

# An Overview of Various Techniques to Improve Receive Diversity in MIMO OFDM

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**Abstract**—MIMO-OFDM system has been currently recognized as one of the most competitive technology for 4G mobile wireless systems. Techniques such as use of multiple antennas can be utilized in order to obtain a multiplexing gain, a diversity gain, or an antenna gain, thus enhancing the throughput (in terms of capacity) or bit rate, the error performance, or the signal-to-noise ratio of wireless systems, respectively. In this paper various receive combining techniques such as MRC, EGC and SC for enhancement of diversity have been discussed. Receive combining techniques with LDPC codes enhance the performance of overall system by providing coding gain. Overview of Alamouti's STBC along with maximum likelihood (ML) decoding algorithms also discussed.

**Keywords**—Multiple-input multiple-output (MIMO), orthogonal frequency division multiplexing (OFDM), low density parity check (LDPC) codes, space time block code (STBC), maximal ratio combining (MRC), selection combining (SC), equal gain combining (EGC).

## I. INTRODUCTION

As the demand for high-data rate multimedia grows, several approaches such as increasing modulation order or employing multiple antennas at both transmitter and receiver have been studied to enhance the spectral efficiency [1]. In today's communication systems Orthogonal Frequency Division Multiplexing (OFDM) is a widespread modulation technique. It has several benefits such as high spectral efficiency, robustness against inter-symbol interference, ease of implementation using the fast Fourier transform (FFT) and simple equalization techniques. Recently, there have been a lot of interests in combining the OFDM systems with the multiple-input multiple- output (MIMO) technique. These systems are known as MIMO OFDM systems [2]. MIMO wireless system combined with OFDM provides easy transmission of symbol in time, frequency and space.

Spatially multiplexed MIMO is known to boost the throughput (in terms of capacity of mimo system) , on the

other hand, when much higher throughputs are aimed at, the multipath character of the environment causes the MIMO channel to be frequency-selective. OFDM can transform such a frequency-selective MIMO channel into a set of parallel frequency-flat MIMO channels and also increase the frequency efficiency.

MIMO system also provides spatial diversity by use of Alamouti STB code [3] for improvement in Bit Error Rate Performances by mitigates the effect of fading. Many other codes have also been proposed [4]–[5] which have been able to achieve some or all of the available diversity in the channel at various transmission rates. MIMO system provides both transmit diversity and received combining schemes along with various decoding algorithms like ML (for decoding purpose at receiver side with less complexity) and ZF equalizer (to reduce effect of intersymbol interferences).

## II. OFDM SYSTEM

OFDM is multicarrier modulation technique where data symbols modulate a parallel collection of regularly spaced sub-carriers. The sub-carriers have the minimum frequency separation required to maintain orthogonality of their corresponding time-domain waveforms, yet the signal spectra corresponding to different sub-carriers overlap in frequency. The spectra overlap result in a waveform that uses the available bandwidth with very bandwidth efficiency.

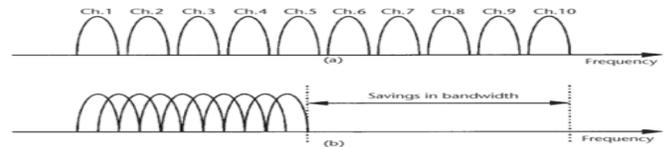


Fig. 1 Concept of OFDM signal (a) conventional multicarrier technique and (b) orthogonal multicarrier modulation technique

By orthogonality of the carriers, means that the carrier frequencies satisfy the following requirement-

$$f_k = f_0 + \frac{k}{T_s}, k=1, 2, \dots, N-1$$

$f_k$  = frequency of  $k^{th}$  carrier,  $f_0$  = fundamental frequency,  $K$  = an integer,  $T_s$  = OFDM symbol duration

