Section 3 analyzes the construction of naif, standard, supercover straight line. Section 4 defines new analytical straight lines based on hexagons and octagons. Section 5 focuses on their recognition with the hough transform method.

## II. PRELIMINARIES

Digital devices use square pixels. There are not really devices capturing data directly in a form of an hexagonal grid or a octogonal grid. But in a simulation purpose, these grids could be used efficiently. For instance, in 2007, Antoine Vacavant, analyzed the applications of irregular grids in digital geometry.

The advantages of an hexagonal grid presented in [16]. For

instance, hexagonal grids bring interesting properties. They consist of :

- having a higher degree of circular symmetry
- uniform fittings of pixels
- reducing complex calculation in processing
- enhancing image quality
- consistent connectivity
- having higher isotropy property

Analytical straight hyperplane is defined by :

*Definition 1:* (Analytical straight hyperplane [1], [5], [7], [13]) Let  $\mathbb{H}$  be an analytical hyperplane in dimension  $n$  noticed  $\mu < \sum_{i=1}^n (A_i x_i) < \mu + \omega$ . with the parameters  $A(A_1, A_2, \dots, A_{n-1}, A_n) \in \mathbb{R}^n$   $\mu \in \mathbb{R}$  and  $\omega \in \mathbb{R}, x_i \in \mathbb{Z}$ . then:

- $\mathbb{H}$  will be called naïf if  $\omega = \max_{1 \leq i \leq n} (|A_i|)$
- $\mathbb{H}$  will be called standard if  $\omega = \sum_{i=1}^n (|A_i|)$
- $\mathbb{H}$  will be called thin if  $\omega < \max_{1 \leq i \leq n} (|A_i|)$
- $\mathbb{H}$  will be called thick if  $\omega > \sum_{i=1}^n (|A_i|)$
- $\mathbb{H}$  will be called \*-connected if  $\max_{1 \leq i \leq n} (|A_i|) < \omega < \sum_{i=1}^n (|A_i|)$ .

Figures 2, 3 illustrates naïf, standard, straight lines.

### III. CONSTRUCTION OF ANALYTICAL STRAIGHT LINE BASED ON SQUARE PIXELS

Let us analyze analytical straight line construction noticed the standard ones. In a two dimensional space, a pixel is represented by a square.

Let  $A(x_a, y_a)$  and  $B(x_b, y_b)$  be respectively the centers of pixels, presented in figure 1. It is easy to go with the input data A and B, to establish a drawing algorithm : we need only two given points, in a two dimensional space. Let  $M(x, y)$  be a point on the segment  $[AB]$  :  $M(x, y) \in [AB]$

We have :

$$\overrightarrow{AB} \begin{pmatrix} x_b - x_a \\ y_b - y_a \end{pmatrix} \quad (1)$$

and

$$\overrightarrow{AM} \begin{pmatrix} x - x_a \\ y - y_a \end{pmatrix} \quad (2)$$

(1) and (2) lead to :

$$\det(\overrightarrow{AB}, \overrightarrow{AM}) = \begin{vmatrix} x_b - x_a & x - x_a \\ y_b - y_a & y - y_a \end{vmatrix} = 0 \quad (3)$$

That means :

$$(3) \Leftrightarrow -(x - x_a)(y_b - y_a) + (x_b - x_a)(y - y_a) = 0 \quad (4)$$

$$(4) \Leftrightarrow (y_b - y_a)x + (x_a - x_b)y - x_a(y_b - y_a) - y_a(x_a - x_b) = 0 \quad (5)$$

Let  $(\vec{v}(\frac{1}{2}, -\frac{1}{2}))$  be a vector. Consider the translation  $t$  defined by :

$$t_{\vec{v}}((AB)) = (D) \quad (6)$$

$$t_{-\vec{v}}((AB)) = (D') \quad (7)$$

The translations gives external straight lines of  $(AB)$ . Suppose that a point  $M(x, y)$  on  $(AB)$  and  $M'(x', y')$  on  $(D)$ . we'll have  $\vec{v} = \overrightarrow{MM'}$  (resp. with a point  $M'(x', y')$  on  $(D')$  we'll have  $-\vec{v} = \overrightarrow{MM'}$ ), we'll obtain :

$$t_{\vec{v}}(M) = (M') \Leftrightarrow \begin{cases} x = x' - \frac{1}{2} \\ y = y' + \frac{1}{2} \end{cases} \quad (8)$$

resp.