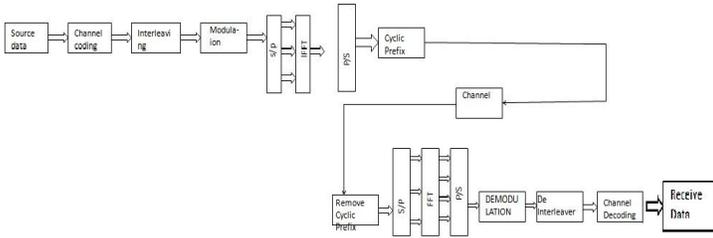


Fig.2 OFDM Building Blocks

Fig shows the block diagram of OFDM system model with SISO configuration. Denote  $X_l(l=0,1,\dots,N-1)$  as the modulated symbols on the  $l^{th}$  transmitting subcarrier of OFDM symbol at transmitter, which are assumed independent, zero-mean random variables, with average power. The complex baseband OFDM signal at output of the IFFT can be written as:

$$X_n = \frac{1}{\sqrt{N}} \sum_{l=0}^{N-1} X_l e^{\frac{j2\pi nl}{N}} \dots (1)$$

Where  $N$  is total numbers of subcarriers and OFDM symbol duration is  $T$ .

At the receiver, the received OFDM signal is mixed with local oscillator signal, with the frequency offset deviated from  $\Delta f$  the carrier frequency of the received signal owing to frequency estimation error or Doppler velocity, the received signal is given by:

$$\hat{x} = (x_n \otimes h_n) e^{\frac{j2\pi n \Delta f T}{N}} + Z_n \dots (2)$$

Where  $h_n$  and  $Z_n$  represent the channel impulse response, the corresponding frequency offset of received signal at the sampling instants:  $\Delta f T$  is the frequency offset to subcarrier frequency spacing ratio, and the AWGN respectively, while denotes the circular convolution. Assuming that a cyclic prefix is employed; the receiver have perfect time synchronization. Note that a discrete Fourier transform (DFT) of the convolution of two signals in time domain is equivalent to the multiplication of the corresponding signals in the frequency domain.

Then the output of the FFT in frequency domain signal on the  $K^{th}$  receiving subcarrier becomes:

$$X_k = \sum_{l=0}^{N-1} X_l H_l Y_{l-k} + Z_k, k=0, 1, \dots, N-1 \dots (3)$$

$$= X_k H_k U_0 + \sum_{l=0, l \neq k}^{N-1} X_l H_l Y_{l-k} + Z_k$$

The first term of (3) is a desired transmitted data symbol  $X_k$ . The second term represents the ICI from the undesired data symbols on other subcarriers in OFDM symbol.  $H_k$  is the

channel frequency response and  $Z_k$  denotes the frequency domain of  $Z_n$ . The term  $Y_{l-k}$  is the coefficient of FFT (IFFT), is given by:

$$Y_{l-k} = \frac{1}{N} \sum_{n=0}^{N-1} e^{2\pi n(l-k+\Delta f T)/N}$$

When the channel is flat,  $Y_{l-k}$  can be considered as a complex weighting function of the transmitted data symbols in frequency domain [6].

### III. CHANNELS

For wireless communication, signal is travel though space or air as a transmission media. The radio propagation is not as smooth as in wire transmission since the received signal is not only coming directly from the transmitter, but the combination of reflected, diffracted, and scattered copies of the transmitted signal.

#### A. AWGN Channel

Additive White Gaussian Noise (AWGN) channel is universal channel model for analyzing modulation scheme. In this channel does nothing but just add white Gaussian noise to signal passing through it. The only distortion is introduced by AWGN. The received signal is simplified to-

$$r(t) = x(t) + n(t)$$

Where  $n(t)$  is additive white Gaussian noise [7]. The whiteness of  $n(t)$  implies that it is stationary random process with flat power spectral density(PSD) for all frequencies. So it is conventional to assume its PSD as follows

$$N(f) = N_0/2, -\infty \leq f \leq \infty$$

This implies that a white process has infinite power.

#### B. RAYLEIGH Fading Channel

Multipath fading results in fluctuations of the signal amplitude because of the addition of signals arriving with different phases. This phase difference is caused due to the fact that signals have traveled different distances by traveling along different paths. Because the phases of the arriving paths are changing rapidly, the received signal amplitude undergoes rapid fluctuation that is often modeled as a random variable with a particular distribution. The most commonly used distribution for multipath fast fading is the RAYLEIGH

distribution, whose probability density function (pdf) is given by

$$f_{ray}(r) = r / \sigma^2 e^{-\frac{r^2}{2\sigma^2}}, r \geq 0$$

The random variable corresponding to the signal amplitude is  $r$ . Here  $\sigma^2$  is the variance [8]. So Rayleigh flat fading channel is commonly used to describe multipath fading channels when there are no Line-Of-Sight (LOS) exits between transmitter and receiver end.

### C. Rician Fading Channel

When strong LOS signal components exist, the distribution is found to be Rician, the pdf of such function is given by:

$$f_{ric}(r) = r / \sigma^2 e^{-\frac{(r^2 + A^2)}{2\sigma^2}} I_0\left(\frac{Ar}{\sigma^2}\right), r \geq 0, A \geq 0$$

Where  $\sigma^2$  is the variance and  $A$  is the amplitude of the signal of the dominant path and  $I_0$  is the zero-order modified Bessel function of the first kind. Normally the dominant path significantly reduces the depth of fading, and in terms of BER Rician fading provides superior performance to Rayleigh fading. The probability of having line-of-sight (LOS) component depends on the size of the cell. The smaller the cell the higher the probability of having LOS path. If there is no dominant path then the Rician pdf reduces to Rayleigh pdf. When  $A$  is large compared with  $\sigma$ , the distribution is approximately Gaussian. Thus, since Rician distribution covers also Gaussian and Rayleigh distribution, mathematically the Rician fading channel can be considered to be general case.

## IV. ANALYTICAL MODEL FOR MIMO

Mimo is a multiple input multiple output system which uses multiple antennas on both transmitter and receiver ends. Mimo technology provides numerous advantages such as spatial multiplexing with multiplexing gain, spatial diversity with diversity gain (both transmit and receive diversity), beam forming, array gains. For the H Channel matrix we have-

$$y = Hx + n$$

$$H = \begin{pmatrix} h_{1,1} & h_{1,2} & \dots & h_{1,N_t} \\ h_{2,1} & h_{2,2} & \dots & h_{2,N_t} \\ \vdots & \vdots & \ddots & \vdots \\ h_{N_r,1} & h_{N_r,2} & \dots & h_{N_r,N_t} \end{pmatrix}$$

Fig.3 Channel State Information Matrix

The proper operation of MIMO systems requires careful design, with the encoded signals received from each transmitting antenna and the multiple communication channels achieving specified orthogonality conditions. [8]

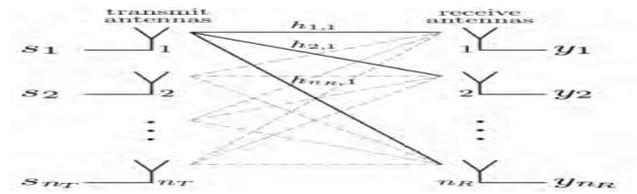


Fig.4 Mimo System with N<sub>t</sub> Transmit And N<sub>r</sub> Receive Antennas

When the signal is transmitted, different signal copies undergo different attenuation, distortion, delays and phase shifts and also the wireless communication channel suffers from much impairment such as

1. Thermal noise often treated as Additive White Gaussian Noise (AWGN).
2. The path loss in power as radio signal propagates.
3. Shadowing mainly occur due to presence of field obstacle in radio path.
4. The fading present as the result of effect of multiple propagation path.

If these impairments are not overcome, the performance of the system is slowly degraded and hence the efficiency. The major problem in all these which makes reliable wireless transmission difficult is multipath fading. There are two techniques proposed to combat the effect of fading.

1. Transmitter power control
2. Diversity technique

In power control method, there are two major drawbacks-First, the transmitter requires a dynamic range. Second, channel information has to be fed back from the receiver to the transmitter. So, Diversity technique is being employed. Diversity refers to transmitting and/or receiving the same information via different independent ways. In such a system, multiple copies of the same information signal are being transmitted to the receiver over two or more real or virtual communication channels. Thus the basic idea of diversity is repetition or redundancy of information. The diversity decisions are made by the receiver and are unknown to the transmitter. Thus, it provides wireless link improvement at a relatively low cost, power savings and increased system capacity. If diversity is not employed; the resulting efficiency would be very low.

## V. DIVERSITY SCHEMES

It can be categorized as time diversity, frequency diversity and spatial diversity. For wireless Mimo system spatial diversity is employed which is the combination of both transmit and receive diversity.