$$t_{-\vec{v}}(M) = (M') \Leftrightarrow \begin{cases} x = x' + \frac{1}{2} \\ y = y' - \frac{1}{2} \end{cases} \quad (9)$$

We pose in (5), by substitution :

$$U = (y_b - y_a), V = (x_a - x_b), T = -x_a(y_b - y_a) - y_a(x_a - x_b) \quad (10)$$

We obtain :

$$(5) \Leftrightarrow U.x + V.y + T = 0 \quad (11)$$

The translations in (8) and (9) applied to (11) respectively, lead to the equations of the straight lines  $(D)$  and  $(D')$  defined by :

- $(D) : U.x' + V.y' + T = \frac{U}{2} - \frac{V}{2}$ .

That means :  $U.x' + V.y' + T = -\frac{U}{2} - \frac{V}{2}$ .

$(D)$  is an increasing straight line, then  $U \leq 0$  and  $V \geq 0$ .

$$(D) : U.x' + V.y' + T = -\frac{|U| + |V|}{2} \quad (12)$$

- $(D') : U.x' + V.y' + T = -\frac{U}{2} + \frac{V}{2}$ .  $(D')$  is also an increasing line, then  $U \leq 0$  and  $V \geq 0$ . We have

$$(D') : U.x' + V.y' + T = \frac{|U| + |V|}{2} \quad (13)$$

Figure 1 gives more details on the straight lines  $(D)$  and  $(D')$ .

Figures 2, 3 present the building of naïf and standard straight lines.

All the pixels will be replaced by hexagons, octagons to obtain new analytical straight lines.

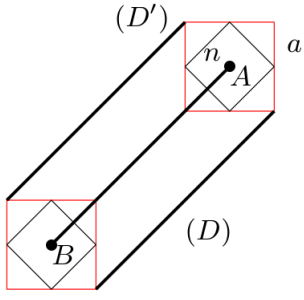


Fig. 1. A square

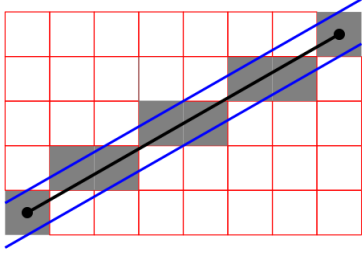


Fig. 2. Naïf :  $-\frac{\max(|4|, |7|)}{2} \leq 4x - 7y + 13.5 < \frac{\max(|4|, |7|)}{2}$

#### IV. PROPOSED ANALYTICAL STRAIGHT LINE

This section is focusing on new analytical straight lines. Here, square pixels contain hexagons or octogons. For instance Figures 4 and 5 present hexagons included in pixels.

If the angle between the straight line (AB) and the horizontal straight line  $\geq \frac{\pi}{3}$ , then the external border straight line will be the line (mk). Else if the angle between the straight line (AB) and the horizontal straight line  $\leq \frac{\pi}{3}$ , then the external border straight line will be the line (nl). Figure 4 and Figure 5 show different representations of (nl) and (mk) .

##### A. Analytical straight line passing through hexagons

This section deals with a new analytical straight line, based on virtual hexagons. Here, hexagons are crossed by a set of continue straight lines. An example of an hexagonal straight line is presented by figure 6. If an object is included in a pixel, its dual will be a subset of the dual of this pixel and the surface of cells in an accumulator to update will be reduced. That means if  $o' \subset o$  then  $\text{dual}(o') \subset \text{dual}(o)$  (in [1]).

In the figure 6, the angle between the straight line (mn) and the horizontal straight line is  $\frac{\pi}{3}$  because of the definition of an hexagon.

Let  $A(x_a, y_a)$  and  $B(x_b, y_b)$  be the center of two pixels. A and B represent the centers of hexagons included in the internal circles of pixels. Suppose that the straight line (AB) is increasing Let d be the length of a pixel side. Let  $k(x_a + \frac{d}{2}, y_a)$ ,  $l(x_a + \frac{d}{4}, y_a - \frac{d\sqrt{3}}{4})$ ,  $m(x_b + \frac{d}{2}, y_b)$   $n(x_b + \frac{d}{4}, y_b - \frac{d\sqrt{3}}{4})$  be hexagon vertices. In this case, the radius of the internal circle is  $\frac{d}{2}$ . Let  $\vec{V}(\frac{d}{4}, -\frac{d\sqrt{3}}{4})$  be a vector. Let  $\alpha$  be the angle formed by the horizontal line and the straight line (AB). Concerning the

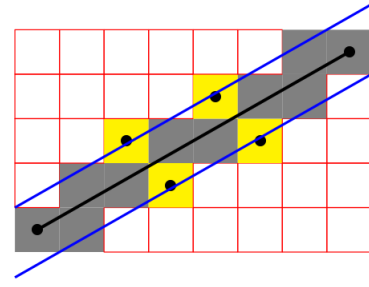


Fig. 3. Standard :  $-\frac{|4| + |7|}{2} \leq 4x - 7y + 13.5 < \frac{|4| + |7|}{2}$

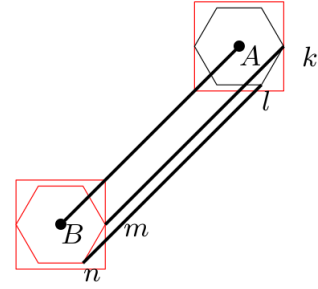


Fig. 4. external straight line (nl)

hexagon of center A, the straight line (kl) and the horizontal straight line form an angle of  $\frac{\pi}{3}$ . Let  $M(x, y)$  be a point on the segment  $[AB]$ . Let  $M'(x', y')$  be the translation of the point  $M(x, y)$  in following the vectors  $\vec{v}$  or the vector  $-\vec{v}$ . We establish analytical straight definitions, in taking into account the angle between the straight line (AB) with the horizontal straight line. Suppose that  $\vec{v} = \overrightarrow{MM'}$  (resp.  $-\vec{v} = \overrightarrow{MM'}$ ), we'll have the relations :

$$\begin{cases} x = x' - \frac{d}{4} \\ y = y' + \frac{d\sqrt{3}}{4} \end{cases} \quad (14)$$

resp.

$$\begin{cases} x = x' + \frac{d}{4} \\ y = y' - \frac{d\sqrt{3}}{4} \end{cases} \quad (15)$$

By substitution of the relations (14) in the equation  $U.x + V.y + T = 0$ , we obtain :

$$(ln) : U.x' + V.y' + T = \frac{d.U}{4} - \frac{d.V\sqrt{3}}{4}. \quad (16)$$

By substitution of the relations (15) in the equation  $U.x + V.y + T = 0$ , we have also :

$$(l'n') : U.x' + V.y' + T = -\frac{d.U}{4} + \frac{d.V\sqrt{3}}{4}. \quad (17)$$

As  $U \leq 0$  and  $V \geq 0$ , then we conclude :

$$(ln) : U.x' + V.y' + T = -(\frac{d.|U|}{4} + \frac{d.|V|\sqrt{3}}{4}) \quad (18)$$

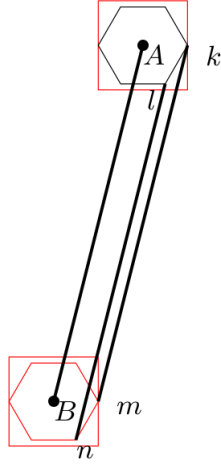


Fig. 5. external straight line (mk)

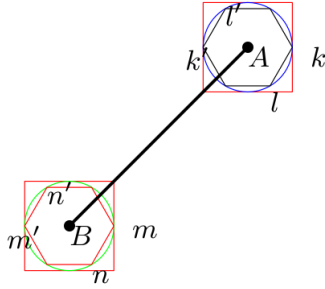


Fig. 6. Hexagons

$$(l'n') : U.x' + V.y' + T = \left( \frac{d \cdot |U|}{4} + \frac{d \cdot |V| \cdot \sqrt{3}}{4} \right) \quad (19)$$