**Space Diversity:** In Space diversity, there are multiple receiving antennas placed at different spatial locations, resulting in different (possibly independent) received signals, M antennas are used to receive M copies of the transmitted signal. The antennas should be spaced far enough apart means there will be  $\frac{\lambda}{2}$  antennas separation, where  $\lambda$  is wavelength of transmitted signal.

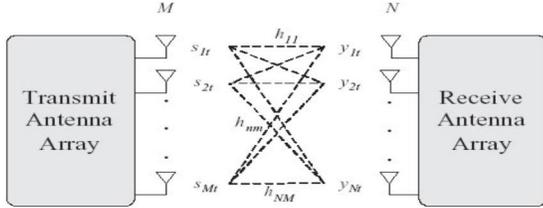


Fig.5 Space Diversity Representation

**A. Selection Combining**

From the number of antennas, the branch that receives the signal with the largest signal-to-noise ratio is selected and connected to the demodulator. Larger the number of available branches, the higher the probability of having a larger signal-to-noise ratio (SNR) at the output. It is seen that among available receive combining techniques at receiver side, SC has worst BER performance [9].

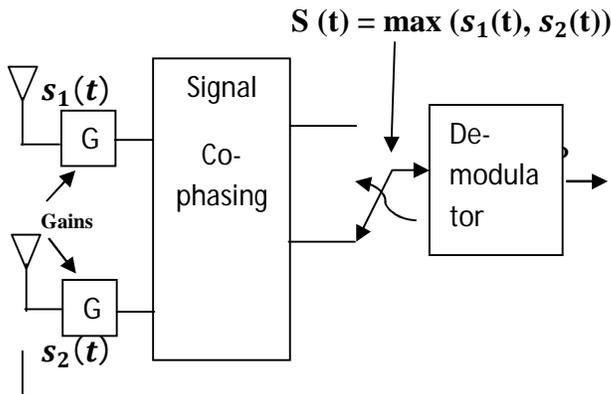


Fig.6 Selection combining technique

The combined output is given by:

$$Y(t) = Ae^{j\theta_{is}}(t) + z(t)$$

$$A = \max(A_0, A_1, A_2, \dots, A_{M-1})$$

The received SNR is

$$\rho = \frac{A^2 E_b}{N_0} = \max(\rho_0, \rho_1, \dots, \rho_{M-1})$$

The CDF of  $\rho$  is

$$P_\rho(\gamma) = \prod_{i=0}^{M-1} P_{\rho_i}(\gamma)$$

The Bit Error Rate Probability

$$P_e = \frac{1}{2} \sum_{k=0}^N (-1)^k \binom{N}{k} \left(1 + \frac{k}{(E_b/N_0)}\right)^{-1/2}$$

**B. Maximal Ration Combining**

Both branches are weighted by their respective instantaneous voltage-to-noise ratios. The branches are then co-phased prior to summing make insure that all branches are added in phase for achieving maximum diversity gain. Then summed signals are used as the received signal and connected to the demodulator. The advantage is that more BER improvements can be achieved with this configuration even when both branches are completely correlated. The disadvantage of maximal ratio is that it is little complicated and requires accurate estimates of the instantaneous signal level and average noise power to established optimum performance with this combining scheme. Maximal ratio combining will always perform better than either selection diversity or equal gain combining because it is an optimum combiner. The information on all channels is used with this technique to get a more reliable received signal. [10]