In following the translation of the vector  $\vec{u} = \overrightarrow{MM'}$  defined by  $\vec{u}(\frac{d}{2}, 0)$ , we have also the relation

$$\begin{cases} x = x' - \frac{d}{2} \\ y = y' \end{cases} \quad (20)$$

resp.

$$\begin{cases} x = x' + \frac{d}{2} \\ y = y' \end{cases} \quad (21)$$

With (20) and (21), by substitution in the equation  $U.x + V.y + T = 0$ , by analogy as in the previous paragraphs, we'll obtain :

$$(km) : U.x' + V.y' + T = -\frac{d \cdot |U|}{4} \quad (22)$$

$$(k'm') : U.x' + V.y' + T = \frac{d \cdot |U|}{4} \quad (23)$$

As a conclusion, an increasing hexagonal straight line is defined, in considering the angle between the straight line (AB) and the horizontal straight line.

**Definition 2:** (Increasing hexagonal straight line)

Let  $A(x_a, y_a)$  and  $B(x_b, y_b)$  be respectively the center of two hexagonal pixels. We set  $U = (y_b - y_a), V = (x_a - x_b)$

and  $T = -x_a(y_b - y_a) - y_a(x_a - x_b)$ . Let  $\alpha$  be the angle created by the straight line (AB) with the horizontal straight line. An increasing hexagonal straight line is a set of points  $M(x, y) \in Z^2$  that verify :

- if  $0 \leq \alpha \leq \frac{\pi}{3}$  then  $-\left(\frac{d \cdot |U|}{4} + \frac{d \cdot |V| \cdot \sqrt{3}}{4}\right) \leq U.x + V.y + T \leq \left(\frac{d \cdot |U|}{4} + \frac{d \cdot |V| \cdot \sqrt{3}}{4}\right)$ ;
- if  $\frac{\pi}{3} \leq \alpha \leq \frac{\pi}{2}$  then  $-\frac{d \cdot |U|}{4} \leq U.x + V.y + T \leq \frac{d \cdot |U|}{4}$ .

Figure 7 shows an example of an increasing hexagonal straight line defined by  $-\left(\frac{4}{4} + \frac{7\sqrt{3}}{4}\right) \leq 4x - 7y + 13,5 \leq \left(\frac{4}{4} + \frac{7\sqrt{3}}{4}\right) = 4.03$  Here, we notice that  $d=1$ .

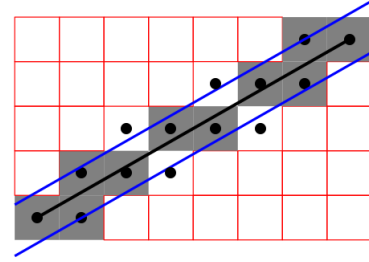


Fig. 7. hexagonal straight line:  $-4.03 \leq 4x - 7y + 13.5 \leq 4.03$

Suppose that (AB) is decreasing. The idea is to build a decreasing straight line definition, in using an increasing straight line. We consider

$$U.x + V.y + T = 0 \quad (24)$$

with  $U \leq 0$  and  $V \leq 0$ , a decreasing straight line equation.

The transformation of the y-line-symmetric relation is defined by:

$$\begin{cases} x = -x' \\ y = y' \end{cases} \quad (25)$$

By a substitution, in taking into account the relation (25) and the equation (24), we'll obtain an increasing straight line defined by :

$$-U.x' + V.y' + T = 0. \quad (26)$$

That means :

$$(26) \Leftrightarrow U.x' - V.y' - T = 0 \quad (27)$$

with  $U \leq 0$  and  $-V \geq 0$ . Then, we conclude immediately :