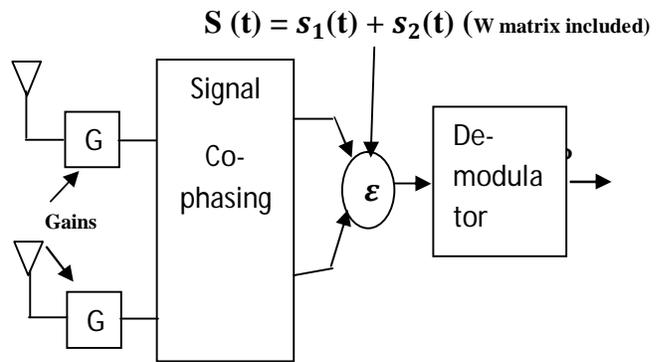


Fig.7 Maximal ration combining technique

Resultant combined output is given as-

$$Y(t) = \sum_{i=0}^{M-1} w_i r_i(t)$$

The received SNR is

$$\rho = \frac{\sum_{i=0}^{M-1} A_i^2 E_b}{N_0} = \sum_{i=0}^{M-1} \rho_i$$

The bit error rate probability

$$P_e = \int_0^\infty \text{erfc}(\sqrt{\gamma}) P(\gamma) d\gamma$$

$$= \int_0^\infty \frac{1}{2} \text{erfc}(\rho) \frac{1}{(N-1)!(E_b/N_0)^N} \gamma^{N-1} e^{-\gamma/E_b/N_0} d\gamma$$

### C. Equal Gain Combining

It is same as that of maximal ratio combining (MRC) except that equal gains are taken in this method. So, In this method the gains of the all branches are set to a single value and are not varied. Then both branch signals are multiplied by the same branch gain (G) and the resulting signals are co-phased and summed. The resultant output signal is connected to the demodulator system [11].

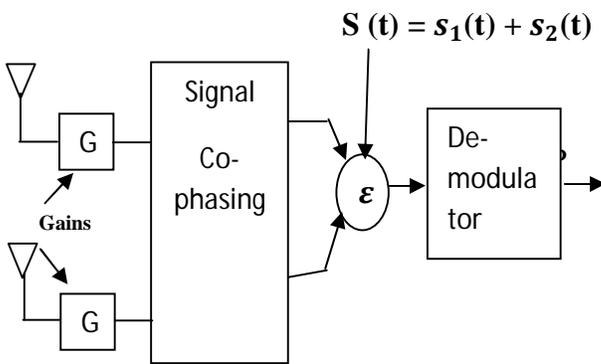


Fig.8 Equal gain combining

The combined output is given by

$$Y(t) = \sum_{i=1}^M e^{-j\theta} r_i(t) = (\sum_{i=1}^M A_i) s(t) + \sum_{i=0}^M e^{-j\theta_i} + z_i(t)$$

The received SNR is

$$\rho = \frac{(\sum_{i=0}^{M-1} A_i)^2 E_b}{MN_0}$$

## VI. LDPC CODE AND LITERATURE

Low density parity check (LDPC) codes [12], are equipped with very fast encoding and decoding algorithm which makes them very useful for practical applications. The main attraction of LDPC codes is that they approach the Shannon limit of channel. There have been simulations that perform within 0.04dB of the Shannon limit. The LDPC codes are characterized by a sparse parity check matrix in which the number of 1's is very less as compared to the number of 0's. LDPC codes have been proved to achieve capacity on binary erasure channels (BEC). Unlike turbo codes, they can be

optimized for good bit-error rate performance over a given channel and for a given code-rate [13].

Satoshi Gounai et al. [14] find result that the SNR thresholds of a regular LDPC codes and the irregular LDPC codes and optimum degree distributions of the irregular LDPC codes for SIMO systems with several diversity orders.

Kwok Hung Li et al.[15] derived that the Low-density parity check (LDPC) codes when combined with Selection ratio combining and Maximal ratio Combining gives better bit error rate (BER) performance than multipath fading channels. We used the GA approach with the LDPC over the i.i.d Nakagami-fading channels to find the BER.

## VII. SPACE TIME BLOCK CODES

Multiple-Input Multiple Output (MIMO) links coupled with space-time codes may combat fading and hence may significantly enhance the channel capacity. Different space-time block coding (STBC) schemes including Alamouti's STBC for 2 transmit antennas as well as orthogonal STBC for 3 and 4 transmit antennas are explored. Space-time coding introduces redundancy in space, though the addition of multiple antennas, and redundancy in time, through channel coding. Two prevailing space-time coding techniques are Space Time Block Codes (STBC) and Space Time Trellis Codes (STTC). STBC provide diversity gain, with very low decoding complexity. This can be achieved by transmitting several replicas of the same information through each antenna. By doing this, the probability of losing the information decreases exponentially. The different replicas sent for exploiting diversity are generated by a space-time encoder which encodes a single stream through space using all the transmit antennas and through time by sending each symbol at different times. This form of coding is called Space-Time Coding (STC). The most dominant form of STCs are space-time block codes (STBC) because of their decoding simplicity. Now we will discuss about Alamouti STBC and OSTBC technique.