**Definition 3:** (decreasing hexagonal straight line) Let  $A(x_a, y_a)$  and  $B(x_b, y_b)$  be respectively the center of two hexagonal pixels. We set  $U = (y_b - y_a), V = (x_a - x_b)$  and  $T = -x_a(y_b - y_a) - y_a(x_a - x_b)$ . Let  $\alpha$  be the angle created by the straight line (AB) with horizontal straight line. A decreasing hexagonal straight line is a set of points  $M(x, y) \in Z^2$  that verify :

- if  $\frac{2\pi}{3} \leq \alpha \leq \pi$  then  $-\left(\frac{d|U|}{4} + \frac{d|V|\sqrt{3}}{4}\right) \leq U.x + V.y + T \leq \left(\frac{d|U|}{4} + \frac{d|V|\sqrt{3}}{4}\right)$ ;
- if  $\frac{\pi}{2} \leq \alpha \leq \frac{2\pi}{3}$  then  $-\frac{d|U|}{4} \leq U.x + V.y + T \leq \frac{d|U|}{4}$ .

Now, we are going to consider only octogons inside pixels. We'll replace hexagons by octogons.

### B. Analytical Straight Line passing through octogons

In this this section, we are focusing on the definition of analytical straight lines based on octogons. We consider octogons included in square pixels in order to get  $a^*$  connected analytical straight line definition.

In a two-dimensional space, we consider an octogon inside a pixel. An octogon is a closed geometric object that has eight identical sides. An irregular octogon will become a regular octogon before being a losange if we increase the length of its sides. Irregular octogons are illustrated by figure 8 (in a gray color or in a blue color).

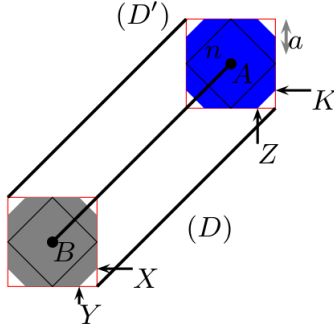


Fig. 8. A square and an octogon

**Proposition:** A square contains one octogon which has a horizontal side and a vertical side as a subset of the side of this square. ■

**Proof:** Let S be a square with c as the length of a side. Let h be an internal octogon of S, having d as the length of one side. We have  $c = a + d + a$  and  $d = \sqrt{a^2 + a^2}$ . That means  $d = a\sqrt{2}$ . Then  $c = 2a + a\sqrt{2}$  and  $a = \frac{c}{2 + \sqrt{2}}$ . We find  $d = \frac{c\sqrt{2}}{2 + \sqrt{2}}$ . For example if  $c = 1$ , we have  $d = \frac{\sqrt{2}}{2 + \sqrt{2}}$ . ■

If we increase the side length  $d = \sqrt{a^2 + a^2}$  to have  $d = \frac{c\sqrt{2}}{2 + \sqrt{2}}$  in order to obtain a regular octogon, d will be inferior to the side of the internal losange : the side n of the internal losange is given by  $n = \sqrt{2}\frac{c}{2}$  and  $d \leq n$  with  $n - d = \frac{c}{2 + \sqrt{2}}$

An octogonal straight line is a subset of  $a^*$ -connected straight line.

We consider the points A and B in figure 8. The segment [ZK] is important in the straight line definition.

Let  $\alpha, \beta$  be respectively, the angle formed by (AB) with the

horizontal straight line and the angle formed by (ZK) with the horizontal straight line. We have  $\beta = \frac{\pi}{4}$  because  $\cos(\beta) = \frac{a}{d}$ .

That means  $\cos(\beta) = \frac{\sqrt{2}}{2}$ . If  $\alpha \geq \beta$  then one computes the equation of the straight line (KX) else the equation of the straight line (ZY) is computed. We are going to define the equations of (KX) and (ZY). Let  $A(x_a, y_a)$  and  $B(x_b, y_b)$  be the centers of the octogons. Let  $\vec{u}(\frac{c}{2}, -\frac{a}{2})$  be a vector such that  $\vec{u} = \vec{AK}$  or  $\vec{u} = \vec{BX}$ . Let  $\vec{g}(\frac{d}{2}, -\frac{c}{2})$  be a vector such as  $\vec{g} = \vec{AZ}$  or  $\vec{g} = \vec{BY}$ . We obtain the equations of (KX) and (ZY) in using respectively the translation vector  $\vec{u}$  and  $\vec{g}$ . We have the translation t defined by :

$$t_{\vec{u}}(M) = M' \Leftrightarrow \begin{cases} x = x' - \frac{c}{2} \\ y = y' + \frac{a}{2} \end{cases} \quad (28)$$

and

$$t_{\vec{g}}(M) = M' \Leftrightarrow \begin{cases} x = x' - \frac{d}{2} \\ y = y' + \frac{c}{2} \end{cases} \quad (29)$$

where  $M(x, y)$  and  $M'(x', y')$ .

One knows (AB) :  $U.x + V.y + T = 0$ . As  $t_{\vec{u}}((AB)) = (KX)$  and  $t_{\vec{g}}((AB)) = (ZY)$ , we have :

- (KX) :  $U.x + V.y + T = c\frac{U}{2} - d\frac{V}{2}$ . As  $d = \frac{c\sqrt{2}}{2 + \sqrt{2}}$ . For example if  $c = 1$ , we have  $d = \frac{\sqrt{2}}{2 + \sqrt{2}}$ . Then we obtain :

$$(KX) : U.x + V.y + T = \frac{U}{2} - \frac{\sqrt{2}V}{2(2 + \sqrt{2})} \quad (30)$$

- (ZY) :  $U.x + V.y + T = \frac{dU}{2} - \frac{cV}{2}$ . As  $d = \frac{c\sqrt{2}}{2 + \sqrt{2}}$ . For example if  $c = 1$ , we have  $d = \frac{\sqrt{2}}{2 + \sqrt{2}}$ . Then we obtain :

$$(ZY) : U.x + V.y + T = \frac{\sqrt{2}U}{2(2 + \sqrt{2})} - \frac{V}{2}. \quad (31)$$

It is easy to compute the orthogonal symetric of (KX) or (ZY). We conclude that:

**Definition 4:** (increasing octogonal straight line) Let  $A(x_a, y_a)$  and  $B(x_b, y_b)$  be centers of two internal octogons in some pixels.

We set  $U = (y_b - y_a), V = (x_a - x_b)$  and  $-x_a(y_b - y_a) - y_b(x_a - x_b)$ . Let  $\alpha$  be the angle created by the straight line (AB) with the horizontal straight line. An increasing octogonal straight line is a set of points  $M(x, y) \in Z^2$  that verify :

- if  $\frac{\pi}{2} \geq \alpha \geq \frac{\pi}{4}$  then  $-\left(\frac{|U|}{2} + \frac{\sqrt{2}|V|}{2(2 + \sqrt{2})}\right) \leq U.x + V.y + T \leq \frac{|U|}{2} + \frac{\sqrt{2}|V|}{2(2 + \sqrt{2})}$
- if  $0 \leq \alpha \leq \frac{\pi}{4}$  then  $-\left(\frac{|V|}{2} + \frac{\sqrt{2}|U|}{2(2 + \sqrt{2})}\right) \leq U.x + V.y + T \leq \frac{|V|}{2} + \frac{\sqrt{2}|U|}{2(2 + \sqrt{2})}$

Figure 9 illustrates an octogonal straight line with  $\frac{|V|}{2} + \frac{\sqrt{2}|U|}{2(2+\sqrt{2})} = 4.33$  where  $U = 4, V = -7$  and  $T = 13.5$ . We see with the octogonal straight line passes through pixels which contain internal octogons.