### A. Alamouti's Scheme

Alamouti's STBC uses two transmit antennas regardless of the number of receive antennas. The Alamouti scheme encoding operation is given by

$$g_2 = \begin{pmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{pmatrix}$$

At a time t, the symbol  $s_1$  and symbol  $s_2$  are transmitted from antenna 1 and antenna 2 respectively. Assuming that each symbol has duration T, then at time t + T, the symbols  $-s_2^*$

and  $s_1^*$ , where  $(.)^*$  denotes the complex conjugate, are transmitted from antenna 1 and antenna 2 respectively. Alamouti STBC reduces the effect of fading at mobile stations while only requiring extra antenna elements at the base station, where it is more economical than having multiple antennas at the receivers. However, if having more antennas at the receivers is not a problem, this scheme can be used with 2 transmit antennas and  $N_r$  receive antennas while accomplishing a  $2N_r$  full diversity.

### B. Orthogonal Space-Time Block Codes

Orthogonality means that the STBC is designed such that the vectors representing any pair of columns taken from the coding matrix are orthogonal. They allow low complexity maximum likelihood decoding and guarantee full diversity. An orthogonal STBC is characterized by a code matrix  $G(p \times n)$  where  $p$  denotes time delay or block length and  $n$  represents the number of transmit antennas. The entries of  $G$  are linear combinations of  $k$  data symbols or their conjugate that belong to an arbitrary signal constellation [16].

Orthogonal Space-Time Block Codes for  $N_t = 3$  with Rate  $1/2$ :

$$G_3 = \begin{pmatrix} s_1 & s_2 & s_3 \\ -s_2 & s_1 & -s_4 \\ -s_3 & s_4 & s_1 \\ -s_4 & -s_3 & s_2 \\ s_1 & s_2 & s_3 \\ -s_2 & s_1 & -s_4 \\ -s_3 & s_4 & s_1 \\ -s_4 & -s_3 & s_2 \end{pmatrix}$$

This code transmits 4 symbols every 8 time intervals, and therefore has rate  $1/2$ .

### VIII. ML EQUALIZER

Maximum-Likelihood (ML) equalizer is a non linear equalizer in which a search is performed over all possible symbols and the most likely one is chosen. It calculates the Euclidean distance between the received signal vector and the product of all possible transmitted signal vectors with the given channel  $H$ , and finds the one with the minimum distance. No noise enhancement takes place and numerical issues are virtually not present, as no matrix inversions or divisions are necessary [17]. The ML equalizer tries to search the most likelihood one which minimizes,

$$J = |y - Hx|^2$$

$$J = \left| \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \begin{bmatrix} h_{1,1} & h_{2,1} \\ h_{1,2} & h_{2,2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right|^2$$

In a BPSK modulation the possible values are 11, 10, 01, and 00. In order to find the Maximum Likelihood one, we have to find the minimum from the all four combinations. In case of QPSK modulation the possible values are  $4^2=16$ . Such method is useful when channel is unknown at transmitter. The computational complexity of ML detector is high. It is optimum in the sense of minimizing bit error rate. If  $[x_1, x_2, \dots, x_N]$  is transmitted symbol stream,  $H$  is channel matrix and  $r$  is the received vector, then the estimated symbol stream is given by-

$$\tilde{x} = \text{arg}_{x_k \in \{x_1, x_2, \dots, x_N\}} \min \|r - Hx_k\|^2$$

Where,  $\tilde{x}$  is the estimated symbol vector.

### IX. CONCLUSION

OFDM is a very efficient method for wireless communications due to its spectrum efficiency and channel robustness. OFDM employ with multiple antenna system gives lot of advantages such as diversity and multiplexing gains, also provide reliable and efficient communication. One of major concern in during such communication is to achieve maximum diversity at receiver side such that all received must be uncorrelated with each other it means loss of signal strength of particular signal at receiver does not affect other replicas of same information bearing signal. In this paper, various crucial aspects is discussed, as well as a mathematical analysis is provided, along with decoding algorithm. Various diversity combining techniques are discussed along with LDPC codes and theoretically observe the performance in terms of reduction in BER for each one.

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