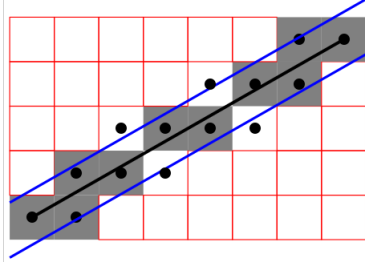


Fig. 9. Octogonal straight line:  $-4.33 \leq 4x - 7y + 13.5 \leq 4.33$

How can we define a decreasing straight line? We obtain an answer to this question, in converting a decreasing straight line into an increasing straight line by a y-line-symmetric. Suppose that  $U.x + V.y + T = 0$  is a decreasing straight line equation and  $\begin{cases} x = -x' \\ y = y' \end{cases}$ , the y-line-symmetric relation. Here, we notice that  $U \leq 0$  and  $V \leq 0$ . By substitution, we have a increasing straight line  $-U.x' + V.y' + T = 0$ . That means  $U.x' + V.y' + T = 0$  with  $U \leq 0$  and  $-V \geq 0$  we conclude that:

- if  $\frac{3\pi}{4} \geq \alpha \geq \frac{\pi}{2}$  then

$$-\left(\frac{|U|}{2} + \frac{\sqrt{2}|V|}{2(2+\sqrt{2})}\right) \leq U.x + V.y + T \leq \frac{|U|}{2} + \frac{\sqrt{2}|V|}{2(2+\sqrt{2})} \quad (32)$$

- if  $\frac{3\pi}{4} \leq \alpha \leq \pi$  then

$$-\left(\frac{|V|}{2} + \frac{\sqrt{2}|U|}{2(2+\sqrt{2})}\right) \leq U.x + V.y + T \leq \frac{|V|}{2} + \frac{\sqrt{2}|U|}{2(2+\sqrt{2})} \quad (33)$$

**Definition 5:** (decreasing octogonal straight line) Let  $A(x_a, y_a)$  and  $B(x_b, y_b)$  be centers of two internal octogons in some pixels.

We set  $U = (y_b - y_a), V(x_a - x_b)$  and  $-x_a(y_b - y_a) - y_b(x_a - x_b)$ . Let  $\alpha$  be the angle created by the straight line  $(AB)$  with the horizontal straight line. A decreasing octogonal straight line is a set of points  $M(x, y) \in Z^2$  that verify :

- if  $\frac{3\pi}{4} \geq \alpha \geq \frac{\pi}{2}$  then  $-\left(\frac{|U|}{2} + \frac{\sqrt{2}|V|}{2(2+\sqrt{2})}\right) \leq U.x + V.y + T \leq \frac{|U|}{2} + \frac{\sqrt{2}|V|}{2(2+\sqrt{2})}$

- if  $\frac{3\pi}{4} \leq \alpha \leq \pi$  then  $-\left(\frac{|V|}{2} + \frac{\sqrt{2}|U|}{2(2+\sqrt{2})}\right) \leq U.x + V.y + T \leq \frac{|V|}{2} + \frac{\sqrt{2}|U|}{2(2+\sqrt{2})}$

## V. DETECTION WITH THE HOUGH TRANSFORM METHOD

This section presents how to detect new analytical straight lines introduced in the previous section 4. In [19], SERE and others presented An extension of the Hough Transform method based on  $p(\theta) = x * \cos(\theta) + y * \sin(\theta)$ . that computes the dual of octogons and hexagons, in using an accumulator to store data.

Here we are explaining these definitions and propositions and their application in this context. As an octogon contains four rectangles while an hexagon has three rectangles inside. The definition of rectangle dual has been proposed by SERE and others in [14]. The dual of a rectangle depends on the dual of its diagonal segments. Octogon dual is defined by:

**Definition 6:** (Octogonl Dual) The dual of an octogon is the union of the dual of its four internal rectangles.

In [14] the dual of a rectangle is the union of the dual of its diagonal segments.

The following proposition facilitates the computing of the dual of an octogon.

**Proposition:** (Octogon Dual)

The dual of an octogon is the union of the dual of its four diagonal segments. ■

**Proof:** Let ABCDEFGH be an octogon and A, B, C, D, E, F, G, H its vertices. We have in ABCDEFGH four internal angles such that ABEF, BCFG, CDGH and DEHA. We also know that an octogon is the union of four rectangles.

Thus,  $\text{dual}(ABCDEFGH) = \text{dual}(ABEF) \cup \text{dual}(BCFG) \cup \text{dual}(CDGH) \cup \text{dual}(DEHA)$ .

The dual of a rectangle is the union of the dual of its diagonal segments. We have then the relation:

$$\text{dual}(ABEF) = \text{dual}(BF) \cup \text{dual}(AE)$$

$$\text{dual}(BCFG) = \text{dual}(CG) \cup \text{dual}(BF)$$

$$\text{dual}(CDGH) = \text{dual}(DH) \cup \text{dual}(CG)$$

$$\text{dual}(DEHA) = \text{dual}(EA) \cup \text{dual}(DH)$$

we conclude:

$$\text{dual}(ABCDEFGH) = \text{dual}(BF) \cup \text{dual}(AE) \cup \text{dual}(CG) \cup \text{dual}(DH)$$

$$\text{dual}(BF) \cup \text{dual}(DH) \cup \text{dual}(CG) \cup \text{dual}(EA) \cup \text{dual}(DH).$$

Finally :

$$\text{dual}(ABCDEFGH) = \text{dual}(BF) \cup \text{dual}(AE) \cup \text{dual}(CG) \cup \text{dual}(DH). \quad \blacksquare$$

We see in this proof that the dual of an octogon is reduced in the union of the dual of its two internal rectangles. Figure 10 shows the proposed octogon. Its dual is presented in Figure 11.

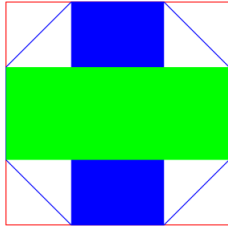


Fig. 10. An octagon

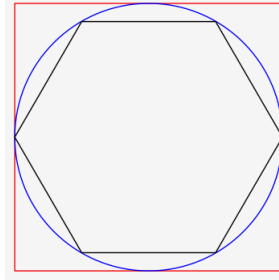


Fig. 12. An hexagon

Any straight line which crosses an octagon passes through one of these two internal rectangles (the first in a blue color and the second in a green color)

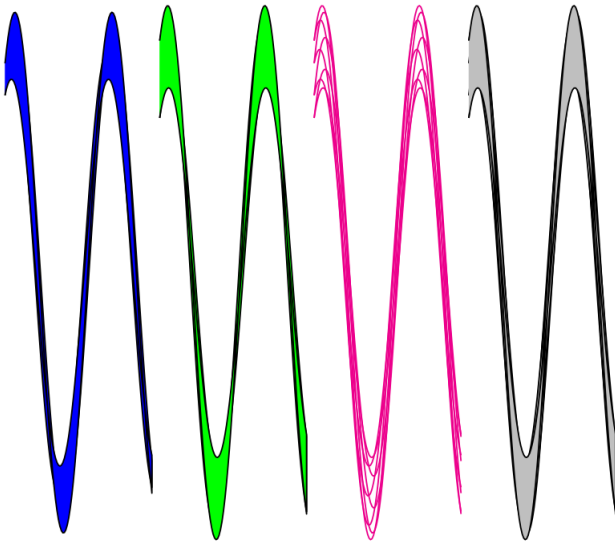


Fig. 11. The dual of an octagon



Fig. 13. The dual of an hexagon

Octagon preimage is the intersection of a set of octagon dual.

*Definition 7:* (Octagon preimage) Let  $S = \{h_1, h_2, \dots, h_n, h_{n-1}\}$  be a set of  $n$  octagons. The preimage of  $S$  is  $\bigcap_{i=1}^n dual(h_i)$ .

The preimage will help to establish later a straight line recognition. Finally, an algorithm will be realized in using this preimage definition.

*Proposition:* [hexagon dual] The dual of a hexagon is the union of the dual of its three diagonal segments. ■

Figure 12 shows the proposed hexagon. Its dual is illustrated in Figure 13.

## VI. CONCLUSION AND PERSPECTIVES

We presented new analytical straight line definitions based on hexagons and octagons. We also proposed the Hough Transform method that leads to the recognition of these straight lines.

The proposed methods are alternatives to triangular, rectangular Hough Transform for straight line recognition.

In perspectives, remaining works will perform applications in 2D and the extension of these definitions in a  $n$  dimensional space. Performance comparisons of each method through a test on real images will give more details on different applications and will lead to create a toolkit of the Hough transform method.

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